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$\mathcal{I}_{m\omega}$-closed Sets

Research Article

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Abstract: In this paper we introduce the notion of $\mathcal{I}_{m\omega}$-closed sets. In Sections 3 and 4, we obtain some basic properties and characterizations of $\mathcal{I}_{m\omega}$-closed sets. In the last section, we define several new subsets in ideal topological spaces which lie between $\star$-closed sets and $\mathcal{I}_{\omega}$-closed sets.

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1. Introduction

Sheik John [11] (= Veera Kumar [12]) introduced the notion of ω-closed sets (= $\hat{g}$-closed sets). Recently many variations of ω-closed sets were introduced and investigated. They are applied to introduce several low separation axioms. Since then, the further generalizations of ω-closed sets are being introduced and investigated. Ravi et al [8] introduced the notion of $\mathcal{I}_{\hat{g}}$-closed sets ($\mathcal{I}_{\omega}$-closed sets) and further properties of $\mathcal{I}_{\hat{g}}$-closed sets are investigated. By combining a topological space (X, τ) and an ideal $\mathcal{I}$ on (X, τ), Dontchev et. al [1] introduced the notion of $\mathcal{I}_g$-closed sets and investigated the properties of $\mathcal{I}_g$-closed sets. By combining an m-space (X, m_x) and an ideal $\mathcal{I}$ on (X, m_x), quite recently Ozbakir and Yildirim [6] have introduced the notion of an ideal minimal spaces. Especially, the notion of m-$\mathcal{I}_g$-closed sets is introduced and investigated.

In this paper we introduce the notion of $\mathcal{I}_{m\omega}$-closed sets. In Sections 3 and 4, we obtain some basic properties and characterizations of $\mathcal{I}_{m\omega}$-closed sets. In the last section, we define several new subsets in ideal topological spaces which lie between $\star$-closed sets and $\mathcal{I}_{\omega}$-closed sets.

2. Preliminaries

Definition 2.1 ([7]). *A subfamily $m_X \subseteq \wp(X)$ is said to be a minimal structure (briefly, m-structure) on X if $\emptyset$, $X \in m_X$. The pair (X, m_X) is called a minimal space (briefly m-space). Each member of m_X is said to be m-open and the complement of an m-open set is said to be m-closed.*

Notice that (X, m_X, $\mathcal{I}$) is called an ideal m-space.

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Remark 2.2. *Let (X, τ) be a topological space. Then $m_X = \tau$, $SO(X)$ and $SPO(X)$ are minimal structures on X.*

Definition 2.3. *Let (X, m_X) be an m-space. For a subset A of X, the m-closure of A and the m-interior of A are defined in [5] as follows:*

(1). $m\text{-}cl(A) = \cap\{F : A \subseteq F,\ F^c \in m_X\}$,

(2). $m\text{-}int(A) = \cup\{U : U \subseteq A,\ U \in m_X\}$.

Lemma 2.4 ([3]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B subsets of X. Then the following properties hold:*

(1). $A \subseteq B \Rightarrow A^\star \subseteq B^\star$,

(2). $A^\star = cl(A^\star) \subseteq cl(A)$,

(3). $(A^\star)^\star \subseteq A^\star$,

(4). $(A \cup B)^\star = A^\star \cup B^\star$,

(5). $(A \cap B)^\star \subseteq A^\star \cap B^\star$.

Lemma 2.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then every ω-closed set is an $\mathcal{I}_\omega$-closed but not conversely ([8], Theorem 2.13).*

Lemma 2.6 ([2]). *Let $\{A_\lambda : \lambda \in \wedge\}$ be a locally finite family of sets in $(X, \tau, \mathcal{I})$. Then $\cup_{\lambda\in\wedge} A_\lambda^\star = (\cup_{\lambda\in\wedge} A_\lambda)^\star$.*

Definition 2.7 ([7]). *A minimal structure m_x on a nonempty set X is said to have property $\mathcal{B}$ if the union of any family of subsets belonging to m_x belongs to m_x.*

3. $\mathcal{I}_{m\omega}$-closed Sets

In this section, let $(X, \tau, \mathcal{I})$ be an ideal topological space and m_X an m-structure on X. We obtain several basic properties of $\mathcal{I}_{m\omega}$-closed sets.

Definition 3.1. *Let (X, τ) be a topological space and m_X an m-structure on X. A subset A of X is said to be m-semiopen [4] if $A \subset m\text{-}cl(m\text{-}int(A))$. The family of all m-semiopen sets in X is denoted by $m\text{-}SO(X)$. The complement of m-semiopen set is said to be m-semiclosed.*

Definition 3.2 ([9]). *Let (X, τ) be a topological space and m_X an m-structure on X. A subset A of X is said to be*

(1). m-ω-closed if $cl(A) \subset U$ whenever $A \subset U$ and U is m-semiopen,

(2). m-ω-open if its complement is m-ω-closed.

Definition 3.3 ([4]). *Let (X, m_X) be an m-space. For a subset A of X, the m-semi-closure of A and the m-semi-interior of A, denoted by $m\text{-}scl(A)$ and $m\text{-}sint(A)$, respectively are defined as follows:*

(1). $m\text{-}scl(A) = \cap\{F : A \subset F,\ F$ is m-semiclosed in $X\}$,

(2). $m\text{-}sint(A) = \cup\{U : U \subset A,\ U$ is m-semiopen in $X\}$.

Remark 3.4. *Let (X, τ) be a topological space and A a subset of X. If $m\text{-}SO(X) = SO(X)$ (resp. τ) and A is m-ω-closed, then A is ω-closed (g-closed).*

Definition 3.5. *A subset A of an ideal m-space $(X, m_X, \mathcal{I})$ is said to be*

(1). $\mathcal{I}_{m\omega}$-closed if $A^\star \subset U$ whenever $A \subset U$ and U is m-semiopen.

(2). $\mathcal{I}_{m\omega}$-open if $X - A$ is $\mathcal{I}_{m\omega}$-closed.

Remark 3.6. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A a subset of X. If m-SO(X) = SO(X) (resp. τ) and A is $\mathcal{I}_{m\omega}$-closed, then A is $\mathcal{I}_\omega$-closed (resp. $\mathcal{I}_g$-closed).*

Proposition 3.7. *Every m-ω-closed set is $\mathcal{I}_{m\omega}$-closed but not conversely.*

Proof. Let A be an m-ω-closed, then cl(A) $\subset$ U whenever A $\subset$ U and U is m-semiopen. By Lemma 2.4, $A^\star \subset$ cl(A). Hence A is $\mathcal{I}_{m\omega}$-closed. □

Example 3.8. *Let $X = \{a, b, c\}$, $m_X = \{\phi, X, \{c\}\}$, $\tau = \{\phi, X, \{c\}, \{b, c\}\}$ and $\mathcal{I} = \{\phi, \{b\}\}$. Then m-semiopen sets are ϕ, X, $\{c\}$, $\{a, c\}$ and $\{b, c\}$; semi-open sets are ϕ, X, $\{c\}$, $\{a, c\}$ and $\{b, c\}$; m-ω-closed sets are ϕ, X, $\{a\}$ and $\{a, b\}$; and $\mathcal{I}_{m\omega}$-closed sets are ϕ, X, $\{a\}$, $\{b\}$ and $\{a, b\}$. It is clear that $\{b\}$ is $\mathcal{I}_{m\omega}$-closed set but it is not m-ω-closed.*

Proposition 3.9. *Let SO(X) $\subset$ m-SO(X). Then every $\mathcal{I}_{m\omega}$-closed set is $\mathcal{I}_\omega$-closed but not conversely.*

Proof. Suppose that A is an $\mathcal{I}_{m\omega}$-closed set. Let A $\subset$ U and U $\in$ SO(X). Since SO(X) $\subset$ m-SO(X), $A^\star \subset$ U and hence A is $\mathcal{I}_\omega$-closed. □

Example 3.10. *Let $X = \{a, b, c\}$, $m_X = \{\phi, X, \{c\}\}$, $\tau = \{\phi, X, \{b\}, \{a, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Then ω-closed sets are the power set of X; m-ω-closed sets are ϕ, X, $\{a\}$, $\{b\}$, $\{a, b\}$ and $\{a, c\}$; $\mathcal{I}_{m\omega}$-closed sets are ϕ, X, $\{a\}$, $\{b\}$, $\{a, b\}$ and $\{a, c\}$; and $\mathcal{I}_\omega$-closed sets are the power set of X. It is clear that $\{c\}$ is $\mathcal{I}_\omega$-closed set but it is not $\mathcal{I}_{m\omega}$-closed.*

Remark 3.11. *Let SO(X) $\subset$ m-SO(X). Then we have the following implications for the subsets stated above.*

$$\begin{array}{ccccc} \text{closed} & \longrightarrow & m\text{-}\omega\text{-closed} & \longrightarrow & \omega\text{-closed} \\ \downarrow & & \downarrow & & \downarrow \\ \star\text{-closed} & \longrightarrow & \mathcal{I}_{m\omega}\text{-closed} & \longrightarrow & \mathcal{I}_\omega\text{-closed} \end{array}$$

The implications in the first line are known in [10]. The three vertical implications follow from Lemma 2.4(2), Proposition 3.7 and Lemma 2.5. It is obvious that every $\star$-closed is $\mathcal{I}_{m\omega}$-closed and by Proposition 3.9, every $\mathcal{I}_{m\omega}$-closed set is $\mathcal{I}_\omega$-closed.

Proposition 3.12. *If $\{A_\lambda : \lambda \in \wedge\}$ is a locally finite family of sets in $(X, \tau, \mathcal{I})$ and A_λ is $\mathcal{I}_{m\omega}$-closed for each $\lambda \in \wedge$, then $(\cup_{\lambda\in\wedge} A_\lambda)$ is $\mathcal{I}_{m\omega}$-closed.*

Proof. Let $(\cup_{\lambda\in\wedge} A_\lambda) \subset$ U where U is m-semiopen. Then $A_\lambda \subset$ U for each $\lambda \in \wedge$. Since A_λ is $\mathcal{I}_{m\omega}$-closed for each $\lambda \in \wedge$, we have $A_\lambda^\star \subset$ U and hence $\cup_{\lambda\in\wedge} A_\lambda^\star \subset$ U. By Lemma 2.6, $(\cup_{\lambda\in\wedge} A_\lambda)^\star \subset$ U. Hence $(\cup_{\lambda\in\wedge} A_\lambda)$ is $\mathcal{I}_{m\omega}$-closed. □

Corollary 3.13. *If A and B are $\mathcal{I}_{m\omega}$-closed sets in $(X, \tau, \mathcal{I})$, then A$\cup$B is $\mathcal{I}_{m\omega}$-closed.*

Proof. Let A$\cup$B $\subset$ U where U is m-semiopen. Then A $\subset$ U and B $\subset$ U. Since A and B are $\mathcal{I}_{m\omega}$-closed, then $A^\star \subset$ U and $B^\star \subset$ U and so $A^\star \cup B^\star \subset$ U. By Lemma 2.4, $A^\star \cup B^\star = (A\cup B)^\star$. Hence A$\cup$B is $\mathcal{I}_{m\omega}$-closed. □

Example 3.14. *Let $X = \{a, b, c, d\}$, $m_X = \{\phi, X, \{a, b\}, \{a, c\}, \{b, d\}\}$ and $\tau = \{\phi, X, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}\}$. Then m-$\omega$-open sets ϕ, X, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{a, d\}$ and $\{a, b, d\}$. It is shown that this collection does not have property $\mathcal{B}$.*

Remark 3.15. *Let (X, τ) be a topological space. Then the families SO(X) and τ^α are all m-structure with property $\mathcal{B}$.*

Proposition 3.16. *Let SO(X) ⊂ m-SO(X) and m-SO(X) have property $\mathcal{B}$. If A is $\mathcal{I}_{m\omega}$-closed in (X, τ, $\mathcal{I}$) and B is closed in (X, τ), then A∩B is $\mathcal{I}_{m\omega}$-closed.*

Proof. Let A∩B ⊂ U where U is m-semiopen. Then we have A ⊂ U∪(X−B). Since τ ⊂ SO(X) ⊂ m-SO(X) and so U∪(X−B) is m-semiopen. Since A is $\mathcal{I}_{m\omega}$-closed, then $A^\star$⊂U∪(X−B) and hence $A^\star$∩B ⊂ U∩B ⊂ U. By Lemma 2.4. $(A\cap B)^\star \subset A^\star\cap B^\star$. Since $\tau \subset \tau^\star$, B is $\star$-closed and $B^\star \subset B$. Therefore, we obtain $(A\cap B)^\star \subset A^\star\cap B^\star \subset A^\star\cap B \subset U$. This shows that A∩B is $\mathcal{I}_{m\omega}$-closed. □

Proposition 3.17. *If A is $\mathcal{I}_{m\omega}$-closed and A ⊂ B ⊂ $cl^\star(A)$, then B is $\mathcal{I}_{m\omega}$-closed.*

Proof. Let B ⊂ U where U is m-semiopen. Then A ⊂ U and A is $\mathcal{I}_{m\omega}$-closed. Therefore $A^\star$ ⊂ U and $B^\star \subset cl^\star(B) \subset cl^\star(A) = A\cup A^\star \subset U$. Hence B is $\mathcal{I}_{m\omega}$-closed. □

Proposition 3.18. *A subset A of X is $\mathcal{I}_{m\omega}$-open if and only if F ⊂ $int^\star(A)$ whenever F ⊂ A and F is m-semiclosed.*

Proof. Suppose that A is $\mathcal{I}_{m\omega}$-open. Let F ⊂ A and F be m-semiclosed. Then X − A ⊂ X − F and X − F is m-semiopen. Since X − A is $\mathcal{I}_{m\omega}$-closed, then $(X - A)^\star \subset X - F$ and $X - int^\star(A) = cl^\star(X - A) = (X - A)\cup(X - A)^\star \subset X - F$ and hence $F \subset int^\star(A)$.

Conversely, let X − A ⊂ G where G is m-semiopen. Then X − G ⊂ A and X − G is m-semiclosed. By hypothesis, we have $X - G \subset int^\star(A)$ and hence $(X - A)^\star \subset cl^\star(X - A) = X - int^\star(A) \subset G$. Therefore, X − A is $\mathcal{I}_{m\omega}$-closed and A is $\mathcal{I}_{m\omega}$-open. □

Corollary 3.19. *Let SO(X) ⊂ m-SO(X) and m-SO(X) have property $\mathcal{B}$. Then the following properties hold.*

(1). Every $\star$-open set is $\mathcal{I}_{m\omega}$-open and every $\mathcal{I}_{m\omega}$-open set is $\mathcal{I}_{\omega}$-open,

(2). If A and B are $\mathcal{I}_{m\omega}$-open, then A∩B is $\mathcal{I}_{m\omega}$-open,

(3). If A is $\mathcal{I}_{m\omega}$-open and B is open in (X, τ), then A∪B is $\mathcal{I}_{m\omega}$-open,

(4). If A is $\mathcal{I}_{m\omega}$-open and $int^\star(A)$ ⊂ B ⊂ A, then B is $\mathcal{I}_{m\omega}$-open.

Proof. This follows from Remark 3.11, Propositions 3.16 and 3.17 and Corollary 3.13. □

Lemma 3.20. *Let (X, m_X) be an m-space and A a subset of X. Then $x \in$ m-scl(A) if and only if $U\cap A \neq \phi$ for every $U \in$ m-SO(X) containing x.*

Lemma 3.21. *Let X be a nonempty set, m_X an m-structure on X and m-SO(X) have property $\mathcal{B}$. For a subset A of X, the following properties hold:*

(1). A ∈ m-SO(X) if and only if m-sint(A) = A,

(2). A is m-semiclosed if and only if m-scl(A) = A,

(3). m-sint(A) ∈ m-SO(X) and m-scl(A) is m-semiclosed.

4. Characterizations of $\mathcal{I}_{m\omega}$-closed sets

In this section, let (X, τ, $\mathcal{I}$) be an ideal topological space and m_X an m-structure on X. We obtain several characterizations of $\mathcal{I}_{m\omega}$-closed sets.

Theorem 4.1. *For a subset A of X, the following properties are equivalent:*

(1). A is $\mathcal{I}_{m\omega}$-closed,

(2). $cl^\star(A) \subset U$ whenever $A \subset U$ and U is m-semiopen,

(3). $cl^\star(A) \cap F = \phi$ whenever $A \cap F = \phi$ and F is m-semiclosed.

Proof. (1) $\Rightarrow$ (2) Let A $\subset$ U where U is m-semiopen. Then by (1), $A^\star \subset U$ and $cl^\star(A) = A \cup A^\star \subset U$.
(2) $\Rightarrow$ (3) Let $A \cap F = \phi$ and F be m-semiclosed. Then $A \subset X - F$ and $X - F$ is m-semiopen. By (2), $cl^\star(A) \subset X - F$. Hence $cl^\star(A) \cap F = \phi$.
(3) $\Rightarrow$ (1) Let A $\subset$ U where U is m-semiopen. Then $A \cap (X - U) = \phi$ and $X - U$ is m-semiclosed. By (3), $cl^\star(A) \cap (X - U) = \phi$ and so $A^\star \subset cl^\star(A) \subset U$. Hence A is $\mathcal{I}_{m\omega}$-closed. □

Definition 4.2. *Let (X, τ) be a topological space, m_X an m-structure on X and A a subset of X. The subset $\wedge_{ms}(A)$ is defined as follows: $\wedge_{ms}(A) = \cap\{U : A \subset U, U \in$ m-SO(X)$\}$.*

Theorem 4.3. *A subset A of X is $\mathcal{I}_{m\omega}$-closed if and only if $cl^\star(A) \subset \wedge_{ms}(A)$.*

Proof. Suppose that A is $\mathcal{I}_{m\omega}$-closed. If $x \notin \wedge_{ms}(A)$, then there exists $U \in$ m-SO(X) such that $A \subset U$ and $x \notin U$. Since A is $\mathcal{I}_{m\omega}$-closed, by Theorem 4.1, $cl^\star(A) \subset U$ and hence $x \notin cl^\star(A)$. Hence we obtain $cl^\star(A) \subset \wedge_{ms}(A)$.
Conversely, suppose that $cl^\star(A) \subset \wedge_{ms}(A)$. Let $A \subset U$ and $U \in$ m-SO(X). Then $cl^\star(A) \subset \wedge_{ms}(A) \subset U$. By Theorem 4.1, A is $\mathcal{I}_{m\omega}$-closed. □

Theorem 4.4. *Let SO(X) $\subset$ m-SO(X) and m-SO(X) have property $\mathcal{B}$. For a subset A of X, the following properties are equivalent:*

(1). A is $\mathcal{I}_{m\omega}$-closed,

(2). $A^\star - A$ contains no nonempty m-semiclosed set,

(3). $A^\star - A$ is $\mathcal{I}_{m\omega}$-open,

(4). $A \cup (X - A^\star)$ is $\mathcal{I}_{m\omega}$-closed,

(5). $cl^\star(A) - A$ contains no nonempty m-semiclosed set,

(6). m-scl($\{x\}$)$\cap A \neq \phi$ for each $x \in cl^\star(A)$.

Proof. (1) $\Rightarrow$ (2) Suppose that A is $\mathcal{I}_{m\omega}$-closed. Let $F \subset A^\star - A$ and F be m-semiclosed. Then $F \subset A^\star$ and $F \not\subset A$. We have $A \subset X - F$ and $X - F$ is m-semiopen. Therefore $A^\star \subset X - F$ and so $F \subset X - A^\star$. Hence $F \subset A^\star \cap (X - A^\star) = \phi$.
(2) $\Rightarrow$ (3) Let $F \subset A^\star - A$ and F be m-semiclosed. By (2), we have $F = \phi$ and so $F \subset int^\star(A^\star - A)$. By Proposition 3.18, $A^\star - A$ is $\mathcal{I}_{m\omega}$-open.
(3) $\Rightarrow$ (1) Let A $\subset$ U where U is m-semiopen. Then $X - U \subset X - A \Rightarrow A^\star \cap (X - U) \subset A^\star \cap (X - A) = A^\star - A$. Since $A^\star$ is closed in (X, τ) and hence $A^\star$ is semi-closed in (X, τ). Since every semi-closed set is m-semiclosed and so $A^\star$ is m-semiclosed.

Since m-SO(X) has property $\mathcal{B}$, then $A^\star \cap (X-U)$ is m-semiclosed and by (3), $A^\star - A$ is $\mathcal{I}_{m\omega}$-open. Therefore by Proposition 3.18, $A^\star \cap (X-U) \subset int^\star(A^\star - A) = int^\star(A^\star \cap (X-A)) = int^\star(A^\star) \cap int^\star(X-A) = int^\star(A^\star) \cap (X - cl^\star(A)) \subset A^\star \cap (A \cup A^\star)^c = A^\star \cap (A^c \cap (A^\star)^c) = \phi$ and hence $A^\star \subset U$. Hence A is $\mathcal{I}_{m\omega}$-closed.

(3) ⇔ (4) This follows from the fact that $X - (A^\star - A) = X \cap (A^\star \cap A^c)^c = X \cap ((A^\star)^c \cup A) = (X \cap (A^\star)^c) \cup (X \cap A) = A \cup (X - A^\star)$.

(2) ⇔ (5) This follows from the fact that $cl^\star(A) - A = (A \cup A^\star) - A = (A \cup A^\star) \cap A^c = (A \cap A^c) \cup (A^\star \cap A^c) = A^\star \cap A^c = A^\star - A$.

(1) ⇒ (6) Suppose that A is $\mathcal{I}_{m\omega}$-closed and $\text{m-scl}(\{x\}) \cap A = \phi$ for some $x \in cl^\star(A)$. We know that m-scl({x}) is m-semiclosed. We have $A \subset X - (\text{m-scl}(\{x\}))$ and $X - (\text{m-scl}(\{x\}))$ is m-semiopen. Therefore by Theorem 4.1, $cl^\star(A) \subset X - (\text{m-scl}(\{x\})) \subset X - \{x\}$. This contradicts that $x \in cl^\star(A)$. Hence $\text{m-scl}(\{x\}) \cap A \neq \phi$ for each $x \in cl^\star(A)$.

(6) ⇒ (1) Suppose $\text{m-scl}(\{x\}) \cap A \neq \phi$ for each $x \in cl^\star(A)$. We have to prove that A is $\mathcal{I}_{m\omega}$-closed. Suppose A is not $\mathcal{I}_{m\omega}$-closed. Then by Theorem 4.1, $\phi \neq cl^\star(A) - U$ for some m-semiopen set U containing A. There exists $x \in cl^\star(A) - U$. Since $x \notin U$, by Lemma 3.20, $\text{m-scl}(\{x\}) \cap U = \phi$ and hence $\text{m-scl}(\{x\}) \cap A \subset \text{m-scl}(\{x\}) \cap U = \phi$. This shows that $\text{m-scl}(\{x\}) \cap A = \phi$ for some $x \in cl^\star(A)$. This is a contradiction. Hence A is $\mathcal{I}_{m\omega}$-closed. □

Corollary 4.5. *Let SO(X) ⊂ m-SO(X) and m-SO(X) have property $\mathcal{B}$. For a subset A of X, the following properties are equivalent:*

(1). A is $\mathcal{I}_{m\omega}$-open,

(2). $A - int^\star(A)$ contains no nonempty m-semiclosed set,

(3). $\text{m-scl}(\{x\}) \cap (X - A) \neq \phi$ for each $x \in X - int^\star(A)$.

Theorem 4.6. *Let SO(X) ⊂ m-SO(X) and m-SO(X) have property $\mathcal{B}$. A subset A of X is $\mathcal{I}_{m\omega}$-closed if and only if $A = F - N$ where F is ⋆-closed and N contains no nonempty m-semiclosed set.*

Proof. If A is $\mathcal{I}_{m\omega}$-closed, then by Theorem 4.4, $N = A^\star - A$ contains no nonempty m-semiclosed set. If $F = cl^\star(A)$, then $A \cup A^\star = cl^\star(A) = F$ and by Lemma 2.4, we obtain $F^\star = (A \cup A^\star)^\star = A^\star \cup (A^\star)^\star \subset A^\star \cup A = F$. Therefore F is ⋆-closed such that $F - N = (A \cup A^\star) - (A^\star - A) = (A \cup A^\star) \cap (A^\star \cap A^c)^c = (A \cup A^\star) \cap (A \cup (A^\star)^c) = A \cup (A^\star \cap (A^\star)^c) = A$.

Conversely, suppose $A = F - N$ where F is ⋆-closed and N contains no nonempty m-semiclosed set. Let U be a m-semiopen set such that $A \subset U$. Then $F - N \subset U \Rightarrow F \cap (X - U) \subset N$. Since $A^\star$ is m-semiclosed, hence $A^\star \cap (X-U)$ is m-semiclosed. Since $A \subset F$ and $F^\star \subset F$, then $A^\star \cap (X-U) \subset F^\star \cap (X-U) \subset F \cap (X-U) \subset N$. Therefore, $A^\star \cap (X-U) = \phi$ and so $A^\star \subset U$. Hence A is $\mathcal{I}_{m\omega}$-closed. □

5. New Forms of Closed Sets in Ideal Topological Spaces

Definition 5.1. *A subset A of a space (X, τ) is called a $\tilde{g}$-semi-preclosed set (briefly $\tilde{g}$sp-closed set) if spcl(A) ⊂ U whenever A ⊂ U and U is $^\#$gs-open in X. The complement of $\tilde{g}$sp-closed set is $\tilde{g}$sp-open in X.*

By SO(X) (resp. $^\#$GSO(X), SGO(X), GSO(X), SPO(X), $\tilde{G}$SPO(X), SPGO(X), GSPO(X)) we denote the collection of all semi-open (resp. $^\#$gs-open, sg-open, gs-open, semi-preopen, $\tilde{g}$sp-open, spg-open, gsp-open) sets of the topological space (X, τ). These collections are m-structures on X. By the definitions, we obtain the following diagram:

Diagram I

$$\begin{array}{ccccccc} \text{semi-open} & \longrightarrow & \tilde{g}s\text{-open} & \longrightarrow & \text{sg-open} & \longrightarrow & \text{gs-open} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{semi-preopen} & \longrightarrow & \tilde{g}sp\text{-open} & \longrightarrow & \text{spg-open} & \longrightarrow & \text{gsp-open} \end{array}$$

For subsets of an ideal topological space (X, τ, $\mathcal{I}$), we can define new types of closed sets as follows:

Definition 5.2. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be $\mathcal{I}_\omega$-closed (resp. $\mathcal{I}_{\tilde{g}sg}$-closed, $\mathcal{I}_{sgg}$-closed, $\mathcal{I}_{gsg}$-closed, $\mathcal{I}_{sog}$-closed, $\mathcal{I}_{\tilde{g}sp}$-closed, $\mathcal{I}_{spg}$-closed, $\mathcal{I}_{gsp}$-closed) if $A^\star \subset U$ whenever $A \subset U$ and U is semi-open (resp. $\tilde{g}s$-open, sg-open, gs-open, semi-preopen, $\tilde{g}sp$-open, spg-open, gsp-open) in (X, τ).*

By Diagram I and Definition 5.2, we have the following diagram:

Diagram II

$$\begin{array}{ccccccccc} \mathcal{I}_\omega\text{-closed} & \longleftarrow & \mathcal{I}_{\tilde{g}sg}\text{-closed} & \longleftarrow & \mathcal{I}_{sgg}\text{-closed} & \longleftarrow & \mathcal{I}_{gsg}\text{-closed} & & \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\ \mathcal{I}_{sog}\text{-closed} & \longleftarrow & \mathcal{I}_{\tilde{g}sp}\text{-closed} & \longleftarrow & \mathcal{I}_{spg}\text{-closed} & \longleftarrow & \mathcal{I}_{gsp}\text{-closed} & \longleftarrow & \star\text{-closed} \end{array}$$

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