International Journal of Mathematics And its Applications
Volume 4, Issue 2–A (2016), 79–83.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Fuzzyfication of Semigroups

Research Article

D.D.Padma Priya[1]*, G.Shobhalatha[2] and R.Bhuvana Vijaya[3]

1 Department of Mathematics, New Horizon College of Engineering, Bangalore, Karnataka, India.

2 Department of Mathematics, Sri Krishnadevaraya University, Anantapuram, India.

3 Department of Mathematics, Jawaharlal Nehru Technological University, Anantapuram, India.

Abstract: The motivation mainly comes from the Fuzzification of sets that are of importance and interest in Semigroups, Gamma-Semigroups etc. In this paper we characterize the properties related to Fuzzy sub semigroups and Fuzzy ideals using identities on Semigroups.

Keywords: Fuzzy sub semigroups, Fuzzy (left, right) ideals, Regular Semigroups, Fuzzy bi ideal.

1. Introduction

Fuzzy set was introduced by Zadeh.L.A [1] and others have found many applications in the domain of Mathematics and elsewhere. After the introduction of fuzzy sets by Zadeh.L.A, reconsideration of classical mathematics began [2]. Fuzzy set has an important impact over the field of mathematical research in both theory and application. It has found manifold applications in Mathematics and related areas [2, 10, 11].

The concept of Fuzzification in Semigroups was first discussed by Kuroki.N.A. He studied fuzzy (left, right) ideals and fuzzy bi ideals in Semigroups [3–5, 9, 12]. The study of Fuzzy algebraic structures with the introduction of the concept of Semigroups and Fuzzy ideals were studied by Rosenfield.A [6]. Later Dib. K.A studied some basic concepts of fuzzy algebra such as fuzzy (left, right) ideals and fuzzy bi ideals in Semigroups using a new approach of fuzzy spaces and fuzzy groups [7]. Wang XuePing, Mo Zhi-Wen and Liu Wang-Jin discussed about Fuzzy ideals generated by fuzzy point in Semigroups [8]. The results in the present communication are obtained by considering some identities of Semigroups with different techniques.

1.1. Fuzzy Sets Definitions

(a). Fuzzy subset of a non empty set is a collection of objects with each object being assigned a value between 0 and 1 by a membership function

(b). Let X be a non empty set. A fuzzy set μ of the set X is a function $\mu : X \to [0, 1]$.

(c). Let S be a semigroup. A map A from S to [0, 1] is called a fuzzy set in S.

* *E-mail: padmapriyadesai@gmail.com*

(d). Let F(S) denote the set of all fuzzy sets in S. For $A, B \in F(S)$, $A \subseteq B$ if and only if $A(x) \leq B(x)$ in the ordering of [0, 1] , $\forall x \in S$.

(e). For $A, B \in F(S)$, the product A o B is defined as

$$A \circ B(x) = \sup\{\min\{A(y), B(z)\}\} \text{ for } y, z \in S,\ x = yz$$
$$= 0;\ \text{ for } y, z \in S,\ x \neq yz$$

(f). A fuzzy set $A \in F(S)$ is said to be a fuzzy point, if $A = \cup x\lambda$, where $0 < \lambda \leq 1$, $x\lambda \in A$ iff $x\lambda \subseteq A$ and

$$x\lambda(y) = \lambda,\ \text{ if } y = x$$
$$= 0,\ \text{ if } y \neq x,\ \forall\ y \in S$$

(g). A fuzzy set $A \in F(S)$ is said to be a fuzzy sub semigroup of S if $A(xy) \geq \min\{A(x), A(y)\}\ \forall\ x, y \in S$.

(h). A fuzzy set $A \in F(S)$ is said to be a fuzzy left ideal of S if $A(xy) \geq A(y)\ \forall\ x, y \in S$.

(i). A fuzzy set $A \in F(S)$ is said to be a fuzzy right ideal of S if $A(xy) \geq A(x)\ \forall\ x, y \in S$.

(j). A fuzzy set $A \in F(S)$ is said to be a fuzzy ideal of S if it is both a fuzzy left and fuzzy right ideal of S.

(k). A fuzzy sub semigroup $A \in F(S)$ is said to be a fuzzy bi ideal of S if $A(xyz) \geq \min\{A(x), A(z)\}\ \forall\ x, y, z \in S$.

2. Fuzzyfication on Semigroup S with Some Identities: For all $a, b \in S$, $aba = ab$

Theorem 2.1. *Let S be a regular semigroup and satisfy an identity $aba = ab\ \forall\ a, b \in S$ then*

(i). $\mu(ab) = \mu(a)$

(ii). $\mu(a^2) = \mu(a)$

(iii). $\mu(a^{n+1}) = \mu(a)$ *for any nonempty fuzzy subset* μ *of S.*

Proof. Let S be a regular semigroup. Then we know that $axa = a\ \forall\ a \in S$ and for some $x \in S$. Given S satisfies $aba = ab\ \forall\ a, b \in S$.

(i). Consider

$$\mu(ab) = \mu(abab) \quad \text{[Since ab=aba]}$$
$$= \mu(a(bab)) \quad \text{[Associativity in S]}$$
$$= \mu(aba) \quad \text{[Since bab=ba]}$$
$$\mu(ab) = \mu(a) \quad \text{[Since aba=a].}$$

(ii). Consider $\mu(a^2) = \mu(a.a) \Rightarrow \mu(a^2) = \mu(a)$ [Put $b = a$ from (i) $\mu(ab) = \mu(a)$; proved].

(iii). Consider $\mu(a^{n+1})$, where $n = 1, 2, 3, \ldots$. Let us prove by mathematical induction

For $n = 1$, $\mu(a^2) = \mu(a.a) = \mu(a)$ [proved above]

For $n = 2$, $\mu(a^3) = \mu(a^2.a) = \mu(a.a^2) = \mu(a.a.a) = \mu(a)$ [Since $axa = a$, given put $x = a$]

. . . .

In general $\mu(a^{n+1}) = \mu(a.a^n) = \mu(a)$. Thus $\mu(a^{n+1}) = \mu(a)$.

□

Theorem 2.2. *Let be a fuzzy bi ideal in a semigroup S and S satisfy an identity $aba = ab$, $\forall\ a, b \in S$ then μ is a right ideal in S.*

Proof. Let μ be a fuzzy bi ideal in a semigroup S. Then for any $x, y, z \in S$, we have

$$\mu(xyz) \geq \min\{\mu(x), \mu(z)\} \tag{1}$$

and is a fuzzy sub semigroup on S. Given S satisfies the identity $aba = ab$, $\forall\ a, b \in S$. To show that μ is a fuzzy right ideal in S. i.e., $\mu(xy) \geq \mu(x)\ \forall\ x, y \in S$. Consider

$$\begin{aligned} \mu(xy) &= \mu(xyx) \text{ [Given } aba = ab, \text{ we write } axa = ax, \forall x, y \in S] \\ &\geq \min\{\mu(x), \mu(x)\} \text{ [From (1) put } z = x] \\ &\geq \mu(x). \end{aligned}$$

Therefore $\mu(xy) \geq \mu(x)\ \forall\ x, y \in S$. Hence μ is a fuzzy right ideal in S. □

Theorem 2.3. *Let μ be a fuzzy left ideal of a semigroup S and S satisfy an identity $aba = ab$, $\forall\ a, b \in S$ then μ is a fuzzy sub semigroup of S.*

Proof. Let μ be a fuzzy left ideal in a semigroup S. Then for all $x, y \in S$, we have

$$\mu(xy) \geq \mu(y) \tag{2}$$

Given S satisfies the identity

$$aba = ab,\ \forall\ a, b \in S \tag{3}$$

To prove μ is a fuzzy sub semigroup. i.e., $\mu(xy) \geq \min\{\mu(x), \mu(y)\}\ \forall\ x, y \in S$ or $\mu(xy) \geq \{\mu(x)\Lambda\mu(y)\}\ \forall\ x, y \in S$. Now consider

$$\begin{aligned} \mu(xy) &= \mu(xyx) \text{ [From (3)]} \\ &= \mu((xy)x) \text{ [Associativity in S]} \end{aligned}$$

$$\mu(xy) \geq \mu(x) \text{ [From (2)]} \tag{4}$$

From (4) we have

$$\begin{aligned} \mu(xy) \wedge \mu(xy) &\geq \mu(x) \wedge \mu(xy) \\ &\geq \mu(x) \wedge \mu(y) \text{ [From (2)]} \\ \mu(xy) &\geq \mu(x) \wedge \mu(y)\ \forall\ x, y \in S. \end{aligned}$$

Thus μ is a fuzzy sub semigroup of S. □

Theorem 2.4. *Let μ be a fuzzy left ideal of a semigroup S and S satisfy an identity $aba = ab$, $\forall\ a, b \in S$ then $\mu(ab) = \mu(ba)$.*

Proof. Let μ be a fuzzy left ideal in a semigroup S. Then for all $x, y \in S$, we have

$$\mu(ab) \geq \mu(b)\ \forall\ a, b \in S \tag{5}$$

Given S satisfies the identity $aba = ab$, $\forall\ a, b \in S$. To prove $\mu(ab) = \mu(ba)$, we prove that $\mu(ab) \geq \mu(ba)$ and $\mu(ba) \geq \mu(ab)$. Now

$$\mu(ab) = \mu(aba) = \mu(a(ba))\ [\text{ Associativity in S}]$$
$$\geq \mu(ba)\ \ [\text{From } (5)]$$

$$\therefore \mu(ab) \geq \mu(ba) \tag{6}$$

Now

$$\mu(ba) = \mu(bab) = \mu(b(ab))\ [\text{Associativity in S}]$$
$$\geq \mu(ab)\ [\text{From } (5)]$$

$$\mu(ba) \geq \mu(ab) \tag{7}$$

Thus from (6) and (7) we have $\mu(ab) = \mu(ba)\ \forall\ a, b \in S$. □

Theorem 2.5. *Let μ be a fuzzy left ideal of a semigroup S and S satisfy an identity $aba = ab$, $\forall\ a, b \in S$ then $\mu \circ \mu \leq \mu$.*

Proof. Let μ be a fuzzy left ideal in a semigroup S. Given S satisfies the identity $aba = ab$, $\forall\ a, b \in S$. Then from result 3 we know that

$$\mu(ab) \geq \mu(a) \wedge \mu(b)\ \forall\ a, b \in S \tag{8}$$

Now

$$\mu \circ \mu(x) = V\{\mu(p) \wedge \mu(q)\}$$
$$= V\{\mu(p) \wedge \mu(q)\}\ [\text{Let } x = ab]$$
$$= \mu(a) \wedge \mu(b) \leq \mu(ab)\ \ [\text{From } (8)]$$
$$i.e., \mu \circ \mu(x) \leq \mu(x)$$
$$\therefore \mu \circ \mu \leq \mu$$

□

References

[1] L.A.Zadeh, *Fuzzy sets*, Inform and Control, 8(1965), 338-353.

[2] D.D.Padma Priya, G.Shobhalatha and R.Bhuvana Vijaya, *Properties of semigroups in (semi-) automata*, International Journal of Mathematics Trends and Technology, 30(1)(2016), 43-47.

[3] N.A.Kuroki, *On normal semi groups*, Czechoslovak Math. Journal, 27(1)(1977), 43-53.

[4] N.A.Kuroki, *On Fuzzy ideals and fuzzy bi-ideals in semi groups*, Fuzzy subsets and Systems, 5(1981), 203-215.

[5] N.A.Kuroki, *On fuzzy semi prime ideals in semi groups*, Fuzzy sets and systems, 8(1982), 71-79.

[6] A.Iampan, *Characterizing fuzzy sets in ordered -semigroups*, J. Math. Res., 2(2010), 52-56.

[7] Khan, Asghar, Tariq Mahmood and M.Irfan Ali, *Fuzzy interior gamma ideals in ordered gamma semigroups*, Journal of applied mathematics & informatics, 28(5-6)(2010), 1217-1225.

[8] Akram, Muhammad, J.Kavikumar and Azme Bin Khamis, *Fuzzy Soft Gamma Semigroups*, Appl. math., 8(2)(2014), 929-934.

[9] Rosenfeld and Azriel, *Fuzzy groups*, Journal of mathematical analysis and applications, 35(3)(1971), 512-517.

[10] K.A.Dib, *On fuzzy spaces and fuzzy group theory*, Information sciences, 80(3)(1994), 253-282.

[11] X.P.Wang, Z.W.Mo and W.J.Liu, *Fuzzy ideals generated by fuzzy point in semigroups*, Sichuan Shifan Daxue Xuebao Ziran Kexue Ban, 15(4)(1992), 17-24.

[12] William, Dr.Prince, K.B.Latha and E.Chandrasekeran, *Fuzzy bi-*Γ*-ideals in* Γ*-semigroups*, Hacettepe Journal of Mathematics and statistics, 38(1)(2009), 1-15.

[13] A.Iampan, *Fuzzification of Ideals and Filters in Gamma Semigroups*, Armenian Journal of Mathematics, 4(1)(2012), 44-48.

[14] Majumder, Samit Kumaar and Surjit Kumar Sardar, *On Cartesian product of fuzzy prime and fuzzy semiprime ideals of semigroups*, Journal of Mathematics and applications, 32(2010), 63-66.

[15] Chinram, Ronnason and Sompob Saelee, *Fuzzy ideals and fuzzy filters of ordered ternary semigroups*, Journal of Mathematics Research, 2(1)(2010).