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Some Result on Balanced Cordial Graphs

Research Article

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Abstract: In this paper we have prove that $P(t \cdot C_{4n})$, $C(t \cdot C_{4n})$ and C^*_{4n} are balanced cordial graphs under certain condition. A balanced cordial graph is a cordial graph and it satisfies $|v_f(0) - v_f(1)| = 0 = |e_f(0) - e_f(1)|$ condition.

MSC: 05C78.

Keywords: Balanced cordial labeling, cycle of graph, star of graph, path of graph.

1. Introduction

Labeled graph has several applications. Gallian [3] survey on graph labeling gives enough amount of literature with comprehensive bibliography of papers on various types of graph labeling. Cahit [1] has introduced a variation of both graceful and harmonious labeling which is cordial labeling. He proved that K_n is admits cordial labeling if and only if $n < 4$. After this, many researcher have studied cordial labeling and similar types of labeling. Kaneria, Makadia and Meera [4] proved that cycle of n copies of K_n and star of K_n are cordial graph for all n. Shee and Ho [5] define path union of graph G, A graph obtained by joining a vertex v of $G^{(i)}$ with the same vertex of $G^{(i+1)}$ by an edge $\forall\ i = 1, 2, ..., n-1$, where $G = G^{(1)} = G^{(2)} = ... = G^{(n)}$. They proved that $P(t \cdot C_n)$ is cordial, $\forall\ t, n \in \mathbb{N}$.

All the graph in this paper are finite, simple and undirected. For a graph G, we take $p = |V(G)|$, number of vertices and $q = |E(G)|$ number of edges in G. We follow Harary [2] for the basic notation and terminology of graph theory. Let f be a function from $V(G)$ to $\{0, 1\}$, for a graph G and for each $e = (x, y) \in E(G)$, the edge label of e assign by $f(x) + f(y)$ $(mod$ 2). f is called cordial labeling for G and G is called a cordial graph if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. Also f is called balanced cordial labeling, edge balanced cordial labeling, vertex balanced cordial labeling and unbalanced cordial labeling if it satisfies 1 to 4 conditions respectively.

(1) $|v_f(0) - v_f(1)| = 0 = |e_f(0) - e_f(1)|$.

(2) $|v_f(0) - v_f(1)| = 1, |e_f(0) - e_f(1)| = 0$.

(3) $|v_f(0) - v_f(1)| = 0, |e_f(0) - e_f(1)| = 1$ and

(4) $|v_f(0) - v_f(1)| = 1 = |e_f(0) - e_f(1)|$.

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2. Main Results

Theorem 2.1. *$P(t \cdot C_{4n})$ is a balanced cordial if t is odd and it is vertex balanced cordial if t is even, where $n \in N$.*

Proof. Let $u_{i,j}(1 \leq \text{j} \leq 4n)$ be the vertices of i^{th} copy $C_{4n}^{(i)}$ in $P(t \cdot C_{4n})$, $\forall\ i = 1, 2, ..., t$. It it obvious that the vertex labeling function $f_1 : V(C_{4n}^{(1)}) \rightarrow \{0, 1\}$ defined by

$$\begin{aligned} f_1(u_{1,j}) &= 0;\ \text{if}\ j \equiv 0, 1 \pmod 4, \\ &= 1;\ \text{if}\ j \equiv 2, 3 \pmod 4,\ \forall\ j = 1, 2, ..., 4n; \end{aligned}$$

is a balanced cordial labeling for $C_{4n}^{(1)}$. For each $i = 1, 2, ..., t-1$, join u_{i,j_0} with u_{i+1,j_0}, by an edge, for some $j_0 \in \{1, 2, ..., 4n\}$ to form the path union $P(t \cdot C_{4n})$. For each $i = 2, 3, ..., t$, define by $f_i : V(C_{4n}^{(i)}) \rightarrow \{0, 1\}$ as follows.

$$\begin{aligned} f_i(u_{i,j}) &= 1 - f_1(u_{1,j})\ \ \text{if}\ j \equiv 2, 3 \pmod 4, \\ &= f_1(u_{1,j})\ \ \ \ \ \ \ \text{if}\ j \equiv 0, 1 \pmod 4,\ \forall\ j = 1, 2, ..., 4n. \end{aligned}$$

It is obvious that above defined labeling function f_i $(i = 2, 3, ..., t)$ is also a balanced cordial labeling. Now define $f : V(P(t \cdot C_{4n})) \rightarrow \{0, 1\}$ as follows. $f(x) = f_i(x)$ if $x \in V(C_{4n}^{(i)})$, for some $i \in \{1, 2, ..., t\}$. Since $V(P(t \cdot C_{4n})) = \bigcup_{i=1}^{t} V(C_{4n}^{(i)})$ and ${f_i}'$s are balanced cordial labelings, $v_f(0) = v_f(1)$. i.e. $|v_f(0) - v_f(1)| = 0$ hold in $P(t \cdot C_{4n})$. For each $i = 1, 2, ..., t-1$, it is observed that the edge label of $(u_{i,j_0}, u_{i+1,j_0}) = 1$ if i is odd and it is 0 if i is even. Thus, $e_f(0) = t \cdot e_{f_1}(0) + \lfloor \frac{t-1}{2} \rfloor$ and $e_f(1) = t \cdot e_{f_1}(1) + \lceil \frac{t-1}{2} \rceil$. Therefore, $|e_f(0) - e_f(1)| = 0$ if t is odd and it is 1 if t is even. So, $P(t \cdot C_{4n})$ is balanced cordial if t is odd and it is vertex balanced cordial if t is even. □

Illustration 2.2. *$P(5 \cdot C_8)$ and its balanced cordial labeling are shown in Figure 1.*

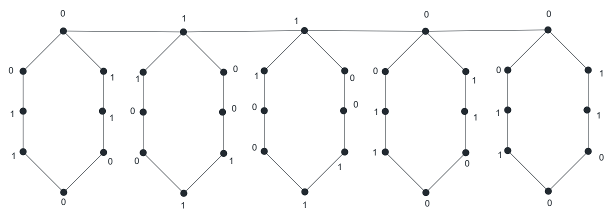

Figure 1. $P(5 \cdot C_8)$ **with its balanced cordial labeling and** $v_f(0) = v_f(1) = 20$ **as well as** $e_f(0) = e_f(1) = 22$.

Theorem 2.3. *$C(t \cdot C_{4n})$ is a balanced cordial if $t \equiv 0 \pmod 4$ and it is a vertex balanced cordial if $t \equiv 1, 3 \pmod 4$, where $n \in N$.*

Proof. Let $u_{i,j}(1 \leq j \leq 4n)$ be the vertices of i^{th} copy $C_{4n}^{(i)}$ in $C(t \cdot C_{4n})$, $i = 1, 2, ..., t$. It is obvious that the vertex labeling function $f_1 : V(C_{4n}^{(1)}) \rightarrow \{0, 1\}$ defined by

$$\begin{aligned} f_1(u_{1,j}) &= 0;\ \text{if}\ j \equiv 0, 1 \pmod 4, \\ &= 1;\ \text{if}\ j \equiv 2, 3 \pmod 4,\ \forall\ j = 1, 2, ..., 4n; \end{aligned}$$

is a balanced cordial labeling for $C_{4n}^{(1)}$. For each $i = 1, 2, ..., t-1$, join u_{i,j_0} with u_{i+1,j_0}. by an edge and also join u_{t,j_0} with u_{1,j_0} by an edge, for some $j_0 \in \{1, 2, ..., 4n\}$ to form the cycle graph $C(t \cdot C_{4n})$. For each $i = 2, 3, ..., t$, define $f_2 : V(C_{4n}^{(i)}) \rightarrow \{0, 1\}$ as follows.

$$\begin{aligned} f_i(u_{i,j}) &= f_1(u_{1,j})\ \ \ \ \ \ \ \text{if}\ j \equiv 0, 1 \pmod 4, \\ &= 1 - f_1(u_{1,j})\ \ \text{if}\ j \equiv 2, 3 \pmod 4,\ \forall\ j = 1, 2, ..., 4n. \end{aligned}$$

It is Obvious that above defined labeling function f_i $(i = 2, 3, ..., t)$ is also a balanced cordial labeling. Now define $f : V(C(t \cdot C_{4n})) \to \{0, 1\}$ as follows. $f(x) = f_i(x)$ if $x \in V(C_{4n}^{(i)})$, for some $i \in \{1, 2, ..., t\}$. Since $V(C(t \cdot C_{4n})) = \bigcup_{i=1}^{t} V(C_{4n}^{(i)})$ and $f_i\ ^{'}$s are balanced cordial labelings, $|v_f(0) - v_f(1)| = 0$ hold in $C(t \cdot C_{4n})$. For each $i = 1, 2, ..., t-1$, it is observed that the edge label of $(u_{i,j_0}, u_{i+1,j_0}) = 1$ if i is odd and it is 0 if i is even. Also the edge label of $(u_{t,j_0}, u_{1,j_0}) = 1$ if $t \equiv 3$ (mod 4) and it is 0 if $t \equiv 0, 1$ (mod 4). Thus, $e_f(0) = t \cdot e_{f_1}(0) + \lceil \frac{t}{2} \rceil, e_f(1) = t \cdot e_{f_1}(1) + \lfloor \frac{t}{2} \rfloor$, if $t \equiv 0, 1$ (mod 4) and $e_f(1) = t \cdot e_{f_1}(1) + (\frac{t+1}{2}), e_f(0) = t \cdot e_{f_1}(0) + (\frac{t-1}{2})$, if $t \equiv 3$ (mod 4). Therefore, $|e_f(0) - e_f(1)| = 1$ if t is odd and it is 0 if $t \equiv 0$ (mod 4) and it is vertex balanced cordial if $t \equiv 1, 3$ (mod 4). □

Illustration 2.4. *$C(9 \cdot C_4)$ and its vertex balanced cordial labeling are shown in Figure 2.*

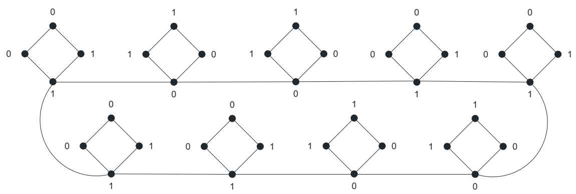

Figure 2. $C(9 \cdot C_4)$ **with its vertex balanced cordial labeling and** $v_f(0) = v_f(1) = 18, e_f(0) = 23, e_f(1) = 22.$

Theorem 2.5. *$C_{4n}^{\star}$ is a balanced cordial graph, $\forall\ n \in N$.*

Proof. Let $u_{i,j} (1 \le j \le 4n)$ be the vertices of i^{th} copy $C_{4n}^{(i)}$ in $C_{4n}^{\star}$, $\forall\ i = 1, 2, ..., 4n$ and $u_{0,j} (1 \le j \le 4n)$ be the vertices of central copy of C_{4n} in $C_{4n}^{\star}$. It is obvious that the vertex labeling function $f : V(C_{4t}^{(0)}) \to \{0, 1\}$ defined by

$$\begin{aligned} f(u_{0,j}) &= 0 \text{ if } j \equiv 0, 1 \ (\text{mod } 4), \\ &= 1 \text{ if } j \equiv 2, 3 \ (\text{mod } 4), \ \forall\ j = 1, 2, ..., 4n; \end{aligned}$$

is a balanced cordial labeling for $C_{4n}^{(0)}$. For each i = 1,2,...,4n, join $u_{0,i}$ with $u_{i,i}$ by an edge to form the of C_{4n}. Now define $g : V(C_{4n}^{\star}) \to \{0, 1\}$ as follows

$$\begin{aligned} g(u_{i,j}) &= 1 - f(u_{0,j}) \text{ if } i \text{ is odd}, \\ &= f(u_{0,j}) \quad \text{ if } i \text{ is even}, \ \forall\ i = 1, 2, ..., 4n, \ \forall\ j = 1, 2, ..., 4n. \end{aligned}$$

Since $V(C_{4n}^{\star}) = \bigcup_{i=0}^{4n} V(C_{4n}^{(i)})$ and $v_f(0) = v_f(1)$ in $C_{4n}^{(0)}$, $|v_g(0) - v_g(1)| = 0$ hold in $C_{4n}^{\star}$. For each $i = 1, 2, ..., 4n$, it is observed that the edge label of $(u_{0,i}, u_{i,i}) = 1$ if i is odd and it is 0 if i is even. Thus, $e_g(0) = e_g(1) = (4n+1)e_f(0) + 2n$. Therefore, $|e_g(0) - e_g(1)| = 0$ hold in $C_{4n}^{\star}$ and So, it is a balanced cordial graph. □

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