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Generalization of Semicompact Spaces via Bioperations

Research Article

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Abstract: In this paper, we introduce $\gamma \vee \gamma'$-semicompact spaces and study some of their properties using the concept of $\gamma \vee \gamma'$-semiconvergence, $\gamma \vee \gamma'$-semifilterbase and $\gamma \vee \gamma'$-semiaccumulation points.

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1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, seperation axioms etc. by utilizing generalized open sets. Kasahara [1] defined the concept of an operation on topological spaces. Ogata and Maki [3] introduced the notion of $\tau_{\gamma\vee\gamma'}$ which is the collection of all $\gamma \vee \gamma'$-open sets in a topological space (X, τ). In this paper, we introduce $\gamma \vee \gamma'$-semicompact spaces and study some of their properties using the concept of $\gamma \vee \gamma'$-semiconvergence, $\gamma \vee \gamma'$-semifilterbase and $\gamma \vee \gamma'$-semiaccumulation points.

2. Preiliminaries

The closure and the interior of a subset A of (X, τ) are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

Definition 2.1 ([1]). *Let (X, τ) be a topological space. An operation γ on the topology τ is function from τ on to power set $\mathcal{P}(X)$ of X such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of τ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.*

Definition 2.2 ([3]). *A topological space (X, τ) equipped with two operations, say, γ and γ' defined on τ is called a bioperation-topological space, it is denoted by $(X, \tau, \gamma, \gamma')$.*

Definition 2.3. *A subset A of a topological space (X, τ) is said to be $\gamma \vee \gamma'$-open set [3] if for each $x \in A$ there exists an open neighbourhood U of x such that $U^\gamma \cup U^{\gamma'} \subset A$. The complement of $\gamma \vee \gamma'$-open set is called $\gamma \vee \gamma'$-closed. $\tau_{\gamma\vee\gamma'}$ denotes set of all $\gamma \vee \gamma'$-open sets in (X, τ).*

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Definition 2.4 ([3]). *For a subset A of (X, τ), $\tau_{\gamma\vee\gamma'}$-Cl(A) denotes the intersection of all $\gamma \vee \gamma'$-closed sets containing A, that is, $\tau_{\gamma\vee\gamma'}\text{-Cl}(A) = \bigcap\{F : A \subset F, X\backslash F \in \tau_{\gamma\vee\gamma'}\}$.*

Definition 2.5. *Let A be any subset of X. The $\tau_{\gamma\vee\gamma'}$-Int(A) is defined as $\tau_{\gamma\vee\gamma'}\text{-Int}(A) = \cup\{U : U$ is a $\gamma \vee \gamma'$-open set and $U \subset A\}$.*

Definition 2.6. *A subset A of a topological space (X, τ) is said to be $\gamma \vee \gamma'$-semiopen [2] if $A \subset \tau_{\gamma\vee\gamma'}\text{-Cl}(\tau_{\gamma\vee\gamma'}\text{-Int}(A))$.*

3. Bioperation-semicompact spaces

In this section, we introduce $\gamma \vee \gamma'$-semicompact spaces and study some of their properties using the concept of $\gamma \vee \gamma'$-semiconvergence, $\gamma \vee \gamma'$-semifilterbase and $\gamma \vee \gamma'$-semiaccumulation points.

Definition 3.1. *A collection $\{A_i : i \in \Delta\}$ of $\gamma \vee \gamma'$-semiopen sets in a bioperation-topological space $(X, \tau, \gamma, \gamma')$ is called a $\gamma \vee \gamma'$-semiopen cover of a subset A in X if $A \subset \bigcup_{i\in\Delta} A_i$.*

Definition 3.2. *A bioperation-topological space $(X, \tau, \gamma, \gamma')$ is said to be $\gamma \vee \gamma'$-semicompact if for each $\gamma \vee \gamma'$-semiopen cover of X has a finite subcover.*

Example 3.3. *Let $X = \mathbb{R}$, $\tau = \{\emptyset, \mathbb{R}\} \cup \{(a, \infty) : a \in \mathbb{R}\}$ and γ, γ' identity operations defined on τ. It is clear that $\gamma \vee \gamma'$-$SO(X, \tau) = \{\emptyset, R\} \cup \{[a, \infty) : a \in R\}$. Clearly, the bioperation-topological space $(X, \tau, \gamma, \gamma')$ is not a $\gamma \vee \gamma'$-semicompact space.*

However, we have the following Theorem.

Theorem 3.4. *Every finite bioperation-topological space is $\gamma \vee \gamma'$-semicompact.*

Proof. Let $X = \{x_1, x_2, ..., x_n\}$. Let $\mathcal{C}$ be a $\gamma \vee \gamma'$-semiopen covering of X. Then each element in X belongs to one of the members of $\mathcal{C}$, say, $x_1 \in G_1$, $x_2 \in G_2$, . . . , $x_n \in G_n$, where $G_i \in \mathcal{C}, i = 1, 2, ..., n$. Then the collection $\{G_1, G_2, ..., G_n\}$ is a finite subcover of X. Hence $(X, \tau, \gamma, \gamma')$ is a $\gamma \vee \gamma'$-semicompact space. □

Theorem 3.5. *If $G_1, G_2, ..., G_n$ are $\gamma \vee \gamma'$-semicompact subsets of a bioperation-topological space $(X, \tau, \gamma, \gamma')$, then $G_1 \cup G_2 \cup ... \cup G_n$ is also $\gamma \vee \gamma'$-semicompact, that is, finite union of $\gamma \vee \gamma'$-semicompact sets is $\gamma \vee \gamma'$-semicompact.*

Proof. Let U and V be any two $\gamma\vee\gamma'$-semicompact subsets of X. Let $\mathcal{C}$ be a $\gamma\vee\gamma'$-semiopen cover of $U\cup V$. Then $\mathcal{C}$ will also be a $\gamma\vee\gamma'$-semiopen cover of both U and V. So by hypothesis, there exists a finite subcollection of $\mathcal{C}$ of $\gamma\vee\gamma'$-semiopen sets, say, $\{U_1, U_2, ..., U_n\}$ and $\{V_1, V_2, ..., V_m\}$ covering U and V, respectively. Clearly, the collection $\{U_1, U_2, ..., U_n, V_1, V_2, ..., V_m\}$ is a finite collection of $\gamma \vee \gamma'$-semiopen sets covering $U \cup V$. By induction, every finite union of $\gamma \vee \gamma'$-semicompact sets is $\gamma \vee \gamma'$-semicompact. □

Theorem 3.6. *Every $\gamma \vee \gamma'$-semiclosed subset of a $\gamma \vee \gamma'$-semicompact space $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact.*

Proof. If G is a $\gamma \vee \gamma'$-semiclosed set in the $\gamma \vee \gamma'$-semicompact space $(X, \tau, \gamma, \gamma')$ and $\mathcal{C}$ is any $\gamma \vee \gamma'$-semiopen cover of G, then the collection $\mathcal{C}^\star = (X\backslash G) \cup \mathcal{C}$ is a $\gamma \vee \gamma'$-semiopen cover of X. Since X is $\gamma \vee \gamma'$-semicompact, the collection $\mathcal{C}^\star$ has a finite subcover. If this finite subcover contains the set $X\backslash G$, discard it, otherwise leave the finite subcover alone, the resulting collection is a finite subcover of $\mathcal{C}$. □

Theorem 3.7. *Every infinite subset of a $\gamma \vee \gamma'$-semicompact space $(X, \tau, \gamma, \gamma')$ has at least one $\gamma \vee \gamma'$-semicluster point in X.*

Proof. Suppose X is a $\gamma \vee \gamma'$-semicompact space and let A be an infinite subset of X. Assume that A has no $\gamma \vee \gamma'$-semicluster points in X. Then for each $x \in X$, there exists a $\gamma \vee \gamma'$-semiopen set U_x such that $U_x \cap A = \{x\}$ or $\emptyset$. Now the collection $\{U_x : x \in X\}$ is a $\gamma \vee \gamma'$-semiopen covering of X. Since X is $\gamma \vee \gamma'$-semicompact, there exist points $x_1, x_2, ..., x_n$ in X such that $\overset{n}{\underset{i=1}{\cup}} U_{x_i} = X$. But $(U_{x_1} \cap A) \cup (U_{x_2} \cap A) \cup .... \cup (U_{x_n} \cap A) = \{x_1\} \cup \{x_2\} \cup ... \cup \{x_n\}$ or $\emptyset$. It follows that $(U_{x_1} \cup U_{x_2} \cup .... \cup U_{x_n}) \cap A = \{x_1, x_2, ..., x_n\}$ or $\emptyset$. Hence $A = \{x_1, x_2, ..., x_n\}$ or $\emptyset$, contradicts that A is infinite. □

Definition 3.8. *A collection η of $\mathcal{P}(X)$ is said to satisfy the finite intersection condition if for every finite subcollection $\{F_1, F_2, ..., F_n\}$ of η, the intersection $F_1 \cap F_2 \cap ...F_n$ is nonempty.*

We will give several characterizations of the $\gamma \vee \gamma'$-semicompact spaces. The first characterization makes use of the finite intersection condition.

Theorem 3.9. *The following statements are equivalent for any bioperation-topological space $(X, \tau, \gamma, \gamma')$:*

(1). $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact.

(2). Given any family $\mathcal{F}$ of $\gamma \vee \gamma'$-semiopen sets, if no finite subfamily of $\mathcal{F}$ covers X, then $\mathcal{F}$ does not cover X.

(3). Given any family $\mathcal{F}$ of $\gamma \vee \gamma'$-semiclosed sets, if $\mathcal{F}$ satisfies the finite intersection condition, then $\cap\{A : A \in \mathcal{F}\} \neq \emptyset$.

(4). Given any family $\mathcal{F}$ of subsets of X, if $\mathcal{F}$ satisfies the finite intersection condition, then $\cap\{\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(A) : A \in \mathcal{F}\} \neq \emptyset$.

Proof. (1) $\Leftrightarrow$ (2) and (2) $\Leftrightarrow$ (3) are obivious.
(3) $\Rightarrow$ (4): If $\mathcal{F} \subset P(X)$ satisfies the finite intersection condition, then $\cap\{\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(A) : A \in \mathcal{F}\}$ is a family of $\gamma \vee \gamma'$-semiclosed sets, which obviously satisfies the finite intersection condition.
(4) $\Rightarrow$ (3): Follows from the fact that $A = \gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(A)$ for every $\gamma \vee \gamma'$-semiclosed subset A of X. □

Theorem 3.10. *A bioperation-topological space $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact if, and only if for every collection of $\gamma \vee \gamma'$-semiclosed sets in X, satisfying the finite intersection condition, the intersection of all the $\gamma \vee \gamma'$-semiclosed sets in the collection is nonempty.*

Proof. Suppose ζ is a collection of subsets of X. Let $\xi = \{X \backslash G : G \in \zeta\}$ be the collection of their complements. Then we have the following equivalent statements:

(1). ζ is a collection of $\gamma \vee \gamma'$-semiopen sets if, and only if ξ is a collection of $\gamma \vee \gamma'$-semiclosed sets in X.

(2). The collection ζ is a $\gamma \vee \gamma'$-semiopen cover of X if, and only if the intersection of all the $\gamma \vee \gamma'$-semiclosed elements in ξ is empty.

(3). The finite subcollection $\{G_1, G_2, ..., G_n\}$ of ζ is a $\gamma \vee \gamma'$-semiopen cover of X if, and only if the intersection of the corresponding $\gamma \vee \gamma'$-semiclosed elements of ξ is nonempty.

Now X is $\gamma \vee \gamma'$-semicompact if, and only if given any collection ζ of $\gamma \vee \gamma'$-semiopen sets, if ζ is a $\gamma \vee \gamma'$-semiopen cover of X, then there is a finite subcollection of ζ covers X if, and only if given any collection ζ of $\gamma \vee \gamma'$-semiopen sets, if there is no finite subcollection of ζ covers X, then ζ is not a $\gamma \vee \gamma'$-semiopen cover of X if, and ony if given any collection ξ of $\gamma \vee \gamma'$-semiclosed sets, if every finite intersection of $\gamma \vee \gamma'$-semiclosed sets in ξ is nonempty, then the intersection of all the $\gamma \vee \gamma'$-semiclosed sets in ξ is nonempty. □

We will give the characterization by nets of $\gamma \vee \gamma'$-semicompact spaces.

Definition 3.11. *Let (X,τ,γ,γ') be a bioperation-topological space. A point $x\in X$ is said to be a $\gamma\vee\gamma'$-semicluster point of a net $\{x_\alpha\}_{\alpha\in\Delta}$ if $\{x_\alpha\}_{\alpha\in\Delta}$ is frequently in every $\gamma\vee\gamma'$-semiopen set containing x. We denote by $\gamma\vee\gamma'$-s-cp$\{x_\alpha\}_{\alpha\in\Delta}$ the set of all $\gamma\vee\gamma'$-semicluster points of a net $\{x_\alpha\}_{\alpha\in\Delta}$.*

Theorem 3.12. *The set of all $\gamma\vee\gamma'$-semicluster points of an arbitrary net in X is $\gamma\vee\gamma'$-semiclosed.*

Proof. Let $\{x_\alpha\}_{\alpha\in\Delta}$ be a net in X. Set $A=\gamma\vee\gamma'$-*s-cp*$\{x_\alpha\}_{\alpha\in\Delta}$. Let $x\in X\backslash A$. Then there exists a $\gamma\vee\gamma'$-semiopen set U_x containing x and $\alpha_x\in\Delta$ such that $x_\beta\notin U_x$ whenever $\beta\in\Delta$, $\beta\geq\alpha_x$. It turns out that $U_x\subset X\backslash A$, hence $x\in\gamma\vee\gamma'$-$s\,\mathrm{Int}(X\backslash A)=X\backslash\gamma\vee\gamma'$-$s\,\mathrm{Cl}(A)$. This shows that $\gamma\vee\gamma'$-$s\,\mathrm{Cl}(A)\subset A$; hence A is $\gamma\vee\gamma'$-semiclosed in (X,τ,γ,γ'). □

Theorem 3.13. *A bioperation-topological space (X,τ,γ,γ') is $\gamma\vee\gamma'$-semicompact if, and only if each net $\{x_\alpha\}_{\alpha\in\Delta}$ in X has at least one $\gamma\vee\gamma'$-semicluster point.*

Proof. Let (X,τ,γ,γ') be a $\gamma\vee\gamma'$-semicompact space. Assume that there exist some net $\{x_\alpha\}_{\alpha\in\Delta}$ in X such that $\gamma\vee\gamma'$-s-cp$\{x_\alpha\}_{\alpha\in\Delta}$ is empty. Then for every $x\in X$, there exist $U(x)\in\gamma\vee\gamma'$-$SO(X,x)$ and $\alpha(x)\in\Delta$ such that $x_\beta\notin U(x)$ whenever $\beta\geq\alpha(x)$, $\beta\in\Delta$. Then the family $\{U(x):x\in X\}$ is a cover of X by $\gamma\vee\gamma'$-semiopen sets and has a finite subcover, say, $\{U_k:k=1,2,...n\}$ where $U_k=U(x_k)$ for $k=$ 1,2,....n, $\{x_k:k=1,2,...n\}\subset X$. Let us take $\alpha\in\Delta$ such that $\alpha\geq\alpha(x_k)$ for all $k\in\{1,2,....n\}$. For every $\beta\in\Delta$ such that $\beta\geq\alpha$ we have, $x_\beta\notin U_k$, $k=$ 1, 2,....n, hence $x_\beta\notin X$, which is a contradiction. Conversely, if X is not $\gamma\vee\gamma'$-semicompact, there exists $\{U_i:i\in\Delta\}$ a cover of X by $\gamma\vee\gamma'$-semiopen sets, which has no finite subcover. Let $F(\Delta)$ be the family of all finite subsets of Δ. Clearly, $(F(\Delta),\subset)$ is a directed set. For each $J\in F(\Delta)$, we may choose $x_J\in X\backslash\cup\{U_i:i\in J\}$. Let us consider the net $\{x_J\}_{J\in F(\Delta)}$. By hypothesis, the set $\gamma\vee\gamma'$-*s-cp*$\{x_J\}_{J\in F(\Delta)}$ is nonempty. Let $x\in\gamma\vee\gamma'$-*s-cp*$\{x_J\}_{J\in F(\Delta)}$ and let $i_0\in\Delta$ such that $x\in U_{i0}$. By the definition of $\gamma\vee\gamma'$-semicluster point for each $J\in F(\Delta)$ there exists $J^\star\in F(\Delta)$ such that $J\subset J^\star$ and $x_{J^\star}\in U_{i0}$. For $J=\{i_0\}$, there exists $J^\star\in F(\Delta)$ such that $i_0\in J^\star$ and $x_{J^\star}\in U_{i0}$. But $x_{J^\star}\in X\backslash\cup\{U_i:i\in J^\star\}\subset X\backslash U_{i_0}$. The contradiction we obtained shows that (X,τ,γ,γ') is $\gamma\vee\gamma'$-semicompact. □

In the following, we will give a characterization of $\gamma\vee\gamma'$-semicompact spaces by means of filterbases.

Let us recall that a nonempty family $\mathcal{F}$ of subsets of X is said to be a filterbase on X if $\emptyset\notin\mathcal{F}$ and each intersection of two members of $\mathcal{F}$ contains a third member of $\mathcal{F}$. Notice that each chain in the family of all filterbases on X (ordered by inclusion) has an upper bound, for example, the union of all members of the chain. Then, by Zorn's Lemma, the family of all filterbases on X has at least one maximal element. Similarly, the family of all filterbases on X containing a given filterbase $\mathcal{F}$ has at least one maximal element.

Definition 3.14. *A filterbase $\mathcal{F}$ on a bioperation-topological space (X,τ,γ,γ') is said to be*

(1). $\gamma\vee\gamma'$-semiconverge to a point $x\in X$ if for each $\gamma\vee\gamma'$-semiopen set U containing x, there exists $B\in\mathcal{F}$ such that $B\subset U$.

(2). $\gamma\vee\gamma'$-semiaccumulate at $x\in X$ if $U\cap B\neq\emptyset$ for every $\gamma\vee\gamma'$-semiopen set U containing x and every $B\in\mathcal{F}$.

Remark 3.15. *A filterbase $\mathcal{F}$ $\gamma\vee\gamma'$-semiaccumulates at x if, and only if $x\in\cap\{\gamma\vee\gamma'$-$s\,\mathrm{Cl}(B):B\in\mathcal{F}\}$. Clearly, if a filterbase $\mathcal{F}$ $\gamma\vee\gamma'$-semiconverges to $x\in X$, then $\mathcal{F}$ $\gamma\vee\gamma'$-semiaccumulates at x.*

Lemma 3.16. *If a maximal filterbase $\mathcal{F}$ $\gamma\vee\gamma'$-semiaccumulates at $x\in X$, then $\mathcal{F}$ $\gamma\vee\gamma'$-semiconverges to x.*

Proof. Let $\mathcal{F}$ be a maximal filterbase which $\gamma\vee\gamma'$-semiaccumulates at $x\in X$. If $\mathcal{F}$ does not $\gamma\vee\gamma'$-semiconverge to x, then there exists a $\gamma\vee\gamma'$-semiopen set U containing x such that $U\cap B\neq\emptyset$ and $(X\backslash U)\cap B\neq\emptyset$ for every $B\in\mathcal{F}$. Then $\mathcal{F}\cup\{U\cap B:B\in\mathcal{F}\}$ is a filterbase which strictly contains $\mathcal{F}$, which is a contradiction. □

One of the most important characterizations of $\gamma \vee \gamma'$-semicompact spaces is the following result:

Theorem 3.17. *For a bioperation-topological space $(X, \tau, \gamma, \gamma')$, the following statements are equivalent:*

(1). $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact.

(2). Every maximal filterbase $\gamma \vee \gamma'$-semiconverges to some point of X.

(3). Every filterbase $\gamma \vee \gamma'$-semiaccumulates at some point of X.

(4). For every family $\{F_\alpha : \alpha \in \Delta\}$ of $\gamma \vee \gamma'$-semiclosed subsets of $(X, \tau, \gamma, \gamma')$, such that $\cap\{F_\alpha : \alpha \in \Delta\} = \emptyset$, there exists a finite subset Δ_0 of Δ such that $\cap\{F_\alpha : \alpha \in \Delta_0\} = \emptyset$.

Proof. (1)⇒(2): Let $\mathcal{F}_0$ be a maximal filterbase on X. Suppose that $\mathcal{F}_0$ does not $\gamma \vee \gamma'$-semiconverge to any point of X. Then, by Lemma 3.16, $\mathcal{F}_0$ does not $\gamma \vee \gamma'$-semiaccumulate at any point of X. For each $x \in X$, there exists a $\gamma \vee \gamma'$-semiopen set U_x containing x and $B_x \in \mathcal{F}_0$ such that $U_x \cap B_x = \emptyset$. The family $\{U_x : x \in X\}$ is a cover of X by $\gamma \vee \gamma'$-semiopen sets. By (1), there exists a finite subset $\{x_1, x_2, ...x_n\}$ of X such that $X = \cup\{U_{x_k} : k = 1, 2, ...n\}$. Since $\mathcal{F}_0$ is a filterbase, there exists $B_0 \in \mathcal{F}_0$ such that $B_0 \subset \cap\{B_{x_k} : k = 1, 2, ...n\} = X\backslash\cup\{U_{x_k} : k = 1, 2, ...n\} = \emptyset$, hence $B_0 = \emptyset$. This is a contradiction.

(2)⇒ (3): Let $\mathcal{F}$ be a filterbase on X. There exists a maximal filterbase $\mathcal{F}_0$ such that $\mathcal{F} \subset \mathcal{F}_0$. By (2), $\mathcal{F}_0$ $\gamma \vee \gamma'$-semiconverges to some point $x_0 \in X$. Let $B \in \mathcal{F}$. For every $U \in \gamma \vee \gamma'$-$SO(X, x_0)$, there exists $B_U \in \mathcal{F}_0$ such that $B_U \subset U$, hence $U \cap B \neq \emptyset$, since it contains the member $B_U \cap B$ of $\mathcal{F}_0$. This shows that $\mathcal{F}$ $\gamma \vee \gamma'$-semiaccumulates at x_0.

(3)⇒ (4): Let $\{F_\alpha : \alpha \in \Delta\}$ be a family of $\gamma \vee \gamma'$-semiclosed subsets of $(X, \tau, \gamma, \gamma')$ such that $\cap\{F_\alpha : \alpha \in \Delta\} = \emptyset$. If possible suppose that every finite subfamily has a nonempty intersection, then $\beta = \{\bigcap_{i=1}^{n} F_{\alpha_i} : n \in \mathbb{N}, F_{\alpha_i} \in \{F_\alpha : \alpha \in \Delta\}\}$ form a filterbase on X. Then by (3) β is $\gamma \vee \gamma'$-semiaccumulates to some points $x \in X$. This implies that for every $\gamma \vee \gamma'$-semiopen set U containing x, $F_\alpha \cap U \neq \emptyset$ for every $F_\alpha \in \beta$ and every $\alpha \in \Delta$. Since $x \notin \cap\{F_\alpha : \alpha \in \Delta\}$, there exists $\alpha_0 \in \Delta$ such that $x \notin F_{\alpha_0}$. Therefore $x \in X\backslash F_{\alpha_0}$, which is $\gamma \vee \gamma'$-semiopen set in X. But $F_{\alpha_0} \cap X\backslash F_{\alpha_0} = \emptyset$, then we get a contradiction of the fact that β is $\gamma \vee \gamma'$-semiaccumulates to x.

(4) ⇒ (1): Let $\{U_\alpha : \alpha \in \Delta\}$ be a $\gamma \vee \gamma'$-semiopen cover of X. Then $\{X\backslash U_\alpha : \alpha \in \Delta\}$ is a family of $\gamma \vee \gamma'$-semiclosed subsets of X such that $\cap\{X\backslash U_\alpha : \alpha \in \Delta\} = \emptyset$. Then by (4) there exists a finite subset Δ_0 of Δ such that $\cap\{X\backslash U_\alpha : \alpha \in \Delta_0\} = \emptyset$. This implies that $X = \cup\{U_\alpha : \alpha \in \Delta_0\}$; hence $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact. □

Definition 3.18. *A point x in a bioperation-topological space $(X, \tau, \gamma, \gamma')$ is said to be a $\gamma \vee \gamma'$-semicomplete accumulation point of a subset A of X if $|S \cap A| = |A|$ for each $S \in \gamma \vee \gamma'$-$SO(X, x)$.*

Definition 3.19. *In a bioperation-topological space $(X, \tau, \gamma, \gamma')$, a point x is said to be a $\gamma \vee \gamma'$-semiadherent point of a filterbase $\mathcal{F}$ on X if it lies in the $\gamma \vee \gamma'$-semiclosure of all sets of $\mathcal{F}$.*

Theorem 3.20. *A bioperation-topological space $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact if, and only if each infinite subset of X has a $\gamma \vee \gamma'$-semicomplete accumulation point.*

Proof. Let $(X, \tau, \gamma, \gamma')$ be a $\gamma \vee \gamma'$-semicompact space and A an infinite subset of X. Let K be the set of all points x in X which are not $\gamma \vee \gamma'$-semicomplete accumulation points of A. Now it is obvious that for each point x in K, we are able to find $U(x) \in \gamma \vee \gamma'$-$SO(X, x)$ such that $|A \cap U(x)| \neq |A|$. If K is the whole space X, then $\mathcal{C} = \{U(x) : x \in X\}$ is a $\gamma \vee \gamma'$-semiopen cover of X. By the hypothesis X is $\gamma \vee \gamma'$-semicompact, so there exists a finite subcover $\mathcal{G} = \{U(x_i) : i = 1, 2, ...n\}$ such that

$A \subset \cup\{U(x_i) \cap A : i = 1, 2, ...n\}$. Then $|A| = max\{|U(x_i) \cap A| : i = 1, 2, ...n\}$ which does not agree with what we assumed. This implies that A has a $\gamma \vee \gamma'$-semicomplete accumulation point. Now assume that X is not $\gamma \vee \gamma'$-semicompact and that every infinite subset A of X has a $\gamma\vee\gamma'$-semicomplete accumulation point in X. It follows that, there exists a $\gamma\vee\gamma'$-semiopen cover $\mathcal{S}$ with no finite subcover. Set $\alpha = min\{|\Psi|$: $\Psi \subset \mathcal{S}$, where Ψ is a $\gamma \vee \gamma'$-semiopen cover of X$\}$. Fix $\Psi \subset \mathcal{S}$ for which $|\Psi| = \alpha$ and $\cup\{U : U \in \Psi\} = X$. Then, by hypothesis $\alpha \geq |\mathbb{N}|$, where $\mathbb{N}$ denotes the set of all natural numbers. By well-ordering of Ψ by some minimal well-ordering "$\sim$", suppose that U is any member of Ψ. By minimal well-ordering "$\sim$" we have $|\{V : V \in \Psi, V \sim U\}| < |\{V : V \in \Psi\}|$. Since Ψ can not have any subcover with cardinality less than α, then for each $U \in \Psi$ we have $X \neq \cup\{V : V \in \Psi, V \sim U\}$. For each $U \in \Psi$, choose a point $x(U) \in X\backslash \cup\{V \cup \{x(V)\} : V \in \Psi, V \sim U\}$. We are always able to do this if not one can choose a cover of smaller cardinality from Ψ. If $H = \{x(U) : U \in \Psi\}$, then to finish the proof we will show that H has no $\gamma\vee\gamma'$-semicomplete accumulation point in X. Suppose that $z \in X$. Since Ψ is a $\gamma\vee\gamma'$-semiopen cover of X, z is a point of some set, say, W in Ψ. By the fact that $U \sim W$, we have $x(U) \in W$. It follows that $T = \{U : U \in \Psi$ and $x(U) \in W\} \subset \{V : V \in \Psi, V \sim W\}$. But $|T| < \alpha$. Therefore, $|H \cap W| < \alpha$. But $|H| = \alpha \geq |\mathbb{N}|$, since for two distinct points U and W in Ψ, we have $x(U) \neq x(W)$. This means that H has no $\gamma \vee \gamma'$-semicomplete accumulation point in X, which contradicts our assumptions. Therefore, X is $\gamma \vee \gamma'$-semicompact. □

Theorem 3.21. *A bioperation-topological space $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact if, and only if every net in X with a well-ordered directed set as its domain $\gamma \vee \gamma'$-semiaccumulates to some point of X.*

Proof. Suppose that X is $\gamma \vee \gamma'$-semicompact and $A = \{x_\alpha : \alpha \in \Lambda\}$ a net with a well-ordered directed set Λ as domain. Assume that A has no $\gamma \vee \gamma'$-semiadherent point in X. Then for each $x \in X$, there exists $V(x) \in \gamma \vee \gamma'$-$SO(X, x)$ and an $\alpha(x) \in \Lambda$ such that $V(x) \cap \{x_\alpha : \alpha \geq \alpha(x)\} = \emptyset$. This implies that $\{x_\alpha : \alpha \geq \alpha(x)\}$ is a subset of $X\backslash V(x)$. Then the collection $\mathcal{C} = \{V(x) : x \in X\}$ is a $\gamma \vee \gamma'$-semiopen cover of X. Since X is $\gamma \vee \gamma'$-semicompact, $\mathcal{C}$ has a finite subfamily $\{V(x_i) : i = 1, 2, ...n\}$ such that $X = \bigcup_{i=1}^{n} V(x_i)$. Suppose that the corresponding elements of Λ be $\{\alpha(x_i)\}$, where $i = 1, 2, ...n$. Since Λ is well-ordered and $\{\alpha(x_i) : i = 1, 2, ...n\}$ is finite, the largest element of $\{\alpha(x_i)\}$ exists. Suppose it is $\{\alpha(x_l)\}$. Then for $\beta \geq \{\alpha(x_l)\}$, we have $\{x_\delta : \delta \geq \beta\} \subset \bigcap_{i=1}^{n}\{X\backslash V(x_i)\} = X\backslash \bigcup_{i=1}^{n} V(x_i) = \emptyset$, which is impossible. This shows that A has at least one $\gamma\vee\gamma'$-semiadherent point in X. Conversely, suppose that S is an infinite subset of X. According to Zorn's Lemma, the infinite set S can be well-ordered. This means that we can assume S to be a net with a domain which is a well-ordered index set. It follows that S has a $\gamma \vee \gamma'$-semiadherent point z. Therefore, z is a $\gamma \vee \gamma'$-semicomplete accumulation point of S. This shows that X is $\gamma \vee \gamma'$-semicompact. □

Theorem 3.22. *A bioperation-topological space $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact if, and only if each filterbase in X has at least one $\gamma \vee \gamma'$-semiadherent point.*

Proof. Suppose that X is $\gamma\vee\gamma'$-semicompact and $\mathcal{F} = \{F_\alpha : \alpha \in \Delta\}$ a filterbase in it. Since all finite intersections of F_α's are nonempty, it follows that all finite intersections of $\gamma \vee \gamma'$-$s\,\mathrm{Cl}(F_\alpha)$'s are also nonempty. By finite intersection property, $\bigcap_{\alpha\in\Delta} \gamma \vee \gamma'$-$s\,\mathrm{Cl}(F_\alpha) \neq \emptyset$. This implies that $\mathcal{F}$ has at least one $\gamma \vee \gamma'$-semiadherent point. Now suppose that $\mathcal{F}$ is any family of $\gamma \vee \gamma'$-semiclosed sets. Let each finite intersection be nonempty, the set F_α with their finite intersection establish the filterbase $\mathcal{F}$. Therefore, $\mathcal{F}$ $\gamma \vee \gamma'$-semiaccumulates to some point z in X. It follows that $z \in \bigcap_{\alpha\in\Delta} F_\alpha$. Now we have by Theorem 3.9 (3), X is $\gamma \vee \gamma'$-semicompact. □

Theorem 3.23. *A bioperation-topological space $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semicompact if, and only if each filterbase on X, with at most $\gamma \vee \gamma'$-semiadherent point, is $\gamma \vee \gamma'$-semiconvergent.*

Proof. Suppose that X is $\gamma\vee\gamma'$-semicompact, $x \in X$ and $\mathcal{F}$ is a filterbase on X. The $\gamma\vee\gamma'$-semiadherent of $\mathcal{F}$ is a subset of $\{x\}$. Then the $\gamma\vee\gamma'$-semiadherent of $\mathcal{F}$ is equal to $\{x\}$ by Theorem 3.22. Assume that there exists $V \in \gamma\vee\gamma'$-$SO(X, x)$ such that for all $F \in \mathcal{F}$, $F \cap (X \backslash V) \neq \emptyset$. Then $\Psi = \{F \backslash V : F \in \mathcal{F}\}$ is a filterbase on X. It follows that the $\gamma \vee \gamma'$-semiadherent of Ψ is nonempty. However, $\bigcap_{F \in \mathcal{F}} \gamma \vee \gamma'\text{-}s\operatorname{Cl}(F \backslash V) \subset (\bigcap_{F \in \mathcal{F}} \gamma \vee \gamma'\text{-}s\operatorname{Cl}(F)) \cap (X \backslash V) = \{x\} \cap (X \backslash V) = \emptyset$, a contradiction. Hence for each $V \in \gamma \vee \gamma'\text{-}SO(X, x)$, there exists $F \in \mathcal{F}$ such that $F \subset V$. This shows that $\mathcal{F}$ $\gamma \vee \gamma'$-semiconverges to x. Conversly, assume that $\mathcal{F}$ is a filterbase on X with no $\gamma \vee \gamma'$-semiadherent point. By hypothesis, $\mathcal{F}$ $\gamma \vee \gamma'$-semiconverges to some point z in X. Suppose F_α is an arbitrary element of $\mathcal{F}$. Then for each $V \in \gamma \vee \gamma'\text{-}SO(X, z)$, there exists a $F_\beta \in \mathcal{F}$ such that $F_\beta \subset V$. Since $\mathcal{F}$ is a filterbase, there exists a δ such that $F_\delta \subset F_\alpha \cap F_\beta \subset F_\alpha \cap V$, where $F_\delta \neq \emptyset$. This means that $F_\alpha \cap V \neq \emptyset$ for every $V \in \gamma \vee \gamma'\text{-}SO(X, z)$ and correspondingly for each α, $z \in \gamma \vee \gamma'\text{-}s\operatorname{Cl}(F_\alpha)$. It follows that $z \in \bigcap_{\alpha} \gamma \vee \gamma'\text{-}s\operatorname{Cl}(F_\alpha)$. Therefore, z is a $\gamma \vee \gamma'$-semiadherent point of $\mathcal{F}$, a contradiction. This shows that X is $\gamma \vee \gamma'$-semicompact. □

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