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Difference Cordial Labeling of 2-tuple Graphs of Some Graphs

Research Article

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Abstract: In this paper, we introduce new notion called 2-tuple graph of an undirected simple graph and we investigate the difference cordiality of 2-tuple graphs of Crown Cr_n, Wheel W_n, Ladder graph $P_n \times P_2$, Triangular snake T_n, Quadrilateral snake Q_n,Middle graph of path P_n and cycle C_n etc.

MSC: 05C78.

Keywords: Triangular snake, Quadrilateral snake,Crown,Ladder,Wheel,Path,Cycle.

1. Introduction

Through out this paper we consider simple, finite, connected and undirected graphs. $G = (V(G), E(G))$ with p vertices and q edges. G is also called a (p, q) graph. Graph labeling, where the vertices are assigned values to certain conditions have been motivated by practical problems as labeled graphs play vital role in the study of X-rays , crystallography , to determine optimal circuit layouts. I.Cahit [1] introduced the concept of cordial labeling of graphs. Subsequently K-product labeling has been introduced by R.Ponraj *et al.* [2] Motivated by these works R.Ponraj *et al.* [3] introduced notion of difference cordial labeling. R.Ponraj , S.Sathish Narayanan and R.Kala [4–6] investigated difference cordial behavior of various graphs in the context of graph operations. For a dynamic survey of various graph labeling problems along with extensive bibliography we refer to Gallian [7].

In this paper we introduce the notion of 2-tuple graphs and we investigate difference cordial behavior of 2-tuple graphs of various graphs. For standard terminology and notations we follow Gross and Yellen [8]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

Definition 1.1. *A function f is called difference cordial labeling of a graph G if $f : V(G) \to \{1, 2, \ldots, p\}$is bijective and the induced function $f^* : E(G) \to \{0\ , 1\}$ defined as $f^*(e = uv) = 1$; if $|f(u) - f(v)| = 1$ and $f^*(e = uv) = 0$; otherwise,such that $|e_{f^*}(0) - e_{f^*}(v)| \leq 1$ .Where $e_{f^*}(0)$ and $e_{f^*}(1)$ denote the number of edges labeled with 0 and 1 respectively. A graph which admits difference cordial labeling is called a difference cordial graph.*

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Definition 1.2. *Let $G = (V, E)$ be a simple graph and $G' = (V', E')$ be another copy of graph G. Join each vertex v of G to the corresponding vertex v' of G' by an edge. The new graph thus obtained we call 2- tuple graph of G. We denote 2-tuple graph of G by the notation $T^2(G)$. Further we note that if $G = (p, q)$ then $\left|V\left(T^2\left(G\right)\right)\right| = 2p$ and $\left|E\left(T^2\left(G\right)\right)\right| = 2q + p$.*

Definition 1.3. *The middle graph $M(G)$ of a graph G is a graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is vertex of G and other is an edge incident on it.*

Definition 1.4. *The crown graph $Cr_n = C_n \odot K_1$ is obtained by joining a pendant edge to each vertex of cycle C_n.*

Definition 1.5. *The triangular snake graph T_n is obtained from the path P_n by replacing each edge v_iv_{i+1} by cycle $C_3(v_iv_i'v_{i+1})$ for $1 \le i \le n-1$.*

Definition 1.6. *The quadrilateral snake graph Q_n is obtained from the path P_n by replacing each edge v_iv_{i+1} by cycle $C_4(v_iv_i'v"_iv_{i+1})$ for $1 \le i \le n-1$.*

Definition 1.7. *The Ladder graph L_n is defined as $P_n \times P_2$.*

Proposition 1.8 ([3])**.** *If G is a (p, q) difference cordial graph then $q \le 2p - 1$.*

2. Main Results

Theorem 2.1. *Let G be a (p, q) graph. If $T^2(G)$ is difference cordial then $2q \le 3p - 1$.*

Proof. Let G be a (p, q) graph. Take two copies of G, in order to obtain $T^2(G)$. For convenience, we say $G' = T^2(G)$. Clearly, $|V(G')| = 2p$ and $|E(G')| = 2q + p$. If we consider the graph G' to be difference cordial then, by Proposition 1.8, $2q + p \le 2(2p) - 1 \Rightarrow 2q + p \le 4p - 1$. Hence $2q \le 3p - 1$. □

Theorem 2.2. *$T^2(M(P_n))$ is difference cordial graph.*

Proof. Consider two copies of $M(P_n)$. Let u_1,e_1, u_2,e_2,..., e_{n-1},u_n be the vertices of the first copy of $M(P_n)$ and v_1,e_1', v_2,e_2',...,e_{n-1}', v_n be the vertices of the second copy of $M(P_n)$. Let G be the graph $T^2(M(P_n))$. Then $|V(G)| = 4n - 2$ and $|E(G)| = 8n - 9$. Define $f : V(G) \to \{1, 2, 3, \dots, 4n - 2\}$ as follows.

$$f(u_i) = 2i - 1;\ \ 1 \le i \le n$$
$$f(e_i) = 2i;\ \ 1 \le i \le n-1$$
$$f(v_i) = 2n - 2 + 2i;\ \ 1 \le i \le n$$
$$f(e_i') = 2n - 1 + 2i;\ \ 1 \le i \le n-1$$

With this labeling pattern we have,

$$e_{f^*}(0) = 4n - 5$$
$$e_{f^*}(1) = 4n - 4$$

In view of the above defined labeling pattern f is difference cordial labeling for $T^2(M(P_n))$. Hence $T^2(M(P_n))$ is difference cordial graph. □

Illustration 2.3. *Difference cordial labeling of the graph $T^2(M(P_4))$ is shown in Figure 1.*

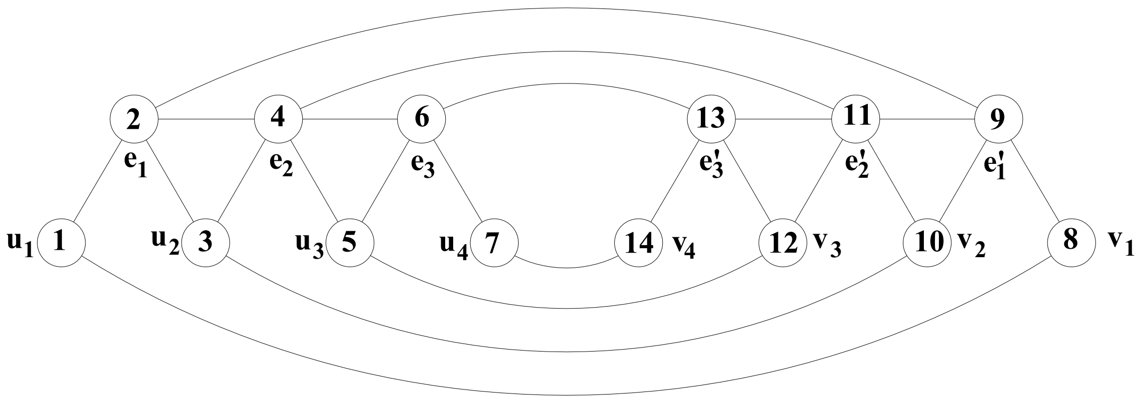

Figure 1. **The graph $T^2(M(P_4))$ and its difference cordial labeling.**

Theorem 2.4. *$T^2(M(C_n))$ is not difference cordial graph.*

Proof. Let $G = M(C_n)$. Then we have $|V(G)| = 2n$ and $|E(G)| = 3n$. Suppose $T^2(G)$ is difference cordial then, by Theorem 2.1, $2q \leq 3p - 1$. This implies $6n \leq 6n - 1$. This is impossible. Hence $T^2(M(C_n))$ is not difference cordial graph. □

Theorem 2.5. *$T^2(W_n)$ is not difference cordial graph.*

Proof. Consider $G = W_n$. Then we have $|V(G)| = n + 1$ and $|E(G)| = 2n$. Suppose $T^2(G)$ is difference cordial. Then, by Theorem 2.1, $2q \leq 3p - 1$. This implies, $4n \leq 3(n+1) - 1$. Hence, $4n \leq 3n + 2$. Thus we have, $n \leq 2$. This is impossible. Hence $T^2(W_n)$ is not difference cordial graph. □

Theorem 2.6. *$T^2(Cr_n)$ is difference cordial graph.*

Proof. Consider two copies of crown Cr_n. Let $u_1, u_2, \ldots, u_n, u'_1, u'_2, \ldots, u'_n$ be the vertices of the first copy of Cr_n and $v_1, v_2, \ldots, v_n, v'_1, v'_2, \ldots, v'_n$ be the vertices of the second copy of Cr_n. Let G be the graph $T^2(Cr_n)$. Then $|V(G)| = 4n$ and $|E(G)| = 6n$. Define $f : V(G) \to \{1, 2, 3, \ldots, 4n\}$ as follows.

$$f(u_i) = i; \;\; 1 \leq i \leq n$$

$$f(u'_i) = 2n + 2i; \;\; 1 \leq i \leq n$$

$$f(v_i) = 2n + 1 - i; \;\; 1 \leq i \leq n$$

$$f(v'_i) = 2n - 1 + 2i; \;\; 1 \leq i \leq n$$

With this labeling pattern we have,

$$e_{f^*}(0) = 3n$$

$$e_{f^*}(1) = 3n$$

In view of the above defined labeling pattern f is a difference cordial labeling for $T^2(Cr_n)$. Hence $T^2(Cr_n)$ is a difference cordial graph. □

Illustration 2.7. *Difference cordial labeling of the graph $T^2(Cr_3)$ is shown in Figure 2.*

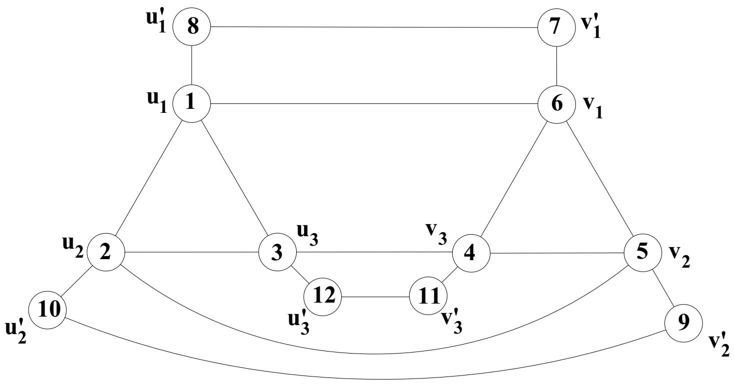

Figure 2. **The graph $T^2(Cr_3)$ and its difference cordial labeling.**

Theorem 2.8. *$T^2(P_n \times P_2)$ is difference cordial graph.*

Proof. Consider two copies of $P_n \times P_2$. Let u_1,u'_1, u_2,u'_2,..., u_n,u'_n be the vertices of the first copy of $P_n \times P_2$ and v_1,v'_1, v_2,v'_2,...,v_n, v'_n be the vertices of the second copy of $P_n \times P_2$. Let G be the graph $T^2(P_n \times P_2)$. Then $|V(G)| = 4n$ and $|E(G)| = 6n - 4$. Define $f : V(G) \to \{1, 2, 3, \ldots, 4n\}$ as follows.

$$f(u_i) = i; \;\; 1 \leq i \leq n$$

$$f(u'_i) = 3n + i; \;\; 1 \leq i \leq n - 2$$

$$f(u'_{n-2+i}) = 4n + 1 - i; \;\; 1 \leq i \leq 2$$

$$f(v_i) = 2n + 1 - i; \;\; 1 \leq i \leq n$$

$$f(v'_i) = 2n + i; \;\; 1 \leq i \leq n$$

With this labeling pattern we have,

$$e_{f^*}(0) = 4n - 2$$

$$e_{f^*}(1) = 4n - 2$$

In view of the above defined labeling pattern f is difference cordial labeling for $T^2(P_n \times P_2)$. Hence $T^2(P_n \times P_2)$ is difference cordial graph. □

Illustration 2.9. *Difference cordial labeling of the graph $T^2(P_4 \times P_2)$ is shown in Figure 3.*

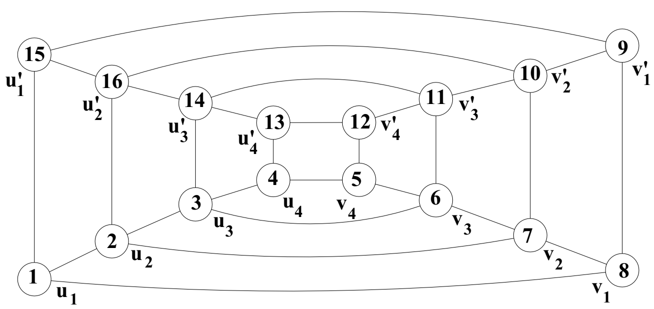

Figure 3. **The graph $T^2(P_4 \times P_2)$ and its difference cordial labeling.**

Theorem 2.10. *$T^2(T_n)$ is difference cordial graph.*

Proof. Consider two copies of Triangular snake T_n. Let u_1,u_1', u_2,...,u_{n-1}', u_n be the vertices of the first copy of T_n and v_1,v_1', v_2,...,v_{n-1}', v_n be the vertices of the second copy of T_n. Let G be the graph $T^2(T_n)$. Then $|V(G)| = 4n-2$ and $|E(G)| = 8n-7$. Define $f : V(G) \to \{1, 2, 3, \ldots, 4n-2\}$ as follows.

$$f(u_i) = 2i - 1; \ \ 1 \le i \le n$$

$$f(u_i') = 2i; \ \ 1 \le i \le n-1$$

$$f(v_i) = 4n - 2i; \ \ 1 \le i \le n$$

$$f(v_i') = 4n - 1 - 2i; \ \ 1 \le i \le n-1$$

With this labeling pattern we have,

$$e_{f^*}(0) = 4n - 4$$

$$e_{f^*}(1) = 4n - 3$$

In view of the above defined labeling pattern f is a difference cordial labeling for $T^2(T_n)$. Hence $T^2(T_n)$ is a difference cordial graph. □

Illustration 2.11. *Difference cordial labeling of the graph $T^2(T_5)$ is shown in Figure 4.*

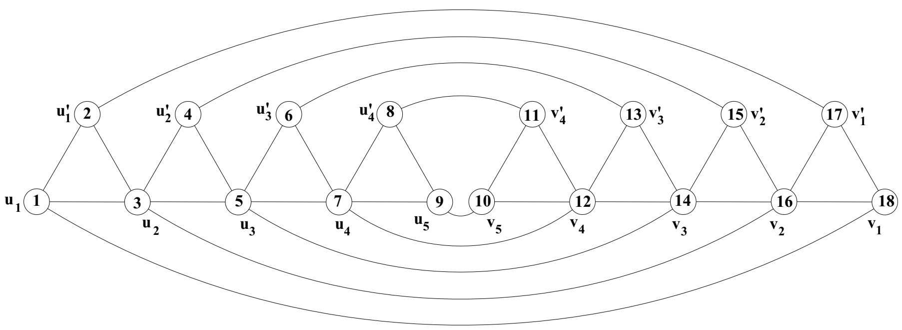

Figure 4. **The graph $T^2(T_5)$ and its difference cordial labeling.**

Theorem 2.12. *$T^2(Q_n)$ is difference cordial graph.*

Proof. Consider two copies of Quadrilateral snake Q_n. Let u_1,u_1',u_1'', u_2,u_2',u_2'',...,u_{n-1},u_{n-1}',u_{n-1}'' , u_n be the vertices of the first copy of T_n and v_1,v_1',v_1'', v_2,v_2',v_2'',...,v_{n-1},v_{n-1}',v_{n-1}'',v_n be the vertices of the second copy of Q_n. Let G be the graph $T^2(Q_n)$. Then $|V(G)| = 6n-4$ and $|E(G)| = 11n-10$. To define $f : V(G) \to \{1, 2, 3, \ldots, 6n-4\}$ we consider following six cases.

Case (i) $n \equiv 0 \,(\mathrm{mod}6)$

$$f(u_i) = 3i - 2 \ ; \ 1 \le i \le n$$

$$f(u_i') = 3i - 1 \ ; \ 1 \le i \le n-1$$

$$f(u_i'') = 3i \ ; \qquad 1 \le i \le n-1$$

$$
\begin{aligned}
&f(v_i) = 3\text{n} - 4 + 3\text{i}; && 1 \le i \le \tfrac{5n}{6}\\
&f\left(v_{\frac{5n}{6}-1+2i}\right) = 6n - 2 - 3i\ ; && 1 \le i \le \left\lceil \tfrac{n}{12} \right\rceil\\
&f\left(v_{\frac{5n}{6}+2i}\right) = \tfrac{11n}{2} - 4 + 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n}{12} \right\rfloor
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i') = 3\text{n} - 3 + 3\text{i}; && 1 \le i \le \tfrac{5n}{6} - 1\\
&f\left(v'_{\frac{5n}{6}-2+2i}\right) = 6n - 1 - 3i\ ; && 1 \le i \le \left\lceil \tfrac{n}{12} \right\rceil\\
&f\left(v'_{\frac{5n}{6}-1+2i}\right) = \tfrac{11n}{2} - 5 + 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n}{12} \right\rfloor
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i'') = 3\text{n} - 2 + 3\text{i}; && 1 \le i \le \tfrac{5n}{6} - 1\\
&f\left(v''_{\frac{5n}{6}-2+2i}\right) = \tfrac{11n}{2} - 6 + 3i\ ; && 1 \le i \le \left\lceil \tfrac{n}{12} \right\rceil\\
&f\left(v''_{\frac{5n}{6}-1+2i}\right) = 6n - 3 - 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n}{12} \right\rfloor
\end{aligned}
$$

Case (ii) $n \equiv 1\,(\text{mod}6)$

$$
\begin{aligned}
&f(u_i) = 3i - 2\ ; && 1 \le i \le n\\
&f(u_i') = 3i - 1\ ; && 1 \le i \le n - 1\\
&f(u_i'') = 3i\ ; && 1 \le i \le n - 1
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i) = 3\text{n} - 4 + 3\text{i}; && 1 \le i \le \tfrac{5n-5}{6}\\
&f\left(v_{\frac{5n-5}{6}-1+2i}\right) = 6n - 1 - 3i\ ; && 1 \le i \le \left\lceil \tfrac{n+5}{12} \right\rceil\\
&f\left(v_{\frac{5n-5}{6}+2i}\right) = \tfrac{11n}{2} - \tfrac{11}{2} + 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n+5}{12} \right\rfloor
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i{}') = 3\text{n} - 3 + 3\text{i}; && 1 \le i \le \tfrac{5n-5}{6}\\
&f\left(v'_{\frac{5n-5}{6}-1+2i}\right) = \tfrac{11n}{2} - \tfrac{13}{2} + 3i\ ; && 1 \le i \le \left\lceil \tfrac{n-1}{12} \right\rceil\\
&f\left(v'_{\frac{5n-5}{6}+2i}\right) = 6n - 3 - 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n-1}{12} \right\rfloor
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i'') = 3\text{n} - 2 + 3\text{i}; && 1 \le i \le \tfrac{5n-5}{6}\\
&f\left(v''_{\frac{5n-5}{6}-1+2i}\right) = 6n - 2 - 3i\ ; && 1 \le i \le \left\lceil \tfrac{n-1}{12} \right\rceil\\
&f\left(v''_{\frac{5n-5}{6}+2i}\right) = \tfrac{11n}{2} - \tfrac{9}{2} + 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n-1}{12} \right\rfloor
\end{aligned}
$$

Case (iii) $n \equiv 2\,(\text{mod}6)$

$$
\begin{aligned}
&f(u_i) = 3i - 2\ ; && 1 \le i \le n\\
&f(u_i') = 3i - 1\ ; && 1 \le i \le n - 1\\
&f(u_i'') = 3i\ ; && 1 \le i \le n - 1
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i) = 3\text{n} - 4 + 3\text{i}; && 1 \le i \le \tfrac{5n-4}{6}\\
&f\left(v_{\frac{5n-4}{6}-1+2i}\right) = 6n - 1 - 3i\ ; && 1 \le i \le \left\lceil \tfrac{n+4}{12} \right\rceil\\
&f\left(v_{\frac{5n-4}{6}+2i}\right) = \tfrac{11n}{2} - 5 + 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n+4}{12} \right\rfloor
\end{aligned}
$$

$$
\begin{aligned}
&f(v_i{}') = 3\text{n} - 3 + 3\text{i}; && 1 \le i \le \tfrac{5n-4}{6}\\
&f\left(v'_{\frac{5n-4}{6}-1+2i}\right) = \tfrac{11n}{2} - 6 + 3i\ ; && 1 \le i \le \left\lceil \tfrac{n-2}{12} \right\rceil\\
&f\left(v'_{\frac{5n-4}{6}+2i}\right) = 6n - 3 - 3i\ ; && 1 \le i \le \left\lfloor \tfrac{n-2}{12} \right\rfloor
\end{aligned}
$$

$$f\left(v_i''\right) = 3\text{n} - 2 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-4}{6}$$
$$f\left(v''_{\frac{5n-4}{6}-1+2i}\right) = 6n - 2 - 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n-2}{12} \right\rceil$$
$$f\left(v''_{\frac{5n-4}{6}+2i}\right) = \tfrac{11n}{2} - 4 + 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n-2}{12} \right\rfloor$$

Case (iv) $n \equiv 3\,(\text{mod}6)$

$$f(u_i) = 3i - 2\ ; \quad 1 \le i \le n$$
$$f(u_i') = 3i - 1\ ; \quad 1 \le i \le n-1$$
$$f(u_i'') = 3i\ ; \quad 1 \le i \le n-1$$

$$f\left(v_i\right) = 3\text{n} - 4 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-3}{6}$$
$$f\left(v_{\frac{5n-3}{6}-1+2i}\right) = \tfrac{11n}{2} - \tfrac{13}{2} + 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n+3}{12} \right\rceil$$
$$f\left(v_{\frac{5n-3}{6}+2i}\right) = 6n - 3 - 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n+3}{12} \right\rfloor$$

$$f\left({v_i}'\right) = 3\text{n} - 3 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-3}{6}$$
$$f\left(v'_{\frac{5n-3}{6}-1+2i}\right) = 6n - 2 - 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n-3}{12} \right\rceil$$
$$f\left(v'_{\frac{5n-3}{6}+2i}\right) = \tfrac{11n}{2} - \tfrac{9}{2} + 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n-3}{12} \right\rfloor$$

$$f\left(v_i''\right) = 3\text{n} - 2 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-9}{6}$$
$$f\left(v''_{\frac{5n-9}{6}-1+2i}\right) = 6n - 1 - 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n+3}{12} \right\rceil$$
$$f\left(v''_{\frac{5n-9}{6}+2i}\right) = \tfrac{11n}{2} - \tfrac{11}{2} + 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n+3}{12} \right\rfloor$$

Case (v) $n \equiv 4\,(\text{mod}6)$

$$f(u_i) = 3i - 2\ ; \quad 1 \le i \le n$$
$$f(u_i') = 3i - 1\ ; \quad 1 \le i \le n-1$$
$$f(u_i'') = 3i\ ; \quad 1 \le i \le n-1$$

$$f\left(v_i\right) = 3\text{n} - 4 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-2}{6}$$
$$f\left(v_{\frac{5n-2}{6}-1+2i}\right) = \tfrac{11n}{2} - 6 + 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n+2}{12} \right\rceil$$
$$f\left(v_{\frac{5n-2}{6}+2i}\right) = 6n - 3 - 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n+2}{12} \right\rfloor$$

$$f\left({v_i}'\right) = 3\text{n} - 3 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-2}{6}$$
$$f\left(v'_{\frac{5n-2}{6}-1+2i}\right) = 6n - 2 - 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n-4}{12} \right\rceil$$
$$f\left(v'_{\frac{5n-2}{6}+2i}\right) = \tfrac{11n}{2} - 4 + 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n-4}{12} \right\rfloor$$

$$f\left(v_i''\right) = 3\text{n} - 2 + 3\text{i}; \quad 1 \le i \le \tfrac{5n-8}{6}$$
$$f\left(v''_{\frac{5n-8}{6}-1+2i}\right) = 6n - 1 - 3i\ ; \quad 1 \le i \le \left\lceil \tfrac{n+2}{12} \right\rceil$$
$$f\left(v''_{\frac{5n-8}{6}+2i}\right) = \tfrac{11n}{2} - 5 + 3i\ ; \quad 1 \le i \le \left\lfloor \tfrac{n+2}{12} \right\rfloor$$

Case (vi) $n \equiv 5\,(\text{mod}6)$

$$f(u_i) = 3i - 2\ ;\ 1 \le i \le n$$
$$f(u_i') = 3i - 1\ ;\ 1 \le i \le n-1$$
$$f(u_i'') = 3i\ ; \quad 1 \le i \le n-1$$

$$f(v_i) = 3n - 4 + 3i; \qquad 1 \le i \le \tfrac{5n-1}{6}$$
$$f\left(v_{\frac{5n-1}{6}-1+2i}\right) = 6n - 2 - 3i\ ; \qquad 1 \le i \le \left\lceil \tfrac{n+1}{12} \right\rceil$$
$$f\left(v_{\frac{5n-1}{6}+2i}\right) = \tfrac{11n}{2} - \tfrac{9}{2} + 3i\ ; \qquad 1 \le i \le \left\lfloor \tfrac{n+1}{12} \right\rfloor$$

$$f(v_i{}') = 3n - 3 + 3i; \qquad 1 \le i \le \tfrac{5n-7}{6}$$
$$f\left(v'_{\frac{5n-7}{6}-1+2i}\right) = 6n - 1 - 3i\ ; \qquad 1 \le i \le \left\lceil \tfrac{n+1}{12} \right\rceil$$
$$f\left(v'_{\frac{5n-7}{6}+2i}\right) = \tfrac{11n}{2} - \tfrac{11}{2} + 3i\ ; \qquad 1 \le i \le \left\lfloor \tfrac{n+1}{12} \right\rfloor$$

$$f(v_i'') = 3n - 2 + 3i; \qquad 1 \le i \le \tfrac{5n-7}{6}$$
$$f\left(v''_{\frac{5n-7}{6}-1+2i}\right) = \tfrac{11n}{2} - \tfrac{13}{2} + 3i\ ; \ 1 \le i \le \left\lceil \tfrac{n+1}{12} \right\rceil$$
$$f\left(v''_{\frac{5n-7}{6}+2i}\right) = 6n - 3 - 3i\ ; \qquad 1 \le i \le \left\lfloor \tfrac{n+1}{12} \right\rfloor$$

With this labeling pattern we have, In view of the above defined labeling pattern f admits difference cordial labeling for

Nature of n	$e_{f^*}(0)$	$e_{f^*}(1)$
$n \equiv 0\,(\mathrm{mod}2)$	$\frac{11n-10}{2}$	$\frac{11n-10}{2}$
$n \equiv 1\,(\mathrm{mod}2)$	$\frac{11n-9}{2}$	$\frac{11n-11}{2}$

$T^2(Q_n)$. Hence $T^2(Q_n)$ is a difference cordial graph. □

Illustration 2.13. *Difference cordial labeling of graph $T^2(Q_4)$ is shown in Figure 5.*

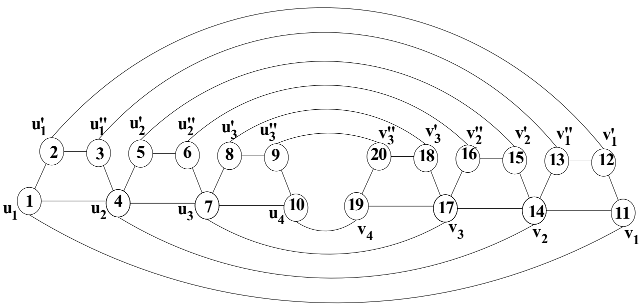

Figure 5. **The graph $T^2(Q_4)$ and its difference cordial labeling.**

3. Concluding Remarks

In this paper, we have defined 2-tuple graphs of simple graphs. We discussed behavior of 2-tuple graphs of some graphs in the context of difference cordial labeling.

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