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A Study on Skin Cancer Using Fractals

Research Article

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Abstract: This paper depicts the study of variations in the skin lesion images. The skin cancer images have been analysed for benign, atypical and malignant melanoma types using fractals.

Keywords: Fractals, Skin cancer, Box-Counting dimension and Sausage Dimension.

1. Introduction

Fractals : A fractal is an object which appears self-similar under varying degrees of magnification, and also an object with its own fractal dimension. Fractals themselves have their own dimension, which is usually a non-integer dimension that is greater than their topological dimension D_T and less than their Euclidean dimension D_E. Self-similarity is the major characteristic of the fractal objects [4]. Fractal objects and process are therefore said to display self-invariant (self-similar or self-affine) properties. Recent studies have attempted with some success to characterize certain parts of the body using fractal geometry [7]. There are many definitions of fractal, among them

(1). A fractal is a shape made of parts similar to the whole in some way.

(2). A fractal is by definition for which the Hausdorff dimension strictly exceeds the Topological dimension.

Skin Cancer : Skin cancer is a disease in which skin cells lose the ability to divide and grow normally. Healthy skin cells normally divide in an orderly way to replace dead cells and grow new skin. Abnormal cells can grow out of control and form a mass or 'tumor'. When abnormal cells originate in the skin, the mass is called a skin tumor [2, 3].

2. Methods

The Fractal Dimension of Color Images and Irregular Border : The concept Fractal was first introduced by Mandelbrot. The fractal dimension has been used in image classification to measure surface roughness where different natural scenes such as mountains, clouds, trees and deserts generate different fractal dimension.
In the box counting method, an image measuring size $R \times R$ pixels is scaled down to $s \times s$ where $1 \leq s \leq \frac{R}{2}$ and s is an integer. Then the image is treated as two dimensional and the coordinates are defined as (x, y). Then (x, y) is partitioned

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into grids which measures $s \times s$. If the minimum and the maximum binary image levels in the (i, j) grid fall into the k^{th} and l^{th} boxes respectively, the contribution of n_r in the (i, j), grid is defined as

$$n_r(i, j) = l - k + 1 \tag{1}$$

In this method N_r is defined as the summation of the contributions from all the grids that are located in a window of the image

$$N_r = \sum_{i,j} n_r(i, j) \tag{2}$$

If N_r is computed for different values of r, then the fractal dimension can be estimated as the slope of the line that best fits pts $\left(\log\left(\frac{1}{r}\right), \log N_r\right)$.

Algorithm :

Step : 1 The image is divided into regular meshes with a mesh size of r.

Step : 2 Count the number of square boxes that intersect with the image N_r.

Step : 3 The number N_r is dependent on the choice of r.

Step : 4 We repeat for several size values and count the corresponding number N_r.

Step : 5 We plot the slope D formed by plotting $\log(N_r)$ against $\log\left(\frac{1}{r}\right)$.

The step 5 indicates the degree of complexity or dimensions of the fractal. Finally a straight line is fitted to the plotted points in the diagram using the least squares method. The linear regression equation used to estimate the fractal dimension is

$$\log(N_r) = \log K + D \log\left(\frac{1}{r}\right). \tag{3}$$

Where K denotes constant and D denotes the dimension of the fractal set. The above algorithm is applied to many patients

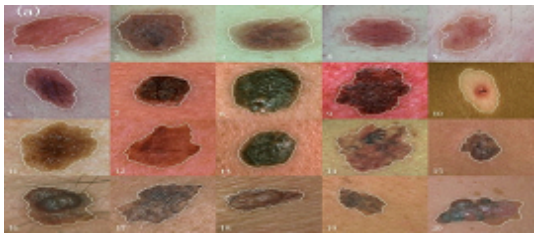

Figure 1. **Twenty images of skin lesions.**

and the dimension of their cancer cells has been found out.

Percolation Process : A single percolation cluster is generated in the way of the cancer spreading in the organ. In a square lattice, each site represents an individual which can be infected with probability (p) and which is immune with probability $(1 - p)$. At an initial time $t = 0$ the individual at the centre of the lattice (cell) is infected. We assume now that in one unit of time this infected site infects all non-immune nearest neighbor sites. In the second unit of time, these infected sites will infect all their non immune nearest neighbor sites, and soon. In this way, after **t** time steps all non immune sites of the l^{th} square grid around the cells are infected, i.e. the maximum length of the shortest path between the infected sites and the cell is $l \equiv t$. This process of randomly infecting individuals is exactly the same as that of randomly occupying sites

in the site percolation, where the sites of a lattice have been occupied randomly, when the bonds between the sites are randomly occupied, we speak of bond percolation. So far, we considered on either site or bond percolation, where either sites or bonds of a given lattice have been chosen randomly. This site bond percolation can be relevant for the spreading of the cancer in the tissue.

Algorithm :

Step : 1 The origin of an empty lattice is occupied.

Step : 2 The nearest neighbor sites from the origin are either occupied with probability (p) or blocked with probability $(1-p)$.

Step : 3 The empty nearest neighbors sites proceed as step 2 with probability (p) and blocked with probability $(1-p)$.

This process continues until no sites are available for growth. Thus percolation clusters are generated with a distribution of cluster sizes s. $sn_s(p)$. The factor s comes from the site of the cluster and has the same chance of being the origin of the cluster, and thus exactly the same cluster can be generated in s ways, enlarging the distribution $sn_s(p)$ by a factor of s.

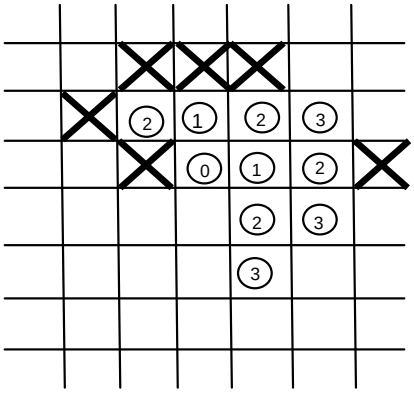

Figure 2. **First four steps for the percolation model.**

The above method is particularly useful for studying the structure and physical properties of single percolation cluster. From this the irregular border of the cancer is formed. [5] The above algorithm has been programmed and run by using C++ [1].

The parameters such as Area, Perimeter, Form factor and Invaslog has been found out using Box-Counting method and Sausage method. The importance of Invaslog has been specifically verified for the invasiveness in the skin cancer.

Boundary Descriptors : There are many features that depend on boundary descriptors of objects such as bending energy, curvature etc. For irregularly shaped object, the boundary direction is the better representation. Consecutive points on the boundary of a shape give relative position or direction. A 4- or 8- connected chain code is used to represent the boundary of an object by a connected sequence of straight lines segments, connected number schemes are used to represent the direction. The direction also represents the slope of the boundary.

The cell growth is continuous in a closed and bounded region in the tissue and is analytic and it is non-constant in the interior of the tissue. Then the cells can attain its maximum on the boundary of the tissue and never in its interior.

Results : The dimension D of the cancer cell images have been estimated using Box-counting Method (D_B) and Sausage Method (or) Boundary Dilation Method (D_s). The amount of invasiveness of the cancer cells in the skin are found out by Form Factor and Invaslog [6].

PICTURE 1 (ATYPICAL)

	Original image						Dermatologist					
Scaling	**Area**	**Perimeter**	**Total area**	**Form factor**	**Invaslog**	**Dimension D_b**	**Area**	**Perimeter**	**Total area**	**Form factor**	**Invaslog**	**Dimension D_b**
2	4631	742	5373	0.106	0.975		1180	375	1555	0.105	0.979	
3	1945	604	2549	0.067	1.174		480	261	741	0.089	1.051	
4	1046	426	1472	0.072	1.143		242	189	431	0.085	1.071	
5	635	352.6	987.6	0.064	1.194		142	152	294	0.077	1.114	
6	423	286	709	0.065	1.187	1.8	92	115	207	0.087	1.060	1.8
7	300	241	541	0.065	1.187		67	86	153	0.1138	0.944	
8	220	190	410	0.077	1.114		46	78	124	0.095	1.022	
9	162	177	339	0.065	1.187		35	61	96	0.118	0.928	
10	128	150.6	278.6	0.071	1.149		25	59	84	0.090	1.046	

Table 1. **Box-Counting Method**

	Original image				Dermatologist			
Scaling	**Area**	**Radius**	K_s	$D_s = 2 - K_s$	**Area**	**Radius**	K_s	$D_s = 2 - K_s$
3	1945	24.882			480	12.361		
5	635	14.217			142	6.723		
7	300	9.772			67	4.618		
9	162	7.181	0.67	1.33	35	3.338	0.33	1.67
11	103	5.726			21	2.585		
13	69	4.687			11	1.871		
15	47	3.868			7	1.493		
17	36	3.385			7	1.493		

Table 2. **Sausage method**

PICTURE 7 (BENIGN)

	Original image						Dermatologist					
Scaling	**Area**	**Perimeter**	**Total area**	**Form factor**	**Invaslog**	**Dimension D_b**	**Area**	**Perimeter**	**Total area**	**Form factor**	**Invaslog**	**Dimension D_b**
2	4693	1982	6675	0.015	1.824		1053	415	1468	0.077	1.114	
3	1813.6664	1442.3336	3256	0.011	1.959		406	277	683	0.066	1.181	
4	940.375	961.75	1902.1250	0.013	1.886		208	185	393	0.076	1.119	
5	563	702	1265	0.014	1.854		123	134	257	0.086	1.066	
6	373	513.1668	886.168	0.018	1.745	1.9	86	104	190	0.100	1	1.9
7	262.4286	396.8571	659.2856	0.021	1.678		59	81	140	0.113	0.947	
8	189.4688	314.6250	504.0938	0.024	1.620		43	62	105	0.141	0.851	
9	150	249.5185	399.5185	0.030	1.523		32	48	80	0.175	0.757	
10	115	210.5	325.5	0.033	1.481		24	44	68	0.156	0.807	

Table 3. **Box-Counting Method**

	Original image				Dermatologist			
Scaling	**Area**	**Radius**	K_s	$D_s = 2 - K_s$	**Area**	**Radius**	K_s	$D_s = 2 - K_s$
3	1813.6664	24.027			406	11.368		
5	563	13.387			123	6.257		
7	2624.4286	9.140			59	4.334		
9	150	6.910	0.66	1.34	32	3.192	0.32	1.68
11	91.379	5.393			21	2.585		
13	64	4.514			13	2.034		
15	46	3.827			8	1.596		
17	37	3.432			5	1.262		

Table 4. **Sausage method**

PICTURE 14 (MALIGNANTMELANOMA)

	Original image						Dermatologist					
Scaling	**Area**	**Perimeter**	**Total area**	**Form factor**	**Invaslog**	**Dimension D_b**	**Area**	**Perimeter**	**Total area**	**Form factor**	**Invaslog**	**Dimension D_b**
2	6090	2200.5	8290.5	0.016	1.80		1635	589	2224	0.059	1.229	
3	2379.33	1632.334	3999.67	0.011	1.96		647	414	1061	0.047	1.328	
4	1194	1186.5	2380.5	0.011	1.96		311	321	634	0.038	1.420	
5	669	910.200	1579.20	0.0101	1.996		181	239	420	0.040	1.398	
6	420	717.500	1137.50	0.0103	1.987	1.8	105	203	308	0.032	1.495	1.8
7	278	580	858	0.0104	1.983		68	164	232	0.032	1.495	
8	200	458.25	658.25	0.012	1.921		49	134	183	0.034	1.469	
9	135	400.1	536	0.011	1.959		32	118	150	0.029	1.538	
10	95	337	432	0.011	1.959		27	93	120	0.039	1.409	

Table 5. **Box-Counting Method**

	Original image				Dermatologist			
Scaling	**Area**	**Radius**	K_s	$D_s = 2 - K_s$	**Area**	**Radius**	K_s	$D_s = 2 - K_s$
3	2379.33	27.520			647	14.351		
5	669	14.593			181	7.590		
7	278	9.407			68	4.652		
9	135	6.555	0.57	1.43	32	3.192	0.25	1.75
11	73	4.820			16	2.257		
13	35	3.338			10	1.784		
15	23	2.706			4	0.128		
17	14	2.111			2	0.798		

Table 6. **Sausage method**

From the above tables we can find that the variation in their dimensions depicts the invasiveness of skin cancer.

3. Conclusion

A new measure of border irregularity for pigmented skin lesions based on the cell potential has been proposed. The Box Counting method gives the maximum boundary value of the irregular border of the original image.

The invasiveness of the cancer cells can be found out form their dimensions. The Sausage method have been analyzed to find the accuracy of invasiveness of cancer cells.

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