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A Time Truncated Group Sampling Plan For Compound Rayleigh Distribution

Research Article

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Abstract: In this paper, group sampling plan is proposed assuming that the life time of the test units follows Compound Rayleigh distribution and the life test is terminated at a prefixed time. The minimum sample size required for ensuring the specified mean life at specified consumer's risk have been determined. The Operating characteristics values for various quality levels are obtained and the results are discussed with the help of tables and examples. The minimum mean ratios are also obtained for a specified level of producer's risk.

Keywords: Acceptance sampling, Compound Rayleigh distribution, Producer's risk, Consumer's risk, Operating Characteristic curve, Truncated life test, Group Sampling Plan.

1. Introduction

Inspection of raw materials, semi finished products, or finished products are one aspect of quality assurance. Whenever a statistical technique is used to control, improve and maintain the quality, it is known as statistical quality control. Acceptance sampling is one of the major components in the field of statistical quality control. When inspection is for the purpose of acceptance or rejection of a product, based on adherence to a standard, the type of procedure employed is usually called acceptance sampling. The probability that a device will function over a specified time period or amount of usage at stated condition is termed as reliability. A typical application of reliability acceptance sampling is as follows:

A company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot, and some quality characteristics of the units in the sample is inspected for a specified period of time. On the basis of the information in this sample, a decision is made regarding lot disposition. Usually, this decision is either to accept or to reject the lot. Accepted lots are put into production; rejected lots may be returned to the vendor or may be subjected to some other lot disposition action. Statistical quality control is the procedure for the control of quality by the application of theory of probability to the results of the inspection of samples of the population. Sampling plans are used in the area of quality and reliability analysis. When the quality of product is related to its lifetime, it is called as life test.

Aslam, Jun and Ahmad (2009) proposed a group sampling plan based on time truncated test for gamma distributed items. Srinivasa Rao (2009) presented a group acceptance sampling plans for life times following a generalized exponential distribution. Aslam and Jun(2009) proposed a group acceptance sampling plan for truncated life tests based on the inverse

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Rayleigh distribution and log-logistic distribution(2009). Group acceptance sampling plan for truncated life tests having Weibull distribution is given by Aslam and Jun (2009). Radhakrishnan and Alagirisamy (2011) constructed attribute group acceptance sampling plan using weighted binomial distribution. Aslam, Jun and Ahmad (2011) presented a group acceptance sampling plan for generalized Rayleigh distribution. Srinivasa Rao (2011) presented a group acceptance sampling plan based on truncated life tests for the Marshall-Olkin Extended Lomax distribution. Priyah Anburajan and Sudamani Ramaswamy (2012) proposed group acceptance sampling plans using weighted Binomial on truncated life tests for Inverse Rayleigh, Log – Logistic and Marshall – Olkin Extended Distributions. Sudamani Ramaswamy & Sutharani (2012) proposed Designing Group Sampling Plan under Generalized Rayleigh Distribution using Minimum Angle Method. Sudamani Ramaswamy & Sutharani (2013) presented on Designing GSP under Generalized Exponential, Weibull and Gamma Distribution using Minimum Angle Method. Sudamani Ramaswamy and Jayasri (2014) presented time truncated group sampling plan using weighted Binomial for Various distributions.

This paper deals with the study of group sampling plan in which a multiple number of items as a group can be tested simultaneously in a tester. In most sampling plans, it is assumed that a single item is put on test in a tester. Sometimes, however, testers accommodating multiple items are available in practice because testing time and cost can be saved by testing those items simultaneously. For this type of testers, the number of items to be equipped in a tester is given by the specification. When designing a sampling plan with this type of testers, determining the sample size is equivalent to determining the number of testers. If we call items in a tester as a group, then we need to determine the number of groups as the group size. Then the sampling plan is called a group acceptance sampling plan.

The selected sample size n is distributed to g groups and r (group size) items are put on test in each groups so that n equals r times g. The r items in a group are tested simultaneously on each different tester for a pre-assigned time. The experiment is truncated if more than the acceptable number c of failures occur in any group during the experiment time. Let us assume that the life time of the items follows Compound Rayleigh distribution. The consumer's risk is given by β. The consumer's confidence level ($P^* = 1 - \beta$) can be obtained from the consumer's risk β. The sampling plan is described such that the mean life of items in a lot (μ) is greater than the specified mean life $_0$, say the lot is accepted if there is enough evidence that $\mu \geq \mu_0$ at certain levels of consumer's risk and producer's risk. Otherwise, reject the lot.

2. Compound Rayleigh Distribution

The Rayleigh distribution plays an important role in modelling the lifetime of random phenomenon. It arises in many areas of applications, including reliability, life testing and survival analysis. Bhupendra Singh, K.K. Sharma and Dushyant Tyagi (2013) have developed a reliability single sampling plan assuming that lifetimes of the test units follow Compound Rayleigh distribution and the life test is terminated at a prefixed time. This type of sampling plan is used to save the test time in practical situations.

Let X denotes a random variable arising from a Rayleigh distribution with p.d.f.

$$f(t;\theta) = 2\theta t e^{-\theta t^2} \tag{1}$$

where t > 0 is the lifetime, and $\theta > 0$. The corresponding hazard function is $h(t) = 2\theta t$, $t > 0$. The mean survival time and the cumulative distribution function of the Rayleigh model are given by

$$E(t) = \frac{1}{2}\sqrt{\frac{\pi}{\theta}} \tag{2}$$

$$F(t) = 1 - e^{-\frac{t^2}{\theta}} \tag{3}$$

In life testing experiments, it is expected that the environmental conditions can not be remained same during the testing time. Therefore, it seems logical to treat the parameters involved in the life time model as random variables. In view of this, if the parameter θ is itself a random variable, then the distribution of lifetime of each item is a Compound Rayleigh distribution. The particular form of θ, which is considered here, is the gamma p.d.f.

$$g(\theta, B, \delta) = \frac{B^{\delta}\theta^{\delta-1}e^{-B\theta}}{\Gamma\delta} \qquad \theta, B, \delta > 0 \tag{4}$$

The parameters β and δ are scale and shape parameters, respectively. The resulting Compound distribution has p.d.f.

$$\begin{aligned} f(t, \alpha, B) &= \int_0^{\infty} 2\theta\, t\, e^{-\theta t^2} \frac{B^{\delta}\theta^{\delta-1}e^{-B\theta}}{\Gamma\delta} d\theta \\ &= 2\delta B^{\delta} t \left(B + t^2\right)^{-(\delta+1)} \end{aligned} \tag{5}$$

The mean survival time and the cumulative distribution function of the Compound Rayleigh model are given by

$$\mu = E(t) = \frac{\sqrt{B\pi}\,\Gamma\left(\delta - \frac{1}{2}\right)}{2\Gamma\delta} \tag{6}$$

and

$$F(t, B, \delta) = 1 - B^{\delta}\left(B + t^2\right)^{-\delta}, \; t > 0 \tag{7}$$

Operating Procedure of Group Acceptance Sampling Plan for Truncated Life Tests

1. Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = rg$.
2. Select the acceptance number (or action limit) c for a group and the experiment time t_0.
3. Perform the experiment for the g groups simultaneously and record the number of failures for each group.
4. Accept the lot if at most c failures occur in each of all groups.
5. Truncate the experiment if more than c failures occur in any group and reject the lot.

This group sampling plan reduces to ordinary sampling plan if $r = 1$, when the sample size $n = g$. The number of groups g and the acceptance number c are determined such that they satisfies both the risks at the same time, whereas the group size and termination time t_0 are assumed to be specified. In practice, but in general c can be determined as well.

Notations

n - Sample size

g - Number of groups

r - Number of items in a group

c - Acceptance number

α - Producer's risk

β - Consumer's risk

t - Termination time

δ - Shape parameter

B - Scale parameter

p - Probability of failure before time t

p_a - Probability of acceptance of lot

μ_0 - Specified mean life

Design of the Sampling Plan

The main objective of this plan is to set a lower consumer's risk β, on the products mean lifetime and to test whether the lifetime of the product is longer than our expectation. It is assumed that the lot size is large enough to use binomial distribution to find the probability of acceptance. The probability of acceptance L(p) for this sampling plan is calculated using the following equation.

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i}\right]^g \tag{8}$$

where p is the probability that an item in a group fails before the termination time. The minimum number of groups required can be determined by considering the consumer's risk when the true mean life equals the specified mean life ($\mu = \mu_0$) (worst case) by means of the following inequality.

$$L(p_0) = \left[\sum_{i=0}^{c} \binom{r}{i} p_0^i (1-p_0)^{r-i}\right]^g \leq \beta \tag{9}$$

According to Bhupendra Singh et.al, (2013) the value of p_0 is given for the compound Rayleigh distribution as

$$\begin{aligned} p_0 &= 1 - B^{\delta}(B+t^2)^{-\delta} \\ &= 1 - \frac{1}{\left(1+\frac{t^2}{B}\right)^{\delta}} \end{aligned} \tag{10}$$

where

$$B = \left(\frac{2\mu\Gamma\delta}{\sqrt{\pi}\Gamma\left(\delta-\frac{1}{2}\right)}\right)^{\frac{1}{2}} \tag{11}$$

Substituting the value of B and $t = a\mu_0$, we get

$$p_0 = 1 - \frac{1}{1+\left(\frac{a\sqrt{\pi}\Gamma\left(\delta-\frac{1}{2}\right)}{2\frac{\mu}{\mu_0}\Gamma\delta}\right)^2} \tag{12}$$

The minimum number of groups satisfying equation (9) are obtained and presented in Table 1 for various values of β and t/μ_0. The shape parameter is fixed as 1, and if some other parameters are involved then they are assumed to be known. By fixing the time termination ratio t/μ_0 as 0.628, 0.942, 1.257, 1.571, 2.356, 3.141 and 4.712, the consumer's risk as 0.25, 0.10, 0.05, 0.01, the failure probability p_0 is calculated such that it satisfies the following worst case ($\mu = \mu_0$).

$$L(p) \leq \beta \tag{13}$$

Operating Characteristic (OC) Curve

The OC function of the sampling plan is the probability of accepting a lot and is given by

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i}\right]^g \tag{14}$$

where $p = F(t, \beta, \delta)$ is treated as a function of lot quality parameter β. The OC values for different combinations of the values of consumer's risk are computed and presented in Table 2. For a given value of the producer's risk α, the minimum value of μ/μ_0 is determined, such that it satisfies the following inequality

$$\left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i}\right]^g \geq 1-\alpha \tag{15}$$

and are presented in Table 3.

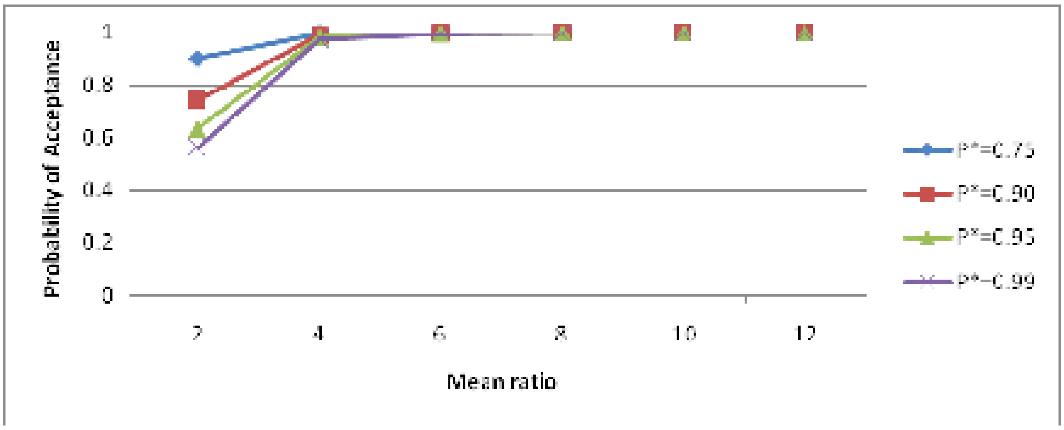

Figure 1. **OC values vs. mean ratio with experiment time ratio a=0.628**

3. Example

Suppose that the lifetime of a product follows Compound Rayleigh distribution with shape parameter $\delta = 1$. Suppose that it is desired to design a group sampling plan to assure that the mean life is greater than 1000 hours, though the experiment to be completed by 1000 hours using testers equipped with four products each. It is assumed that the consumer's risk is 25% when the true mean is 1000 hours and the producer's risk is 5% when the true mean is 2000 hours. For $\beta = 0.25$, $r = 4$, $a = 0.628$, the minimum number of groups and acceptance number can be found as $g = 4$ and $c = 2$ from Table 1.
This means that a total of 16 products are needed and that four items will be allocated to each of the four testers. The lot will be accepted if no more than two failures occurs before 1000 hours in each of the four groups. For this sampling plan under the Compound Rayleigh distribution, the number of groups decreases and the OC values increases, which is summarized as follows.

a = t/μ_0	0.628	0.942	1.257	1.571	2.356	3.141
g	4	2	1	1	1	1
μ/μ_0	2	4	6	8	10	12
L(p) at a=0.628	0.901791	0.997127	0.999716	0.999968	0.999984	0.999992

4. Conclusion

It is observed from Table 2 and from Figure 1, that the Operating Characteristic values of Compound Rayleigh distribution increases and it tends to unity when the mean ratio μ/μ_0 increases. The minimum ratio increases and the group size decreases, with an increase in confidence level. For various experiment time ratio, the minimum number of groups required to make a decision decreases with an increase in the confidence level. It is concluded that this group sampling plan would be beneficial in terms of test time and cost since a group of items can be tested simultaneously.

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β	r	c	a = t/μ_0							
			0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	2	0	2	1	1	1	1	1	1	1
	3	1	3	1	1	1	1	1	1	1
	4	2	4	2	1	1	1	1	1	1
	5	3	8	2	2	1	1	1	1	1
	6	4	13	3	2	1	1	1	1	1
	7	5	24	4	2	2	1	1	1	1
0.10	4	0	1	1	1	1	1	1	1	1

β	r	c	a = t/μ_0							
			0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	5	1	2	1	1	1	1	1	1	1
	6	2	3	1	1	1	1	1	1	1
	7	3	4	2	1	1	1	1	1	1
	8	4	6	2	1	1	1	1	1	1
	9	5	9	2	1	1	1	1	1	1
0.05	5	0	1	1	1	1	1	1	1	1
	6	1	2	1	1	1	1	1	1	1
	7	2	3	1	1	1	1	1	1	1
	8	3	4	2	1	1	1	1	1	1
	9	4	5	2	1	1	1	1	1	1
	10	5	7	2	1	1	1	1	1	1
0.01	7	0	1	1	1	1	1	1	1	1
	8	1	2	1	1	1	1	1	1	1
	9	2	2	1	1	1	1	1	1	1
	10	3	3	2	1	1	1	1	1	1
	11	4	4	2	1	1	1	1	1	1
	12	5	6	2	1	1	1	1	1	1

Table 1. Minimum Number of groups required for Group sampling plan when the life time of the items follows Compound Rayleigh distribution when $\delta = 1$.

β	r	g	a=t/μ_0	μ/μ_0					
				2	4	6	8	10	12
0.25	4	4	0.628	0.901791	0.997127	0.999716	0.999968	0.999984	0.999992
	4	2	0.942	0.757144	0.987378	0.998563	0.99972	0.999922	0.999984
	4	1	1.257	0.697439	0.974396	0.996551	0.99927	0.999795	0.999929
	4	1	1.571	0.519071	0.933656	0.989227	0.997554	0.999277	0.999742
	4	1	2.356	0.222292	0.743672	0.933712	0.981032	0.993657	0.997557
	4	1	3.141	0.09836	0.519334	0.817132	0.93374	0.974452	0.989243
	4	1	3.927	0.047733	0.341656	0.666839	0.850884	0.93369	0.969272
	4	1	4.712	0.025371	0.222292	0.519246	0.743672	0.869951	0.933712
0.10	6	3	0.628	0.745045	0.990145	0.998974	0.999802	0.999949	0.999982
	6	1	0.942	0.639986	0.97375	0.996704	0.99933	0.999813	0.999934
	6	1	1.257	0.35634	0.906256	0.985169	0.996691	0.999032	0.999656
	6	1	1.571	0.174654	0.788305	0.957053	0.989301	0.996693	0.998786
	6	1	2.356	0.026404	0.419508	0.788453	0.928227	0.973696	0.989312
	6	1	3.141	0.0047	0.174862	0.537753	0.788525	0.906435	0.957115
	6	1	3.927	0.001051	0.067518	0.318622	0.60094	0.788393	0.889954
	6	1	4.712	0.000288	0.026404	0.174793	0.419508	0.639592	0.788453
0.05	7	3	0.628	0.635164	0.983525	0.99824	0.999664	0.999913	0.99997
	7	1	0.942	0.52423	0.958122	0.994478	0.998857	0.999679	0.999888
	7	1	1.257	0.237653	0.859176	0.975915	0.994454	0.998354	0.99941
	7	1	1.571	0.093088	0.701898	0.932789	0.982478	0.99446	0.99794
	7	1	2.356	0.008205	0.295466	0.702083	0.890467	0.958039	0.982497
	7	1	3.141	0.000919	0.093231	0.413025	0.702175	0.85943	0.932881
	7	1	3.927	0.000139	0.027255	0.204874	0.480709	0.702009	0.836424
	7	1	4.712	0.000026	0.008205	0.093184	0.295466	0.523785	0.702083
0.01	9	2	0.628	0.562476	0.975842	0.997292	0.999474	0.999856	0.99995
	9	1	0.942	0.329151	0.916297	0.987847	0.99739	0.999252	0.999737
	9	1	1.257	0.096588	0.749329	0.95016	0.987797	0.996268	0.998638
	9	1	1.571	0.023828	0.529581	0.870584	0.963134	0.987807	0.995347
	9	1	2.356	0.000704	0.134576	0.529816	0.799074	0.916145	0.963171
	9	1	3.141	0.00003	0.023882	0.225876	0.529933	0.749723	0.870746
	9	1	3.927	0.000002	0.003962	0.077177	0.286772	0.529722	0.714587
	9	1	4.712	0.000002	0.000701	0.023865	0.134576	0.328705	0.529816

Table 2. Probability of acceptance for group sampling plan with c = 2, when the life time of the items follows Compound Rayleigh distribution

β	c	r	a = t/μ_0							
			0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	0	2	8.679	9.176	12.245	15.303	22.950	30.596	3.252	45.899
	1	3	3.410	3.739	4.988	6.234	9.349	12.464	15.583	18.698
	2	4	2.318	3.005	3.431	4.288	6.431	8.574	10.719	12.862
	3	5	1.989	2.346	3.130	3.417	5.124	6.832	8.541	10.248
	4	6	1.748	2.109	2.633	2.910	4.363	5.817	7.273	8.726
	5	7	1.617	1.920	2.309	2.886	3.856	5.140	6.427	7.711
0.10	0	4	8.679	13.019	17.372	21.711	32.560	43.409	54.271	65.120
	1	5	4.164	5.141	6.860	8.574	12.858	17.142	21.431	25.715
	2	6	2.919	3.478	4.641	5.800	8.698	11.596	14.498	17.396
	3	7	2.328	3.107	3.660	4.574	6.859	9.144	11.432	13.718
	4	8	2.029	2.593	3.094	3.867	5.799	7.731	9.665	11.597
	5	9	1.833	2.259	2.720	3.399	5.098	6.796	8.497	10.195
0.05	0	5	9.710	14.565	19.435	24.290	36.470	48.563	60.717	72.853
	1	6	4.620	5.711	7.621	9.525	14.284	19.043	23.808	28.568
	2	7	3.220	3.848	5.134	6.416	9.622	12.828	16.038	19.244
	3	8	2.557	3.420	4.037	5.046	7.567	10.087	12.612	15.133
	4	9	2.166	2.846	3.406	4.257	6.384	8.511	10.640	12.767
	5	10	1.940	2.476	2.990	3.737	5.604	7.470	9.340	11.206
0.01	0	7	11.497	17.246	23.013	28.761	43.132	57.504	71.893	86.265
	1	8	4.471	6.706	8.948	11.183	16.771	22.359	27.955	33.543
	2	9	3.458	4.494	5.996	7.494	11.238	14.984	18.732	22.476
	3	10	2.827	3.968	4.698	5.871	8.805	11.738	14.675	17.609
	4	11	2.421	3.292	3.953	4.940	7.408	9.876	12.348	14.816
	5	12	2.186	2.856	3.462	4.327	6.489	8.650	10.815	12.977

Table 3. Minimum ratio of true value to specified μ_0 for the acceptability of a lot in a group sampling plan with producer's risk 0.05, when the life time of the items follows Compound Rayleigh distribution.