International Journal of Mathematics And its Applications
Volume 4, Issue 2–A (2016), 135–142.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Heat Transfer Analysis of MHD Stagnation Point Flow Over a Stretching Sheet with Convective Boundary Conditions

Research Article

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Abstract: The problem of a steady two dimensional MHD stagnation point flow and heat transfer analysis over a stretching sheet with CBC and NH is considered. The governing partial differential equation are transformed in to ordinary differential equation using a similarity transformation. Numerical solution are obtained for the skin friction coefficient, the surface temperature, nusselt number as well as the velocity temperature and velocity profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number, stretching parameter, magnetic parameter and conjugate parameter are analyzed and discussed.

Keywords: Similarity solution, convective boundary condition, heat transfer, mathematical modeling, stagnation point flow, MHD, stretching sheet.

1. Introduction

The steady two-dimensional boundary-layer flow of a viscous incompressible fluid over a non-linearly stretching sheet is very important and it has many practical applications in several industries such as polymer sheet extrusion from a dye, aerodynamic extrusion of plastic sheets, glass-fiber production and many others. The two-dimensional boundary-layer flow caused by a moving rigid surface was first investigated by Sakiadis [1]. Later, Crane [2], extended this idea for the two dimensional flow over a stretching sheet problem. Gupta and Gupta [3] using similar solution method, analyzed heat and mass transfer in the boundary layer over a stretching sheet subject to suction or blowing. Wang [4] obtained similarity solutions to the axisymmetric case. Mahapatra and Gupta [6] investigated the magnetohydrodynamic stagnation-point flow towards isothermal stretching sheet and pointed that velocity decreases/increases with the increase in magnetic field intensity when free stream velocity is smaller/greater, respectively than the stretching velocity. Mahapatra and Gupta [7] studied heat transfer in stagnation-point flow towards stretching sheet with viscous dissipation effect. Effect of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet was studied by Cortell [8].

Wang [9] investigated the steady two-dimensional flow and axisymmetric stagnation point flow with heat transfer over a shrinking/sretching sheet and found that solutions do not exist for the larger shrinking rates. Jat and Chaudhary [10–12] studied the MHD boundary layer flow over a stretching sheet for stagnation point, heat transfer with and without viscous

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dissipation and Joule heating. Ishak A [17, 18], who have studied the problem of boundary layer flow and heat transfer over a flat plate with a convective surface boundary condition. Ishak, Nazar etc [19] studied MHD stagnation point flow towards a stretching sheet with prescribed surface heat flux. Bhattacharyya [20] studied the existence of dual solutions, unique solution and non-existence of solution for self-similar equations of the flow and heat transfer are analyzed numerically. Nik Long et al. [21] found that the solution is unique over the stretching sheet. Nik Long [22] The unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet with suction/injection is studied Numerical results show that the range of dual solutions increases with mass suction and decreases with mass injection. Yacob and Ishak [23] studied The effects of the material parameter and the convective parameter on the fluid flow and heat transfer characteristics are discussed of a steady laminar two-dimensional stagnation point flow towards a stretching/shrinking sheet in a micro polar fluid with a convective surface boundary condition. Bhattacharyya [25, 26] heat transfer analysed in unsteady boundary layer stagnation-point flow and towards a shrinking/stretching sheet. Jat, Chand [27] studied the viscous dissipation and radiation effects on MHD flow and heat transfer over stretching sheet. Further, Jat et al. [29] studied the above problem with micro polar fluid. [28] Ishak A et al. studied for stagnation point flow over a stretching surface with convective boundary conditions is considered.

1.1. Nomenclature

a, b	: constant	v	: kinematic viscosity
$u_e(x)$	: external velocity	k	: stretching parameter
$u_w(x)$	: stretching velocity	κ	: thermal conductivity
T	: temperature	γ	: Conjugate parameter
u, v	: velocity component along x and y direction	α	: thermal diffusivity
C_f	: skin friction coefficient	h_f	: heat transfer coefficient
T	: temperature	η	: similarity variable
T_∞	: fluid ambient temperature	ψ	: stream function
Nu_x	: local Nusselt number	ρ	: fluid density
Re_x	: local Reynolds number	τ_w	: surface shear stress
Pr	: Prandtl number	q_w	: surface heat flux
B	: Heat Source/Sink parameter	θ	: dimensionless temperature

2. Mathematical Formulation

A study two dimensional MHD stagnation-point flow over a stretching plate immersed in an incompressible viscous fluid ambient temperature T_∞ is considered. It is assumed that the external velocity $u_e(x)$ and the stretching velocity $u_w(x)$ are of the forms $u_e(x) = ax$ and $u_w(x) = bx$, where a and b are constants. It is further assumed that the the plate is subjected to a convective boundary condition. The boundary layer equation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0^2}{\rho}(u - u_e) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) \tag{3}$$

Subject to the boundary condition $u = u_w(x)$; $v = 0$.

$$\frac{\partial T}{\partial y} = -h_f T(NH), \ -\kappa\frac{\partial T}{\partial y}(x,o) = h_f\left(T_f - T(x,0)\right)(CBC) \ \text{ at } \ y=0 \tag{4}$$

$u = u_e(x)T \to T_\infty$ as $y \to \infty$, where u and v are the velocity components along the x and y direction, respectively. Further T is temperature of the hot fluid, v is the kinematic viscosity, k is the thermal conductivity α is the thermal diffusivity and h_f is the heat transfer coefficient. We now introduce the following similarity variables

$$\eta = \left(\frac{u_e}{vx}\right)^{1/2} y, \qquad \psi = (vxu_e)^{1/2} f(\eta)$$
$$\theta(\eta) = \frac{T - T_\infty}{T_\infty}(NH) \ \text{ or } \ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}(CBC) \tag{5}$$

Where ψ is the strem function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, which identically satisfies equation (1). Thus we have

$$u = axf'(\eta) \quad v = -(av)^{1/2} f(n) \tag{6}$$

Where prime denotes differentiation with respect to η. Substituting (5) and (6) in to equations (2) and (3), we obtain the following non linear ordinary differential equations.

$$f''' + ff'' + 1 - f'^2 + M(f' - 1) = 0 \tag{7}$$
$$\frac{1}{\Pr}\theta'' + (f + B)\theta' = 0 \tag{8}$$

Where $\Pr = \frac{v}{\alpha}$ is the Prandtl number, $M = \sigma\beta_o^2/\rho a$ is Magnetic parameter and $B = Q_o/\rho c_p a$ is the heat source ($B < 0$) or sink ($B > 0$) parameter. The boundary condition (4) become

$$f(0) = 0, \quad f'(0) = k \tag{9}$$

$\theta'(0) = -\gamma\,(1 + \theta(0))(\text{NH})$ or $\theta'(0) = -\gamma\,(1 - \theta(0))(\text{CBC})$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 1 \ \text{ as } \ \eta \to \infty. \tag{10}$$

Where $k = \frac{b}{a} \geq 0$ is the stretching parameter. Further $\gamma = h_f\left(\frac{v}{a}\right)^{1/2}(NH)$ or $\gamma = h_f\left(\frac{v}{a}\right)^{1/2}\kappa^{-1}(CBC)$ is the conjugate parameter for the convective boundary condition. It is noticed that $\gamma = 0$ is for the insulated plate and $\gamma \to \infty$ is when the surface temperature is prescribed. The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are given by

$$C_f = \frac{\tau_w}{\rho u_e^2}; \quad Nu_x = \frac{xq_w}{\kappa(T_w - T_\infty)} \tag{11}$$

Where ρ is the fluid density. The surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}; \quad q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{12}$$

With $\mu = \rho v$ and k being the dynamic viscosity and the thermal conductivity, respectively Using the similarity variables in (5) gives

$$C_f Re_x^{1/2} = f''(0); \quad \frac{Nu_x}{Re_x^{1/2}} = \gamma\left(\frac{1}{\theta(0)} + 1\right)(NH)$$
$$\frac{Nu_x}{Re_x^{1/2}} = \gamma\left(\frac{1}{\theta(0)} - 1\right)(CBC) \tag{13}$$

Where $Re_x = \frac{u_e x}{v}$ is the local Reynolds number and Nu_x is the local Nusselt number.

3. Method of Solution

The system of boundary value problem (BVP) (7)-(8) was solved numerically via the shooting technique by converting it into an equivalent initial value problem(IVP). In this technique, we choose a suitable finite value of η_∞ (where η_∞ corresponds to $\eta \to \infty$) which depends on the value of parameter considered.

$$f' = pp' = q$$

$$q' + fq + 1 - p^2 + M(p-1) = 0 \tag{14}$$

$$\theta' = r; \quad \frac{1}{\mathrm{Pr}} r' + (f+B)r = 0 \tag{15}$$

With the boundary conditions

$$f(0) = 0; \quad p(0) = k$$

$$r(0) = -\gamma[1+\theta(0)](NH) \quad \text{or} \quad r(0) = -\gamma[1-\theta(0)](CBC) \tag{16}$$

$$p(\eta_\infty) = 1; \quad \theta(\eta_\infty) = 0$$

Now we have to set of partial initial conditions

$$f(0) = 0; \;\; p(0) = k; \;\; q(0) = \beta_1; \;\; \theta(0) = \beta_1$$

$$r(0) = -\gamma[1+\beta_2](NH); \quad r(0) = -\gamma[1-\beta_2](CBC) \tag{17}$$

In order to integrate (14) and (15) as an initial value problem one requires a value for $q(0)$ i.e. $f''(0)$ and $\theta(0)$ but no such values are given in the boundary conditions. The suitable guesses for $f''(0)$ and $\theta(0)$ is chosen by the shooting technique and the fourth order Runge–Kutta method is applied to obtain the solution. Then we compare the calculated values for f' and θ at η_∞ (a suitable gauss of η) with the given boundary conditions $\theta(\eta_\infty) = 0$ and $p(\eta_\infty) = 1$ and adjust the estimated values of $f''(0)$ and $\theta(0)$ using the Secant method to give a better approximation for the solution. The step-size is taken $h = 0.001$. The above procedure is repeated until we get the converged results within a tolerance limit of 10^{-7}. All the computations are done in the Matlab software which uses a symbolic and computational language.

4. Result and Discussion

The set of non linear ordinary differential equation (8) and (9) with boundary conditions (10) were solved numerically using Runga-Kutta forth order algorithm with a systematic guessing of $f''(0)$ and $\theta(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution and accuracy up to the seventh decimal place i.e. 1×10^{-7}, which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta \to \infty$ say η_∞ which depend on the values of the parameter used. The computation are done by generating a program in Matlab. For the validation of numerical results, and compared to those of Wang [7], Yacob and Ishak [23] and Mohamed MKA,Ishak A etc. [29]. Table presents the comparison between the present results with the previously reported results by Wang [7], Yacob and Ishak [23] and Mohamed MKA,Ishak A etc. [29]. for various values of the stretching parameter k. It has been found that they are in good agreement. We can conclude e that this method work sufficiently for the present problem, and we are also confident that the results presented here are accurate

These quantitative comparisons are shown in table 1 for the variation of k and found to be in favorable agreement. The computation through employed numerical scheme has been carried out for various values of the Magnetic parameter M, Prandtl number Pr Conjugate parameter G.

It is observed from the figures that the boundary conditions are satisfied asymptotically in all the cases, which supporting the accuracy of the numerical results obtained. The velocity profile $f^{'}(\eta)$ for different values of the k is shown in Figure 1. shows the velocity profiles for different values of k which produce $f'(0) = k$. When $k > 1$, the flow has an inverted boundary layer structure and the thickness of the boundary layer decreases with k. On the other hand, when $k < 1$, the flow has a boundary layer structure, which results from the fact that when $b/a < 1$, the Figure 2, the fluid velocity decreases with the increase in magnetic parameter M. This is happening due to the increasing value of M tends to the increasing of Lorentz force, which produces more resistance to the transport phenomena. As the fluid velocity decrease the fluid temperature increases with the increase in magnetic parameter M Figure 5 presents the temperature profiles for various values of Pr. It has been found that as Pr increases, the temperature e in the boundary layer decreases, and the thermal boundary layer thickness also decreases. This is because for small values of the Prandtl number, the fluid is highly thermal conductive. Physically, if Pr increases, the thermal diffusivity decreases, and this phenomenon leads to the decreasing of energy ability that reduces the thermal boundary layer. Similar to the results presented in Figure 3 The temperature profiles with various values of k are presented in Figure 3, and it has been found again that as k increases, the temperature decreases, and the thermal boundary layer thickness also decreases.

Figure 4 exhibits the temperature profile for various values of conjugate parameter G in cases of stretching sheet. It is found that the temperature profile as well as thermal boundary layer thickness increases with increasing values of G. We now illustrate the in?uence of heat source $(B < 0)$ or sink $(B > 0)$ parameter on the dimensionless temperature profiles by Figure 6. Due to increase in the strength of the heat source the ?uid temperature increases as the thermal boundary layer thickness enlarges. On the other hand, for the increase of heat sink strength the temperature decreases because it lessens the thermal boundary layer.

K	wang [7]	Yacob and Ishak [23]	Mohamed MKA,Ishak A etc. [29]	present
5	-10.26475	-	-	-10.264749
3	-	-	-	-4.276541
2	-1.88731	-1.88731	-1.88731	-1.88731
1	0	0	0	0
0.5	0.7133	0.713295	0.713295	0.713295
0	1.232588	1.232588	1.232588	1.23259

Table 1.

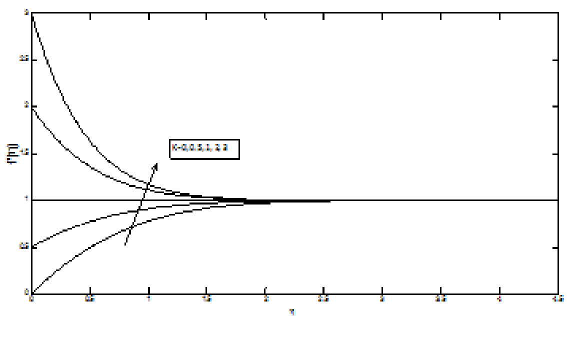

Figure 1. **Velocity profiles $f'(\eta)$ for various value of k.**

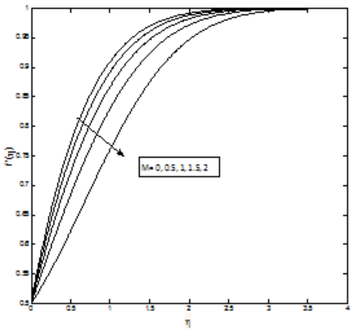

Figure 2. Velocity profiles $f'(\eta)$ for various value of M when k=0.5.

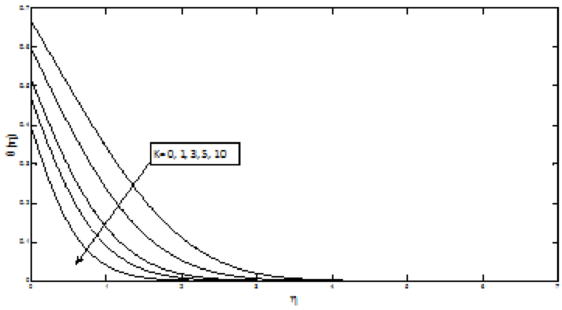

Figure 3. Temperature profiles $\theta(\eta)$ for various values of k when γ=1, Pr=0.72.

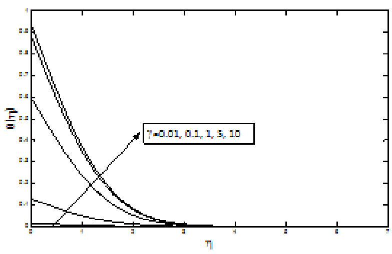

Figure 4. Temperature profiles $\theta(\eta)$ for various values of γ when k=1 and Pr=0.72.

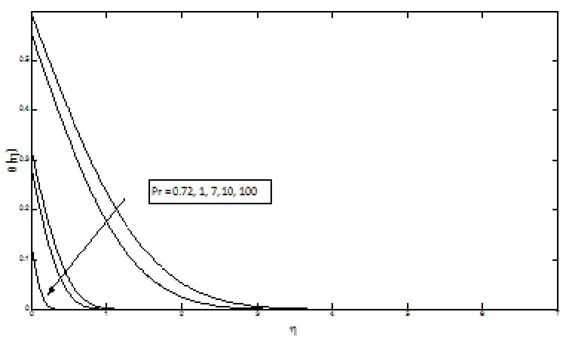

Figure 5. Temperature profiles $\theta(\eta)$ for various values of Pr. When γ=1, k=1.

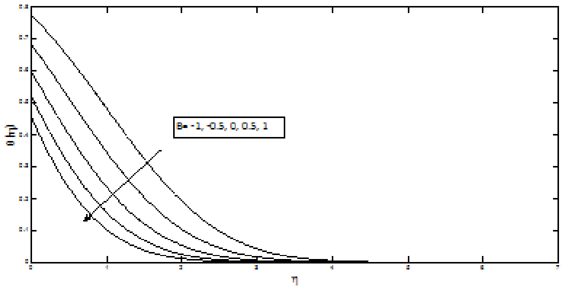

Figure 6. **Temperature profiles $\theta(\eta)$ for various values of B when Pr=0.72.**

5. Conclusions

In this paper, we have theoretically and numerically studied the problem of stagnation point ?ow over a stretching sheet with the convective boundary condition. It is shown how the Prandtl number Pr, stretching parameter k and conjugate parameter G affect the values of the surface temperature $\theta(0)$ and skin friction coefficient $f''(0)$.

Acknowledgements

This work has been carried out with the financial support of UGC in the form of J.R.F (F-41-4/NET/RES/JRF-1296) awarded to one of the author (Navin Kumar).

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