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Sub-Trident Ranking Using Fuzzy Numbers

Research Article

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Abstract: This paper deals with the new ranking technique by using Sub-Trident Form along with the help of fuzzy numbers. Here the Sub-Trident Form is obtained from Fuzzy Sub-Triangular Form using fuzzy numbers. The average Sub-Trident Form for each possible path is determined and the minimum of it gives the ranking for the shortest path.

Keywords: Sub-Trident, Ranking, Average, Fuzzy Number, Pascal's Triangle.

1. Introduction

In the year 1965, Lotfi.A.Zadeh introduced the Fuzzy Set Theory [1]. Dubois and Prade introduced the Shortest Path Problem in the year 1980 [2]. They analyzed this problem using Floyd's algorithm. Okada and Gen introduced the same problem with the help of Dijkstra's algorithm in the year 1994 [3]. Chen and Hsieh introduced the Graded Mean Integration Representation for fuzzy numbers [5]. Okada and Soper worked on the shortest path problems using fuzzy numbers [4]. In this paper, the new ranking technique is introduced using Sub-Trident Form with the help of fuzzy numbers. This ranking technique is called the Sub-Trident Ranking.This paper consists of six sections: preliminaries in the first section, Graded Mean Integration Representation together with Pascal's Triangle Graded Mean in the second section, explaining the proposed method in the third section, Algorithm for our study in the fourth section, explaining the Sub-Trident Ranking Technique with a suitable numerical example in the fifth section and finally the conclusion based on our study.

2. Preliminaries

2.1. Basic Definitions

Definition 2.1. *A fuzzy set $\widetilde{A}$ in X is characterized by a membership function $\mu_{\widetilde{A}}(x)$ represents grade of membership of $x \in \mu_{\widetilde{A}}(x)$. More general representation for a fuzzy set is given by*

$$\widetilde{A} = \left\{ \left(x, \mu_{\widetilde{A}}(x) \right) / x \in X \right\}.$$

Definition 2.2. *The α-cut of a fuzzy set $\widetilde{A}$ of the Universe of discourse X is defined as $\widetilde{A} = \left\{ x \in X / \mu_{\widetilde{A}}(x) \geq \alpha \right\}$, where $\alpha \in [0,1]$.*

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Definition 2.3. *A fuzzy set $\widetilde{A}$ defined on the set of real numbers $\Re$ is said to be a fuzzy number if its membership function $\widetilde{A} : \Re \to [0, 1]$ has the following characteristics:*

a). $\widetilde{A}$ is convex if $\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_{\widetilde{A}}(x_1),\ \mu_{\widetilde{A}}(x_2)\},\ \ \forall\ x_1,\ x_2 \in X, \lambda \in [0, 1]$.

b). $\widetilde{A}$ is normal if there exists an $x \in \Re$ such that if $\max \mu_{\widetilde{A}}(x) = 1$.

c). $\mu_{\widetilde{A}}(x)$ is piecewise continuous [10].

2.2. Representation of Generalized (Trapezoidal) Fuzzy Number

In general, a generalized fuzzy number A is described at any fuzzy subset of the real line R, whose membership function $\mu_{\widetilde{A}}$ satisfies the following conditions:

1). $\mu_{\widetilde{A}}$ is a continuous mapping from R to [0, 1],

2). $\mu_{\widetilde{A}}(x) = 0,\ -\infty \prec x \leq c$,

3). $\mu_{\widetilde{A}}(x) = L(x)$ is strictly increasing on [c, a]

4). $\mu_{\widetilde{A}}(x) = w,\ a \leq x \leq b$,

5). $\mu_{\widetilde{A}}(x) = R(x)$ is strictly decreasing on [b, d],

6). $\mu_{\widetilde{A}}(x) = 0,\ d \leq x \prec \infty$, where $0 \prec w \leq 1$ and a, b, c and d are real numbers.

Here denote this type of generalized fuzzy number as $A = (c, a, b, d; w)_{LR}$. When $w = 1$, denote this type of generalized fuzzy number as $A = (c, a, b, d)_{LR}$. When $L(x)$ and $R(x)$ are straight line, then A is Trapezoidal fuzzy number and denote it as (c, a, b, d).

2.3. Different Forms of Fuzzy Numbers

The Following are the different forms of fuzzy numbers defined by its membership functions:

1. ***Triangular Fuzzy Number:*** A Triangular fuzzy number is defined as $\widetilde{A} = (a_1, a_2, a_3)$, where all a_1, a_2, a_3 are real numbers and its membership function is given below:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0, & \text{for } x < a_1; \\ \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2; \\ \frac{a_3-x}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3; \\ 0, & \text{for } x > a_3. \end{cases}$$

2. ***Trapezoidal Fuzzy Number:*** A Trapezoidal fuzzy number is defined as $\widetilde{A} = (a_1, a_2, a_3, a_4)$, where all a_1, a_2, a_3, a_4 are real numbers and its membership function is given below:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0, & \text{for } x < a_1; \\ \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2; \\ 1, & \text{for } a_2 \leq x \leq a_3; \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4; \\ 0, & \text{for } x > a_4. \end{cases}$$

3. ***Pentagonal Fuzzy Number:*** A Pentagonal fuzzy number is defined as $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5)$, where all a_1, a_2, a_3, a_4, a_5 are real numbers and its membership function is given below:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0, & \text{for} \quad x < a_1; \\ \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \le x \le a_2; \\ \frac{x-a_2}{a_3-a_2}, & \text{for } a_2 \le x \le a_3; \\ 1, & \text{for } x = a_3; \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \le x \le a_4; \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_4 \le x \le a_5; \\ 0, & \text{for } x > a_5. \end{cases}$$

This can also be extended to any fuzzy number.

3. Graded Mean Approach

3.1. Graded Mean Integration Representation

In 1998, Chen and Hsieh [6] and [7] proposed graded mean integration representation for representing generalized fuzzy number. Suppose L^{-1}, R^{-1} are inverse functions of L and R respectively, and graded mean h-level value of generalized fuzzy number $A = (c, a, b, d; w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$. Then the graded mean integration representation of generalized fuzzy number based on integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right) dh}{\int_0^w h dh} = \frac{\mathrm{c} + 2\mathrm{a} + 2\mathrm{b} + \mathrm{d}}{6},$$

where h is between 0 and w, $0 < w \le 1$.

3.2. Pascal's Triangle Graded Mean Approach

The Graded Mean Integration Representation for generalized fuzzy number by Chen and Hsieh [5–7]. Later Sk.Kadhar Babu and B.Rajesh Anand introduces Pascal's triangle graded mean in statistical optimization [9]. But the present approach is very simple for analyzing fuzzy variables to get the optimum shortest path. This procedure is taken from the following Pascal's triangle. Here the coefficients of fuzzy variables as Pascal's triangle numbers. Then just add and divide by the total of Pascal's number and call it as Pascal's Triangle Graded Mean Approach.

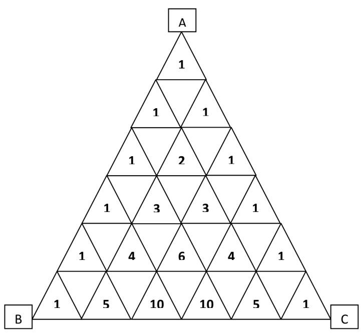


Figure 1. **Pascal's Triangle**

4. Proposed Method

(a). **Fuzzy Sub-Triangular Form of Fuzzy Numbers:** The Fuzzy Sub-Triangular form for different fuzzy number is given below:

(A). **Triangular Fuzzy Numbers:** The Pascal's Triangle for Triangular Fuzzy Number is given in Figure 2 and the Sub -Triangles for Triangular Fuzzy Number is given in Figure 3 (*Set: I*) as follows:

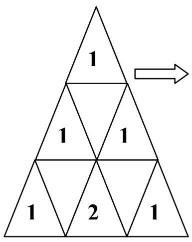


Figure: 2

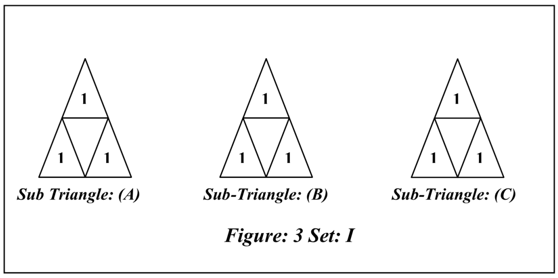


Figure: 3 Set: I

$$P_1 = P(A) = \frac{a_1 + a_2}{2}, \quad q_1 = P(B) = \frac{a_1 + a_2}{2}, \quad r_1 = P(C) = \frac{a_1 + a_2}{2}.$$
$$P_2 = P(A) = \frac{a_1 + a_2}{2}, \quad q_2 = P(B) = \frac{a_1 + 2a_2}{3}, \quad r_2 = P(C) = \frac{2a_1 + a_2}{3}.$$
$$P_3 = P(A) = \frac{a_1 + 2a_2}{3}, \quad q_3 = P(B) = \frac{2a_1 + a_2}{3}, \quad r_3 = P(C) = \frac{a_1 + a_2}{2}.$$

The Fuzzy Sub-Triangular Form for Triangular Fuzzy Number is given by

$$FST_f = (p_p, q_q, r_r), \text{ where } p_p = \frac{p_1 + p_2 + p_3}{3}, \; q_q = \frac{q_1 + q_2 + q_3}{3}, \; r_r = \frac{r_1 + r_2 + r_3}{3}$$

(B). **Trapezoidal Fuzzy Numbers:** The Pascal's Triangle for Trapezoidal Fuzzy Number is given in Figure 4 and the Sub-Triangles for Trapezoidal Fuzzy Number is given in Figure 5 (*Set: II*) as follows:

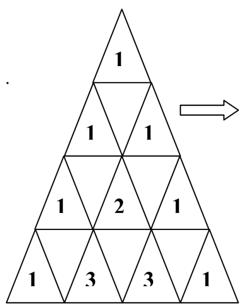


Figure: 4

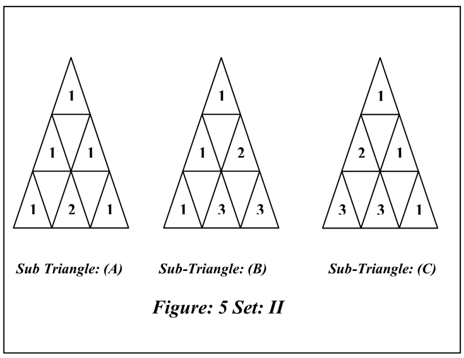


Figure: 5 Set: II

$$P_1 = P(A) = \frac{a_1 + a_2 + a_3}{3}, \quad q_1 = P(B) = \frac{a_1 + 2a_2 + a_3}{4}, \quad r_1 = P(C) = \frac{a_1 + a_2 + a_3}{3}.$$
$$P_2 = P(A) = \frac{a_1 + a_2 + a_3}{3}, \quad q_2 = P(B) = \frac{a_1 + 3a_2 + 3a_3}{7}, \quad r_2 = P(C) = \frac{3a_1 + 2a_2 + a_3}{6}.$$
$$P_3 = P(A) = \frac{a_1 + 2a_2 + 3a_3}{6}, \quad q_3 = P(B) = \frac{3a_1 + 3a_2 + a_3}{7}, \quad r_3 = P(C) = \frac{a_1 + a_2 + a_3}{3}.$$

The Fuzzy Sub-Triangular Form for Trapezoidal Fuzzy Number is given by

$$FST_f = (p_p, q_q, r_r), \text{ where } p_p = \frac{p_1 + p_2 + p_3}{3}, \; q_q = \frac{q_1 + q_2 + q_3}{3}, \; r_r = \frac{r_1 + r_2 + r_3}{3}$$

(C). **Pentagonal Fuzzy Numbers:** The Pascal's Triangle for Pentagonal Fuzzy Number is given in Figure 6 and the Sub-Triangles for Pentagonal Fuzzy Number is given in Figure 7 (*Set: III*) as follows:

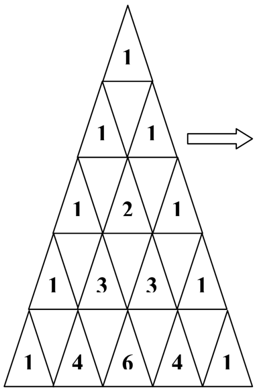
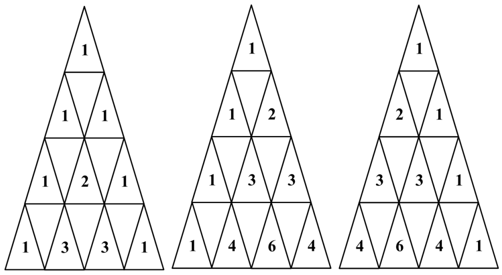


Figure: 6



Sub Triangle: (A) *Sub-Triangle: (B)* *Sub-Triangle: (C)*

Figure: 7 Set: III

$$P_1 = P(A) = \frac{a_1 + a_2 + a_3 + a_4}{4},\ q_1 = P(B) = \frac{a_1 + 3a_2 + 3a_3 + a_4}{8},\ r_1 = P(C) = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$
$$P_2 = P(A) = \frac{a_1 + a_2 + a_3 + a_4}{4},\ q_2 = P(B) = \frac{a_1 + 4a_2 + 6a_3 + 4a_4}{15},\ r_2 = P(C) = \frac{4a_1 + 3a_2 + 2a_3 + a_4}{10}.$$
$$P_3 = P(A) = \frac{a_1 + 2a_2 + 3a_3 + 4a_4}{10},\ q_3 = P(B) = \frac{4a_1 + 6a_2 + 4a_3 + a_4}{15},\ r_3 = P(C) = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

The Fuzzy Sub-Triangular Form for Pentagonal Fuzzy Number is given by

$$FST_f = (p_p, q_q, r_r),\ \text{ where } p_p = \frac{p_1 + p_2 + p_3}{3},\ q_q = \frac{q_1 + q_2 + q_3}{3},\ r_r = \frac{r_1 + r_2 + r_3}{3}$$

Similarly we can extend this to different fuzzy numbers.

(b). **Sub-Trident Form of Fuzzy Numbers:** The Sub-Trident Form of Fuzzy Number is given by $ST_{ri} = \frac{1}{3}\left[p_p^{\frac{1}{3}} + q_q^{\frac{1}{3}} + r_r^{\frac{1}{3}}\right]$, where p_p, q_q, r_r is the Graded Means of the Pascal's Triangle from the Fuzzy Triangular Form.

(c). **Sub-Trident Ranking Technique:** The Sub-Trident ranking is given by $ST_{ri}rank = \min(Avg\ ST_{ri})$ in order to give the ranking for the Shortest path.

5. Algorithm

The working rule for the Sub-Trident Form to find the Shortest Path and Optimum Solution is given by the following algorithm:

Step: 1 Start by choosing all the possible paths whose and edge weights as fuzzy numbers.

Step: 2 Obtain the Fuzzy Sub–Triangular form (FST_f) for all fuzzy number.

Step: 3 Obtain the Sub–Trident Form (ST_{ri}) for all fuzzy numbers.

Step: 4 Obtain Average Sub–Trident form ($Avg\ ST_{ri}$) in each possible path.

Step: 5 Minimum of Average Sub–Trident Form gives the Sub–Trident Ranking (i.e.) $ST_{ri}rank = \min\ (Avg\ ST_{ri})$.

6. Numerical Example

The numerical example is given below to illustrate the above procedure whose edge length is represented as Trapezoidal Fuzzy Number [8]:

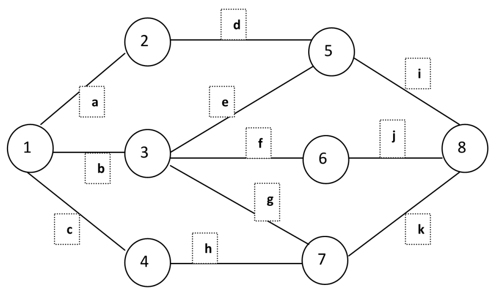


Figure 8

The Fuzzy numbers for the edges of Figure 8 is given below in the Table: 1

Edge	Triangular	Trapezoidal	Pentagonal
(1, 2)=a	(0.2, 0.4, 0.6)	(0.1, 0.2, 0.3, 0.4)	(0.1, 0.3, 0.5, 0.7, 0.9)
(1, 3)=b	(0.3, 0.5, 0.7)	(0.2, 0.4, 0.6, 0.8)	(0.2, 0.4, 0.6, 0.8, 1)
(1, 4)=c	(0.1, 0.2, 0.3)	(0.1, 0.3, 0.5, 0.7)	(0.1, 0.2, 0.3, 0.4, 0.5)
(2, 5)=d	(0.4, 0.5, 0.6)	(0.2, 0.4, 0.8, 1)	(0.2, 0.3, 0.4, 0.5, 0.6)
(3, 5)=e	(0.2, 0.4, 0.8)	(0.1, 0.3, 0.6, 0.8)	(0.3, 0.6, 0.7, 0.9, 1)
(3, 6)=f	(0.1, 0.4, 0.5)	(0.2, 0.5, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8, 0.9)
(3, 7)=g	(0.3, 0.6, 0.9)	(0.4, 0.6, 0.8, 1)	(0.3, 0.4, 0.5, 0.6, 0.7)
(4, 7)=h	(0.5, 0.7, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.2, 0.3, 0.5, 0.7, 0.9)
(5, 8)=i	(0.1, 0.4, 0.6)	(0.3, 0.4, 0.5, 0.6)	(0.1, 0.2, 0.5, 0.6, 0.7)
(6, 8)=j	(0.1, 0.4, 0.7)	(0.4, 0.6, 0.8, 1)	(0.4, 0.6, 0.8, 0.9, 1)
(7, 8)=k	(0.3, 0.3, 0.2)	(0., 0.5, 0.7, 0.9)	(0.2, 0.5, 0.6, 0.9, 1)

Table 1.

Sub-Trident form of fuzzy numbers is given below in the Tables 2, 3, and 4.

Edge	p_p	q_q	r_r	$FST_f = (p_p, q_q, r_r)$	Sub-Trident Form $ST_{ri} = \frac{1}{3}\left(p_p^{\frac{1}{3}} + q_q^{\frac{1}{3}} + r_r^{\frac{1}{3}}\right)$
(1, 2)=a	0.3111	0.3	0.2889	(0.3111, 0.3, 0.2889)	0.6694
(1, 3)=b	0.4111	0.4	0.3889	(0.4111, 0.4, 0.3889)	0.7368
(1, 4)=c	0.1557	0.15	0.1444	(0.1557, 0.15, 0.1444)	0.5313
(2, 5)=d	0.4556	0.45	0.4444	(0.4556, 0.45, 0.4444)	0.7663
(3, 5)=e	0.3111	0.3	0.2889	(0.3111, 0.3, 0.2889)	0.6694
(3, 6)=f	0.2667	0.25	0.2333	(0.2667, 0.25, 0.2333)	0.6298
(3, 7)=g	0.4667	0.45	0.4333	(0.4667, 0.45, 0.4333)	0.7662
(4, 7)=h	0.6111	0.6	0.5889	(0.6111, 0.6, 0.5889)	0.8198
(5, 8)=i	0.2667	0.25	0.2333	(0.2667, 0.25, 0.2333)	0.5422
(6, 8)=j	0.2667	0.25	0.2333	(0.2667, 0.25, 0.2333)	0.5422
(7, 8)=k	0.3	0.3	0.3	(0.3, 0.3, 0.3)	0.6694

Table 2. **Triangular Fuzzy Numbers**

Edge	p_p	q_q	r_r	$FST_f = (p_p, q_q, r_r)$	Sub-Trident Form $ST_{ri} = \frac{1}{3}\left(p_p^{\frac{1}{3}} + q_q^{\frac{1}{3}} + r_r^{\frac{1}{3}}\right)$
(1, 2)=a	0.2111	0.2	0.1889	(0.2111, 0.2, 0.1889)	0.5847
(1, 3)=b	0.4222	0.4	0.3778	(0.4222, 0.4, 0.3778)	0.7366
(1, 4)=c	0.3222	0.3	0.2778	(0.3222, 0.3, 0.2778)	0.6692
(2, 5)=d	0.5000	0.5048	0.4334	(0.5000, 0.5048, 0.4334)	0.4794
(3, 5)=e	0.3611	0.3274	0.3055	(0.3611, 0.3274, 0.3055)	0.6916
(3, 6)=f	0.4945	0.4726	0.4389	(0.4945, 0.4726, 0.4389)	0.7766
(3, 7)=g	0.6222	0.6	0.5778	(0.6222, 0.6, 0.5778)	0.8433
(4, 7)=h	0.6111	0.6	0.5889	(0.6111, 0.6, 0.5889)	0.8434
(5, 8)=i	0.4111	0.4	0.3889	(0.4111, 0.4, 0.3889)	0.7368
(6, 8)=j	0.6222	0.6	0.5778	(0.6222, 0.6, 0.5778)	0.8433
(7, 8)=k	0.5222	0.5	0.4778	(0.5222, 0.5, 0.4778)	0.7936

Table 3. **Trapezoidal Fuzzy Numbers**

Edge	p_p	q_q	r_r	$FST_f = (p_p, q_q, r_r)$	Sub-Trident Form $ST_{ri} = \frac{1}{3}\left(p_p^{\frac{1}{3}} + q_q^{\frac{1}{3}} + r_r^{\frac{1}{3}}\right)$
(1, 2)=a	0.4333	0.4	0.3667	(0.4333, 0.4, 0.3667)	0.7364
(1, 3)=b	0.5333	0.5	0.4667	(0.5333, 0.5, 0.4667)	0.7934
(1, 4)=c	0.2667	0.25	0.2333	(0.2667.0.25, 0.2333)	0.6298
(2, 5)=d	0.3667	0.35	0.3333	(0.3667, 0.35, 0.3333)	0.7046
(3, 5)=e	0.6567	0.6347	0.5933	(0.6567, 0.6347, 0.5933)	0.8563
(3, 6)=f	0.6667	0.65	0.6333	(0.6667, 0.65, 0.6333)	0.8662
(3, 7)=g	0.4667	0.45	0.4333	(0.4667, 0.45, 0.4333)	0.7662
(4, 7)=h	0.4533	0.4153	0.3967	(0.4533, 0.4153, 0.3967)	0.7497
(5, 8)=i	0.38	0.35	0.32	(0.38, 0.35, 0.32)	0.7043
(6, 8)=j	0.7033	0.6847	0.6467	(0.7033, 0.6847, 0.6467)	0.8785
(7, 8)=k	0.5867	0.5494	0.5133	(0.5867, 0.5494, 0.5133)	0.8190

Table 4. **Pentagonal Fuzzy Numbers**

7. Sub-Trident Ranking Technique

The Sub-Trident Ranking Technique for fuzzy numbers is given by the following Tables 6, 7 and 8.

Possible paths	Average Sub-Trident Form $Avg\ ST_{ri}$	Ranking
$1 \xrightarrow{a} 2 \xrightarrow{d} 5 \xrightarrow{i} 8$	0.6693	3
$1 \xrightarrow{b} 3 \xrightarrow{e} 5 \xrightarrow{i} 8$	0.6495	2
$1 \xrightarrow{b} 3 \xrightarrow{f} 6 \xrightarrow{j} 8$	0.6363	1
$1 \xrightarrow{b} 3 \xrightarrow{g} 7 \xrightarrow{k} 8$	0.7241	5
$1 \xrightarrow{c} 4 \xrightarrow{h} 7 \xrightarrow{k} 8$	0.6735	4

Table 5. **Ranking for Triangular Fuzzy Number**

Possible paths	Average Sub-Trident Form $Avg\ ST_{ri}$	Ranking
$1 \xrightarrow{a} 2 \xrightarrow{d} 5 \xrightarrow{i} 8$	0.6003	1
$1 \xrightarrow{b} 3 \xrightarrow{e} 5 \xrightarrow{i} 8$	0.7217	2
$1 \xrightarrow{b} 3 \xrightarrow{f} 6 \xrightarrow{j} 8$	0.7855	4
$1 \xrightarrow{b} 3 \xrightarrow{g} 7 \xrightarrow{k} 8$	0.7912	5
$1 \xrightarrow{c} 4 \xrightarrow{h} 7 \xrightarrow{k} 8$	0.7687	3

Table 6. Ranking for Trapezoidal Fuzzy Number

Possible paths	Average Sub-Trident Form $Avg\ ST_{ri}$	Ranking
$1 \xrightarrow{a} 2 \xrightarrow{d} 5 \xrightarrow{i} 8$	0.7151	1
$1 \xrightarrow{b} 3 \xrightarrow{e} 5 \xrightarrow{i} 8$	0.7847	3
$1 \xrightarrow{b} 3 \xrightarrow{f} 6 \xrightarrow{j} 8$	0.8460	5
$1 \xrightarrow{b} 3 \xrightarrow{g} 7 \xrightarrow{k} 8$	0.7929	4
$1 \xrightarrow{c} 4 \xrightarrow{h} 7 \xrightarrow{k} 8$	0.7328	2

Table 7. Ranking for Pentagonal Fuzzy Number

8. Conclusion

This paper concludes from the Sub-Trident ranking that, the path $1 \xrightarrow{b} 3 \xrightarrow{f} 6 \xrightarrow{j} 8$as the shortest path for triangular fuzzy numbers and the path $1 \xrightarrow{a} 2 \xrightarrow{d} 5 \xrightarrow{i} 8$ as the shortest path for trapezoidal, pentagonal fuzzy numbers.

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