



Thermal Convection of Micropolar Fluid in the Presence of Suspended Particles in Hydromagnetics

Research Article

Veena Sharma¹ and Sumit Gupta^{2*}

1 Department of Mathematics, Himachal Pradesh University, Shimla, India.

2 Department of Mathematics, Government Degree College, Diggal, Solan, India.

Abstract: The onset of instability is investigated in a micropolar fluid layer heated from below in the presence of suspended particles (fine dust) and uniform vertical magnetic field $H(0, 0, H)$. Using the Boussinesq approximation, the linearized stability theory and normal mode analysis method, the exact solutions are obtained for the case of two free boundaries. It is found that the presence of coupling between thermal and micropolar effects, the suspended particles number density, the magnetic field intensity and the micropolar coefficients bring oscillatory modes and over stability in the system which were non-existent in their absence. The behaviour of the Rayleigh numbers for the stationary convection and the case of over stability are computed numerically using Newton-Raphson method through the software Fortran-90 and Mathcad. The graphs show that Rayleigh number for the case of over stability and stationary convection increase with increase in magnetic field intensity $H(0, 0, H)$ and decrease with increase in micropolar coefficients (the dynamic microrotation viscosity κ and coefficient of angular viscosity γ'), for a fixed wave-number, implying thereby the stabilizing effect of magnetic field intensity and destabilizing effect of micropolar coefficients on the thermal convection of micropolar fluids.

Keywords: Micropolar fluid, Magnetic field, Suspended particles (fine dust), Microrotation viscosity, Coefficient of angular viscosity.
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1. Introduction

Eringen [1] gave a continuum theory of microfluids which takes into accounts the local motions and deformations of the permissive elements of the fluids. Micromorphic or microstructure theory, in which each element of the fluid is associated with two sets of degrees of freedom: (a) translatory degrees of freedom giving rise to classical mean motion (or velocity) and (b) rotation and stretch allowing the particles of the fluid to undergo independent intrinsic spins and homogeneous deformation. Eringen [2] introduced a theory of micropolar fluids, that is, fluids whose behaviour is determined in part by their microstructure, in particular by spin inertia and by the presence of stress moments and body moments. Eringen [3, 4] showed that if the skew-symmetric property of the gyration tensor is imposed; in addition to a condition of microisotropy, the simple microfluid system of nineteen equations reduces to seven equations in seven unknowns; which is the case of micropolar fluids. Under these assumptions deformation of the fluid, microelements are ignored; nevertheless, microrotational effects are still present and surface and body couples are permitted.

Thus in micropolar fluids, rigid particles contained in a small volume element can rotate about the centroid of the volume element in an average sense described by the microrotational vector. Micropolar theory was introduced by Eringen [3] in order to describe some physical systems which do not satisfy the Navier Stokes equations. These fluids are able to describe

* E-mail: sumittgupta@gmail.com

the behaviour of colloidal solutions, liquid crystals, animal blood etc. For example fluids consisting of bar-like elements, dumb-bell molecules, other polymeric fluids. The equations governing the flow of micropolar fluids theory involve a spin vector and a microinertia tensor in addition to velocity vector. A generalization of the theory including thermal effects has been developed by Kazakia and Ariman [5] and Eringen [6].

Micropolar fluids stabilities have become an important field of research these days. A particular stability problem is the Rayleigh–Bénard instability in a horizontal thin layer of fluid heated from below. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below has been given by Chandrasekhar [7]. Ahmadi [8] and Pérez–Garcia et al. [9] have studied the effects of the microstructures on the thermal convection and have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stabilities may not be fulfilled and consequently micropolar fluids introduce oscillatory motions. The existence of oscillatory motions in micropolar fluids has been depicted by Lekkerkerker in liquid crystals [10], Bradley in dielectric fluids [11] and Laidlaw in binary mixture [12]. In the study of problems of thermal convection, it is frequent practice to simplify the basic equations by introducing an approximation which is attributed to Boussinesq [13].

In geophysical situations, the fluid is often not pure but contains several suspended particles. Motivation for the study of certain effect of particles immersed in the fluid such as particle heat capacity, particle mass fraction and thermal force is due to the fact that the knowledge concerning fluid–particles mixture is not commensurate with their industrial and scientific importance. Saffman [14] has considered the stability of laminar flow of a dusty gas. Sharma et al. [15] have considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics and have found that the critical Rayleigh number is reduced because of the heat capacity of particles thereby destabilizing the system. The suspended particles were thus found to destabilize the layer. Palaniswami and Purushotham [16] have studied the stability of shear flow of stratified fluids with the fine dust and found that the presence of dust particles increases the region of instability.

On the other hand, multiphase fluid systems are concerned with the motion of liquid or gas containing immiscible inert identical particles of all multiphase fluid systems observed in nature, blood flow in arteries, flow in rocket tubes, dust-in-gas cooling system to enhance heat transfer processes, movement of inset solid particles in atmosphere, and sand or other particles on sea or ocean beaches are the most common examples of multiphase fluid systems. Sharma and Kumar ([17], [18]) have studied the effect of rotation and magnetic field separately, on the thermal convection in micropolar fluids.

The constitutive equations for micropolar fluids which are polar and isotropic with stress tensor T and couple stress tensor C given by Eringen [3] and Petrosyan [19] are

$$T_{ij} = (-p + \lambda e_{kk}) \delta_{ij} + 2\mu e_{ij} + 2\kappa W_{ij} - 2\kappa \varepsilon_{mij} G_m \quad (1)$$

and

$$C_{ij} = \varepsilon' \omega_{k,k} \delta_{ij} + 2\beta' W_{[ij]} + \gamma' W_{(ij)}. \quad (2)$$

Here $e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$, $2W_{ij} = v_{j,i} - v_{i,j}$, $2W_{[ij]} = \omega_{i,j} + \omega_{j,i}$ and $2W_{(ij)} = \omega_{i,j} - \omega_{j,i}$ and T_{ij} , e_{ij} , W_{ij} , C_{ij} , $W_{[ij]}$, $W_{(ij)}$, G_m , ε_{mij} , v and λ , μ are stress tensor, symmetric part of T_{ij} , the vorticity tensor, the couple-stress tensor, symmetric part of spin tensor, antisymmetric part of spin tensor, vorticity vector, the alternating unit tensor, velocity field and material constants, respectively. The dimensions of λ and μ are those of viscosity. Also $\mu \geq 0$, $3\lambda + 2\mu \geq 0$. The positive constant κ in equation (1) represents the dynamic microrotation viscosity. In equation (2) ε' , β' , γ' are constants called coefficients of angular viscosity. The problem of hydromagnetics of micropolar fluids has relevance and importance in chemical engineering, bio-mechanics, astrophysics and electrically conducting colloidal suspensions. Sharma and Kumar [20] have studied the stability of micropolar fluids heated from below in the presence of suspended

particles (fine dust) and have found that suspended particles number density has destabilizing effect on the convection of micropolar fluids. Sharma and Gupta [21] have studied the effect of rotation on the thermal convection of micropolar fluid in the presence of suspended particles.

The present paper, therefore, deals with the stability of electrically conducting micropolar fluid heated from below in the presence of suspended particles in a uniform vertical magnetic field.

2. Mathematical Formulation of the Problem

An infinite horizontal layer of an incompressible electrically conducting micropolar fluids of thickness d permeated with suspended particles (or fine dust) is considered. A uniform vertical magnetic field $H(0, 0, H)$ pervades the system. This fluid-particles layer is heated from below but convection sets in when the temperature gradient ($\beta = \left| \frac{dT}{dz} \right|$) between the lower and upper boundaries exceeds a certain critical value. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid.

Let $v, \vartheta, H, p, \rho, T, g, k_T, c_{pt}, c_v, \mu_e, \eta, \hat{e}_z, u, \delta$ and j denote the velocity, the spin, the magnetic field intensity, the pressure, the density, the temperature, the acceleration due to gravity, the thermal conductivity, the heat capacity of particles, the specific heat at constant volume, the magnetic permeability, the electrical resistivity, the unit vector in z -direction, the particle velocity, the coefficient giving account of coupling between spin and heat flux and microinertial constant, respectively. Assume that external couples and heat sources are not present. If N is the number density and mN is the mass of suspended particles per unit volume, $K = 6\pi\mu r'$, r' being the particle radius, is the Stoke's drag coefficient. Assuming dust particles of uniform size, spherical shape and small relative velocities between the two phases (fluid and particles), the net effect of the particles on the fluid is equivalent to an extra body force term per unit volume $KN(u - v)$, as has been taken in equations of motion. The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distance between the particles is assumed to be so large compared with their diameter that inter particle reactions are ignored. The buoyancy force on the particles is also neglected. The equation of state is

$$\rho = \rho_0 [1 - \alpha (T - T_0)], \quad (3)$$

where ρ_0, T_0 are reference density, reference temperature at the lower boundary and α is the coefficient of thermal expansion. The steady state solution is $v = 0, u = 0, \vartheta = 0, N = N_0$ (constant), $T = T_0 - \beta z, \rho = \rho_0 (1 + \alpha \beta z), p = p_0 - g\rho_0 \left(z + \frac{\alpha \beta z^2}{2} \right)$, where p_0 is the pressure at $z = 0$ and $\beta = \frac{T_0 - T_1}{d}$ ($T_0 > T_1$) is the magnitude of uniform temperature gradient.

Let $v(u, v, w), u(\ell, r, s), \omega, N, \delta p, \delta \rho, \theta$ and $h(h_x, h_y, h_z)$ denote, respectively, the perturbations in fluid velocity $v(0, 0, 0)$, particles velocity $u(0, 0, 0)$, spin ϑ , particles number density N_0 , pressure p , density ρ , temperature T and magnetic field $H(0, 0, H)$ so that the change in density $\delta \rho$ caused by the perturbation θ in temperature is given by

$$\delta \rho = -\rho_0 \alpha \theta.$$

Then the perturbation equations relevant to the problem, using the Boussinesq approximation are

$$\nabla \cdot v = 0, \quad (4)$$

$$\rho_0 \left(\frac{\partial}{\partial t} + v \cdot \nabla \right) v = -\nabla \delta p + (\mu + \kappa) \nabla^2 v + \kappa \nabla \times \omega + \alpha \rho_0 g \theta \hat{e}_z + KN_0 (u - v) + \frac{\mu_e}{4\pi} (\nabla \times h) \times H, \quad (5)$$

$$\rho_0 j \left(\frac{\partial}{\partial t} + v \cdot \nabla \right) \omega = (\varepsilon' + \beta') \nabla (\nabla \cdot \omega) + \gamma' \nabla^2 \omega + \kappa \nabla \times v - 2 \kappa \omega, \quad (6)$$

$$H_1 \left(\frac{\partial}{\partial t} + v \cdot \nabla \right) \theta = \beta (w + h_1 s) + \kappa_T \nabla^2 \theta + \frac{\delta}{\rho_0 c_v} [\nabla \theta \cdot (\nabla \times \omega) - (\nabla \times \omega)_z \cdot \beta], \quad (7)$$

$$\frac{\partial h}{\partial t} = \nabla \times (v \times h) + \eta \nabla^2 h, \quad (8)$$

$$\nabla \cdot h = 0, \quad (9)$$

$$m N_0 \left(\frac{\partial}{\partial t} + u \cdot \nabla \right) u = K N_0 (v - u), \quad (10)$$

$$\frac{\partial M}{\partial t} + \nabla \cdot u = 0, \quad (11)$$

where $H_1 = 1 + h_1$, $h_1 = \frac{f c_{pt}}{c_v}$, $f = \frac{m N_0}{\rho_0}$ and $M = \frac{N}{N_0}$. Using the non-dimensional numbers

$$z = z^* d, \quad \theta = \beta d \theta^*, \quad t = \frac{\rho_0 d^2}{\mu} t^*, \quad v = \frac{\kappa_T}{d} v^*, \quad u = \frac{\kappa_T}{d} u^*,$$

$$p = \frac{\mu \kappa_T}{d^2} p^*, \quad \omega = \frac{\kappa_T}{d^2} \omega^*, \quad h = \left(\frac{\mu \kappa_T}{d^2} \right)^{\frac{1}{2}} h^*, \quad \nabla = \frac{\nabla^*}{d} \quad (12)$$

and then removing the stars for convenience, the non-dimensional forms of equations (4)–(11) become

$$\nabla \cdot v = 0, \quad (13)$$

$$\left(\frac{\partial}{\partial t} + v \cdot \nabla \right) \mathbf{v} = -\nabla \delta p + (1 + K_1) \nabla^2 \mathbf{v} + K_1 \nabla \times \omega + R \theta \hat{e}_z + N_2 (\mathbf{u} - \mathbf{v}) + \frac{\mu e}{4\pi} (\nabla \times h) \times H, \quad (14)$$

$$j_1 \left(\frac{\partial}{\partial t} + v \cdot \nabla \right) \omega = C'_1 \nabla (\nabla \cdot \omega) - C'_0 \nabla \times (\nabla \times \omega) + K_1 (\nabla \times v - 2 \omega), \quad (15)$$

$$H_1 p_1 \left(\frac{\partial}{\partial t} + v \cdot \nabla \right) \theta = \beta (w + h_1 s) + \kappa_T \nabla^2 \theta + \bar{\delta} [\nabla \theta \cdot (\nabla \times \omega) - (\nabla \times \omega)_z], \quad (16)$$

$$\frac{\partial h}{\partial t} = \nabla \times (v \times \vec{h}) + \frac{1}{p_2} \nabla^2 h, \quad (17)$$

$$\nabla \cdot h = 0, \quad (18)$$

$$\left[a \left(\frac{\partial}{\partial t} + v \cdot \nabla \right) + 1 \right] u = v, \quad (19)$$

where new dimensionless coefficients are

$$K_1 = \frac{\kappa}{\mu}, \quad j_1 = \frac{j}{d^2}, \quad \bar{\delta} = \frac{\delta}{\rho_0 c_v d^2}, \quad C'_0 = \frac{\gamma'}{\mu d^2}, \quad C'_1 = \frac{\varepsilon' + \beta' + \gamma'}{\mu d^2}, \quad N_2 = K N_0 \frac{d^2}{\mu}, \quad \kappa_T = \frac{k_T}{\rho_0 c_v}$$

and the dimensionless Rayleigh number R , thermal Prandtl number p_1 , the magnetic Prandtl number p_2 are $R = \frac{g \alpha \beta \rho_0 d^4}{\mu \kappa_T}$, $p_1 = \frac{v}{\kappa_T}$, $p_2 = \frac{v}{\eta}$. Let us assume both the boundaries to be free, perfectly heat conducting and the medium adjoining the fluid is electrically non-conducting. The case of two free boundaries, though little artificial is the most appropriate for stellar atmosphere. Since the surfaces are fixed and are maintained at fixed temperature, $w = 0 = \theta$ at $z = 0$ and $z = d$. Then the appropriate boundary conditions are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial}{\partial z} (\nabla \times v)_z = 0, \quad (\nabla \times h)_z = (\nabla \times \omega)_z = 0, \quad \theta = \frac{\partial h_z}{\partial z} = 0 \quad \text{at } z = 0 \quad \text{and } z = d. \quad (20)$$

2.1. Linear Theory: Dispersion Relation

Under the linearized theory, second and higher order terms are neglected and only the linear terms are retained. Accordingly, the non-linear terms $(v \cdot \nabla) v, (v \cdot \nabla) \theta, (v \cdot \nabla) \omega, \nabla \theta \cdot (\nabla \times \omega)$ in equations (14)–(16) are neglected. Eliminating s between equations (16) and (19) and applying the curl operator twice to resulting equation, we obtain

$$L_2 \left[H_1 p_1 \frac{\partial}{\partial t} - \nabla^2 \right] \theta = \left(a \frac{\partial}{\partial t} + H_1 \right) \beta w - L_2 \bar{\delta} \Omega_z. \quad (21)$$

Eliminating u between (14) and (19), we obtain

$$L_1 v = L_2 \left[-\nabla \delta p + (1 + K_1) \nabla^2 v + K_1 \nabla \times \omega + R \theta \hat{e}_z + \frac{\mu_e}{4\pi} (\nabla \times h) \times H \right], \quad (22)$$

where $L_1 = a \frac{\partial^2}{\partial t^2} + F \frac{\partial}{\partial t}$, $L_2 = a \frac{\partial}{\partial t} + 1$ and $F = f + 1$. Applying the curl operator twice to equation (14) and taking z -component, we get

$$L_1 \nabla^2 w = L_2 \left[R \nabla_1^2 \theta + (1 + K_1) \nabla^4 w + K_1 \nabla^2 \Omega_z + \frac{\mu_e H}{4\pi} \frac{\partial}{\partial z} \nabla^2 h_z \right], \quad (23)$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad \Omega_z = (\nabla \times \omega)_z. \quad (24)$$

Applying the curl operator to equations (14), (15) and (17), taking z -component, we get

$$L_2 \frac{\partial}{\partial t} \zeta_z + n_1 \zeta_z (L_2 - 1) = (1 + K_1) \nabla^2 \zeta_z L_2 + \frac{\mu_e H}{4\pi} \frac{\partial \xi_z}{\partial z} L_2, \quad (25)$$

$$j_1 \frac{\partial \Omega_z}{\partial t} = C'_0 \nabla^2 \Omega_z - K_1 (\nabla^2 w + 2\Omega_z), \quad (26)$$

$$\frac{\partial \xi_z}{\partial t} = H \frac{\partial}{\partial t} \zeta_z + \frac{1}{p_2} \nabla^2 \xi_z, \quad (27)$$

where $\xi_z = (\nabla \times h)_z$, $\zeta_z = (\nabla \times v)_z$ are the z -components of current density and vorticity, respectively. K_1 and C'_0 account for coupling between vorticity and spin effects and spin diffusion, respectively. Taking the z -component of equation (17), we get

$$\frac{\partial h_z}{\partial t} = H \frac{\partial w}{\partial z} + \frac{1}{p_2} \nabla^2 h_z. \quad (28)$$

Analyzing the disturbances into normal modes, we ascribe to all quantities describing the perturbation a dependence on x, y, z and t of the form

$$[w, \Omega_z, \zeta_z, \xi_z, \theta, h_z] = [W(z), \Omega(z), Z(z), G(z), \Theta(z), B(z)] \exp(ik_x x + ik_y y + nt), \quad (29)$$

where $k = (k_x^2 + k_y^2)^{\frac{1}{2}}$ is the resultant wave-number, k_x and k_y are real constants and n is the stability parameter which is complex, in general. Then the equations (21), (23) and (25)–(28) using expression (29) become

$$(an + 1) \{H_1 p_1 n - (D^2 - k^2)\} \Theta = (an + H_1) W - (an + 1) \bar{\delta} \Omega, \quad (30)$$

$$(D^2 - k^2) \{ (an^2 + Fn) - (an + 1)(1 + K_1)(D^2 - k^2) \} W = \left\{ (an + 1) - Rk^2 \Theta + K_1 (D^2 - k^2) \Omega + \frac{\mu_e H}{4\pi} (D^2 - k^2) DB \right\}, \quad (31)$$

$$\{(an^2 + Fn) - (an + 1)(D^2 - k^2)(1 + K_1)\} Z = \frac{\mu_e H}{4\pi} (an + 1) DG, \quad (32)$$

$$\{\ell_1 n + 2A - (D^2 - k^2)\} \Omega = -A(D^2 - k^2) W, \quad (33)$$

$$\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\} G = H DZ, \quad (34)$$

$$\left\{n - \frac{1}{p_2}(D^2 - k^2)\right\} B = H DW, \quad (35)$$

where $A = \frac{K_1}{C_0}$, $\ell_1 = j_1 \frac{A}{K_1}$. Eliminating Θ , Z , B , Ω from equations (30)–(35), we get

$$\begin{aligned} (D^2 - k^2) \{(an^2 + Fn) - (an + 1)(1 + K_1)(D^2 - k^2)\} \{H_1 p_1 n - (D^2 - k^2)\} \{\ell_1 n + 2A - (D^2 - k^2)\} \\ \{\ell_1 n + 2A - (D^2 - k^2)\} W = -Rk^2 \{\ell_1 n + 2A - (D^2 - k^2)\} \left\{n - \frac{1}{p_2}(D^2 - k^2)\right\} (an + H_1) W \\ - Rk^2 \left\{n - \frac{1}{p_2}(D^2 - k^2)\right\} (an + 1) \bar{\delta} A (D^2 - k^2) W - AK_1 (D^2 - k^2)^2 (an + 1) \{H_1 p_1 n - (D^2 - k^2)\} \\ \left\{n - \frac{1}{p_2}(D^2 - k^2)\right\} W + \frac{H^2}{4\pi} (D^2 - k^2) \{H_1 p_1 n - (D^2 - k^2)\} (an + 1) \{\ell_1 n + 2A - (D^2 - k^2)\} D^2 W. \end{aligned} \quad (36)$$

The boundary conditions (20) transform to

$$W = 0, \quad D^2 W = 0, \quad DZ = 0, \quad G = 0, \quad \Omega = 0, \quad \Theta = 0, \quad DB = 0 \quad \text{at } z = 0 \text{ and } 1. \quad (37)$$

Using boundary equations (36), equations (30)–(35) give

$$D^2 \Theta = 0, \quad D^2 \Omega = 0, \quad D^3 Z = 0, \quad D^3 G = 0, \quad D^3 B = 0. \quad (38)$$

The proper solution of equation (36) satisfying the boundary conditions (37), (38) and characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (39)$$

where W_0 is a constant. Substituting the value of W from (39) in equation (36) and putting $b = \pi^2 + k^2$, we obtain the dispersion relation

$$\begin{aligned} Rk^2 \left\{n + \frac{b}{p_2}\right\} \{(an + H_1)(\ell_1 n + 2A + b) - (an + 1)\bar{\delta} Ab\} = b \{(an^2 + Fn) + (an + 1)(1 + K_1)b\} \\ (H_1 p_1 n + b)(\ell_1 n + 2A + b) \left\{n + \frac{b}{p_2}\right\} - K_1 A b^2 (an + 1)(H_1 p_1 n + b) \left(n + \frac{b}{p_2}\right) \\ + \frac{H^2 \pi}{4} (H_1 p_1 n b + b^2) (an + 1)(\ell_1 n + 2A + b) \end{aligned} \quad (40)$$

2.2. The Case of Oscillatory Modes

Equating the imaginary parts of equation (40), we have

$$\begin{aligned} n_i \left[n_i^4 (ab H_1 p_1 \ell_1 - abn) + n_i^2 \left(-2A H_1 p_1 \frac{ab^2}{p_2} - H_1 p_1 \frac{ab^3}{p_2} - \ell_1 n_i^2 \frac{ab^3}{p_2} - 2AFb - F H_1 p_1 b^2 - F \ell_1 b^2 \right. \right. \\ \left. \left. - F H_1 p_1 \ell_1 \frac{b^2}{p_2} - H_1 p_1 K_1 b^3 - ab^3 \ell_1 K_1 - a H_1 p_1 \ell_1 K_1 \frac{b^3}{p_2} - 2A H_1 p_1 ab^2 - H_1 p_1 ab^4 - ab^3 \ell_1 - 2A H_1 p_1 a K_1 b^2 \right. \right. \\ \left. \left. - H_1 p_1 \ell_1 b^2 - H_1 p_1 \ell_1 \frac{ab^3}{p_2} + Rk^2 a \ell_1 \right) + \frac{ab^5}{p_2} (K_1 + 1) + \frac{b^4}{p_2} (aK_1 + 2Aa + \ell_1 + H_1 p_1 + F) + \frac{b^3}{p_2} (H_1 p_1 2A + 2AF) \right. \\ \left. + \frac{b^2}{p_2} (-Rk^2 a + Rk^2 a \bar{\delta} A) + \frac{b}{p_2} (-2Rk^2 a A - Rk^2 H_1 \ell_1) + b (-Rk^2 H_1 - \bar{\delta} A) + b^4 + 2Ab^3 - 2Rk^2 H_1 A \right] = 0. \end{aligned} \quad (41)$$

Equation (41) yields that either $n_i = 0$ or $n_i \neq 0$, which means that the modes are either non-oscillatory or oscillatory. In the absence of suspended particles number density, magnetic field intensity and magnetic permeability, equation (41) reduces to

$$n_i (b^4 \ell_1 + Rk^2 \bar{\delta} Ab) = 0 \quad (42)$$

and term within the brackets is definitely positive, which implies that $n_i = 0$. Therefore, the oscillatory modes are not allowed and principal of exchange of stabilities is valid in the absence of suspended particles and magnetic field.

2.3. The Case of Overstability

Since for overstability, we wish to determine Rayleigh number for the onset of overstability via a state of pure oscillations, it suffices to find conditions for which equation (40) will admit of solution with n_i real. Substituting $n = in_i$ in equation (40), the real and imaginary parts of equation (40), yield

$$\begin{aligned} & Rk^2 \left[\frac{b^2}{p_2} \{2H_1 A + b(1 - \bar{\delta} A)\} - n_i^2 \left\{ bl_1 \left(1 + \frac{a}{p_2}\right) + \{2aAH_1 + b(1 - \bar{\delta} A)\} \right\} \right] = \\ & n_i^4 \left[H_1 p_1 \ell_1 b^2 \{1 + a(1 + K_1)\} + ab \left\{ bl_1 \left(1 + \frac{H_1 p_1}{p_2}\right) \right\} + H_1 p_1 (2aA + F\ell_1) \right] - n_i^2 \left[\left\{ (2A + b) \left(1 + \frac{H_1 p_1}{p_2}\right) + \frac{H_1 b \ell_1}{p_2} \right\} \right. \\ & \left. \{1 + a(1 + K_1)\} b^3 + b^3 (2aA + F\ell_1) \left\{ H_1 p_1 (1 + K_1) + \frac{a}{p_2} \right\} + b^4 a \ell_1 (1 + K_1) \left(1 + \frac{H_1 p_1}{p_2}\right) - K_1 A b^3 \right. \\ & \left. \left\{ p_1 H_1 + a \left(1 + \frac{p_1 H_1}{p_2}\right) \right\} \right] + \frac{\pi H^2 b}{4} [-n_i^2 \{bl_1 (H_1 p_1 + a) + H_1 p_1 (2A + b)\} + (2A + b) b^2] \\ & + \left[\frac{1}{p_2} (1 + K_1) b^6 + \frac{A}{p_2} (2 + K_1) b^5 \right] \end{aligned} \quad (43)$$

and

$$\begin{aligned} & Rk^2 \left[-a \ell_1 n_i^3 + 2A n_i H_1 + n_i H_1 b - n_i \bar{\delta} Ab + \frac{2Ab}{p_2} a n_i + \frac{b^2}{p_2} a n_i + \frac{b}{p_2} H_1 \ell_1 n_i - \frac{1}{p_2} a n_i \bar{\delta} A b^2 \right] \\ & = ab H_1 p_1 \ell_1 n_i^5 - 2A ab^2 n_i^4 - ab n_i^5 - 2A H_1 p_1 n_i^3 \frac{ab^2}{p_2} - H_1 p_1 n_i^3 \frac{ab^3}{p_2} - \ell_1 n_i^3 \frac{ab^3}{p_2} - 2A F n_i^3 b \\ & - F n_i^3 H_1 p_1 b^2 - F n_i^3 \ell_1 b^2 - F H_1 p_1 \ell_1 n_i^3 \frac{b^2}{p_2} + 2A F n_i \frac{b^3}{p_2} + F n_i \frac{b^4}{p_2} - H_1 p_1 n_i^3 K_1 b^3 - a n_i^3 b^3 \ell_1 K_1 \\ & - a H_1 p_1 \ell_1 n_i^3 K_1 \frac{b^3}{p_2} + n_i a K_1 \frac{b^4}{p_2} + a n_i K_1 \frac{b^5}{p_2} - 2H_1 p_1 n_i^3 ab^2 A - H_1 p_1 n_i^3 ab^4 - ab^3 \ell_1 n_i^3 \\ & - H_1 p_1 \ell_1 n_i^3 \frac{ab^3}{p_2} + 2A a n_i \frac{b^4}{p_2} + \frac{b^5}{p_2} a n_i - 2A H_1 p_1 n_i^3 a K_1 b^2 - H_1 p_1 \ell_1 n_i^3 b^2 + 2Ab^3 n_i + b^4 n_i + \frac{b^4}{p_2} \ell_1 n_i \\ & + 2A H_1 p_1 n_i \frac{b^3}{p_2} + H_1 p_1 n_i \frac{b^4}{p_2}. \end{aligned} \quad (44)$$

Eliminating R between equations (43) and (44), we get

$$\begin{aligned} & n_i^6 \left[-a^2 \ell_1^2 \{1 + H_1 p_1 (1 + K_1)\} b^2 + ab \ell_1 H_1 p_1 (H_1 \ell_1 - ab \bar{\delta} A - F\ell_1) - b^3 a \ell_1 H_1 \left(\frac{1}{p_2} + (1 + K_1) \right) \right] \\ & + n_i^4 \left[b^5 \left\{ H_1 p_1 a^2 (1 - \bar{\delta} A) + H_1 p_1 \ell_1 \bar{\delta} A a \frac{1}{p_2} (1 + K_1) + \frac{p_1 \bar{\delta} A (H_1 - 1)}{p_2} \right\} \right. \\ & \left. + b^4 \left\{ 2H_1 p_1 a^2 (1 + K_1) A - a^2 \bar{\delta} A \ell_1 (1 + K_1) - a^2 H_1^2 p_1 (H_1 - 1) \frac{p_1}{p_2} \right\} \right. \\ & \left. + b^3 \left\{ H_1 p_1 F a (1 - \bar{\delta} A) + H_1 p_1 \ell_1^2 \frac{a^2}{p_2} (H_1 - 1) - H_1 p_1 \ell_1 K_1 a (2 - \bar{\delta} A) + \frac{1}{p_2^2} H_1 p_1 a \ell_1^2 (F - a K_1) \right\} \right. \\ & \left. + b^2 \left\{ \frac{-2a}{p_2} A F \ell_1 (H_1 - 1) - \frac{H^2 \pi}{4} a \ell_1 \left(a \ell_1 - \frac{p_1}{p_2} + p_1 \bar{\delta} A \right) - 2A a \ell_1 H_1 (H_1 - 1) \right\} \right. \\ & \left. + b \left\{ -2A^2 a H_1^2 p_1 (H_1 - 1) - \frac{\pi H^2}{4} a \ell_1 2A \left(a \ell_1 - \frac{p_1}{p_2} a \ell_1 + p_1 \bar{\delta} A \right) \right\} \right] \end{aligned}$$

$$\begin{aligned}
 & + n_i^2 \left[b^7 \left\{ -H_1 p_1 \frac{a}{p_2^2} (1 - \bar{\delta}A) - \bar{\delta}A \frac{a}{p_2^2} (1 + K_1) \right\} + b^6 \left\{ \frac{a^2}{p_2} \bar{\delta}A (1 + K_1) - H_1 p_1 \ell_1 \frac{1}{p_2^2} (2 - \bar{\delta}A) \right. \right. \\
 & - 2H_1^2 \frac{p_1}{p_2} aA (1 + K_1) - \frac{H^2 \pi}{4} H_1 p_1 \frac{1}{p_2} (2 - \bar{\delta}A) - H_1 p_1 \bar{\delta}A \frac{1}{p_2} (F - aK_1) \left. \right\} + b^5 \left\{ -H_1 p_1 \ell_1 \frac{a}{p_2} (H_1 - 1) \right. \\
 & + H_1 p_1 a^2 \frac{1}{p_2^2} (1 + K_1) + \frac{a \bar{\delta}A}{p_2} (1 + K_1) - \frac{\ell_1^2}{p_2^2} (2Aa + F\ell_1) - 2A^2 \frac{\bar{\delta}}{p_2} (H_1 - F) + H_1 p_1 \frac{a}{p_2^2} (F - aK_1) \\
 & + \frac{K_1 Aa}{p_2^2} (\ell_1 + H_1 p_1 \bar{\delta}A) + \frac{2Aa}{p_2^2} \{2 + 2H_1 p_1 (1 + K_1) - K_1 p_1\} + 2 \{1 + H_1 p_1 (1 + K_1)\} (1 - \bar{\delta}A) \left. \right\} \\
 & + b^4 \left\{ H_1 p_1 - \frac{1}{p_2} (F\bar{\delta} - \bar{\delta} - aK_1) + \frac{H_1 p_1 \ell_1}{p_2^2} (F\ell_1 + H_1 p_1) - \frac{H^2 \pi}{4} \left(a^2 \ell_1 - \frac{p_1}{p_2} a \ell_1 + p_1 \bar{\delta}A \right) \right. \\
 & + \frac{\ell_1^2}{p_2^2} (H_1 - F) - \frac{H^2 \pi}{4} \left(\left\{ F \left(1 - \frac{p_1}{p_2} \right) - \frac{H_1 p_1 \ell_1}{p_2} \right\} (1 - \bar{\delta}A) + \frac{F\ell_1}{p_2} (H_1 p_1 + \bar{\delta}A) \right) + 4A^2 \{1 + H_1 p_1 (1 + K_1)\} \\
 & + \frac{2A^2 F}{p_2^2} \{2 + 2H_1 p_1 (1 + K_1) - K_1 p_1\} \left. \right\} \\
 & + b^3 \left\{ \frac{1}{p_2} 2H_1^2 p_1 A \ell_1 F (H_1 - 1) + \frac{H^2 \pi}{4} \left[H_1 p_1 (2 - \bar{\delta}A) \left(1 - \frac{H_1 p_1}{p_2} \right) \right] - \frac{H^2 \pi}{4} \left[F \left(1 - \frac{H_1 p_1}{p_2} \right) \frac{H_1 p_1 \ell_1}{p_2} \right] \right. \\
 & + 2H_1 p_1 A \frac{1}{p_2} (F\bar{\delta} - \bar{\delta} - aK_1) - \frac{K_1 a \ell_1}{p_2} (H_1 - 1) + \frac{H_1 \ell_1}{p_2^2} (\ell_1 + H_1 p_1 \bar{\delta}A) + 2H_1^2 p_1 F \frac{1}{p_2} A (1 - \bar{\delta}A) \left. \right\} \\
 & + b^2 \left\{ \frac{H^2 \pi}{4} H_1 p_1 \ell_1 \frac{a}{p_2} (2 - \bar{\delta}A) + \frac{H^2 \pi}{4} p_1 H_1 \ell_1 (1 - \bar{\delta}A) + H_1 p_1 \frac{1}{p_2} (2aA + F\ell_1) - \frac{\pi H^2}{4} H_1 p_1 \ell_1 F \left(1 - \frac{p_1}{p_2} \right) \right. \\
 & + b \left\{ \frac{2H^2 \pi}{4} H_1 p_1 \ell_1 F A \left(1 - \frac{p_1}{p_2} \right) \right\} + b^8 \left[\frac{1}{p_2^2} \{1 + H_1 p_1 (1 + K_1) (2 - \bar{\delta}A) - F\ell_1 (1 + K_1) \bar{\delta}Aa\} \right] \\
 & + b^7 \left\{ \frac{4A}{p_2^2} \{1 + H_1 p_1 (1 + K_1)\} (1 - \bar{\delta}A) + 2aA \{1 + H_1 p_1 (1 + K_1)\} \right\} + b^6 \left[\frac{2A^2 F}{p_2^2} \{1 + H_1 p_1 (1 + K_1)\} (1 - \bar{\delta}A) \right. \\
 & + \frac{1}{p_2^2} H_1 p_1 (H_1 - 1) + \frac{H^2 \pi}{4} \left\{ \left(\frac{H_1 p_1}{p_2} - 1 \right) (2 - \bar{\delta}A) - \frac{H_1 p_1 \ell_1}{p_2} \bar{\delta}AF \right\} + b^5 \left[4H_1 p_1 A^2 \frac{1}{p_2^2} (2 - \bar{\delta}A) \right. \\
 & + \frac{2}{p_2^2} AH_1 (F - aK_1) + \frac{H^2 \pi}{4} \left\{ 2AF \left(\frac{H_1 p_1}{p_2} - 1 \right) (2 - \bar{\delta}A) - \frac{H_1 p_1 F}{p_2} \bar{\delta}AF \right\} + b^4 \left[\frac{-4}{p_2} A^2 H_1 (1 + K_1) \right. \\
 & + \frac{H^2 \pi}{4} \left\{ (1 - \bar{\delta}A) \left(\frac{H_1 p_1}{p_2} - 1 \right) + 2A^2 \frac{a}{p_2^2} K_1 (H_1 - 1) \right\} + b^3 \left[\frac{-H^2 \pi}{2} \frac{Aa}{p_2} (H_1 - 1) + \frac{H_1 a}{p_2} (1 + K_1) \right] \\
 & + b^2 \left[\frac{H^2 \pi}{4} \left\{ (2 - \bar{\delta}A) \frac{\ell_1 H_1}{p_2} \right\} - \frac{H^2 \pi}{4} \frac{H_1 a}{p_2} \right] = 0. \tag{45}
 \end{aligned}$$

It is evident from the equation (45) that overstable modes will not be present for all values of parameters. For example, in the absence of coupling between spin and heat flux ($\bar{\delta} = 0$), magnetic field ($H = 0$) and in the absence of suspended particles ($a = 0 = f = h_1$), equation (45) allows only $n_i = 0$ and so overstable solution will not take place if $K_1 p_1 < 2$. Thus for stationary convection i.e. $n_i = 0$ and in the presence of coupling between spin and heat fluxes ($\bar{\delta} \neq 0$), equation (43) reduces to

$$R = \frac{b^4 (1 + K_1) + A b^3 (2 + K_1) + \frac{H^2 \pi}{4} (2A + b) b p_2}{k^2 \{ H_1 2A + b (1 - \bar{\delta}A) \}}. \tag{46}$$

In the absence of magnetic field intensity ($H = 0$), equation (46) reduces to

$$R = \frac{b^4 (1 + K_1) + A b^3 (2 + K_1)}{k^2 \{ 2H_1 A + b (1 - \bar{\delta}A) \}}, \tag{47}$$

a result in good agreement with Sharma and Kumar [20].

3. Results and Discussions

Equation (45) has been examined numerically using the Newton–Raphson method through the software Fortran 90 and Mathcad. The variation of Rayleigh number with respect to wave-numbers using equation (43) satisfying equation (45) for overstable case and equation (47) for stationary case, for the fixed permissible values of the dimensionless parameters $K_1 = 1$, $A = 0.5$, $\bar{\delta} = 1$, $\ell_1 = 1$, $p_1 = 5$, $p_2 = 1$, $a = 10$, $F = 1.005$ and $H_1 = 1.01$. These values are the permissible values

for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar [7] while describing various hydrodynamic and hydromagnetic stability problems.

Figures 1-4 represent the behaviour of the Rayleigh number for both the cases of overstability and stationary convection w.r.t. k for four different values of the magnetic field intensity $H = 70, 100, 120$ and 0 Gauss, respectively. From the curves we observe that Rayleigh number increases with increase in magnetic field intensity for a fixed wave-number depicting thereby the stabilizing effect of magnetic field intensity.

Figures 5-7 correspond to three values of micropolar coefficient $\kappa = 0.5, 0.7$ and 1.0 , respectively, accounting for dynamic microrotation viscosity. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient κ for a fixed wave-number implying thereby the destabilizing effect of dynamic microrotation viscosity.

Figures 8-10 correspond to three values of micropolar coefficient $\gamma' = 1.0, 1.2$ and 1.4 , respectively, accounting for coefficient of angular viscosity. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient γ' for a fixed wave-number implying thereby the destabilizing effect of coefficient of angular viscosity.

Thus there is a competition between the large enough stabilizing effect of magnetic field intensity and the destabilizing effect of the micropolar coefficients. The presence of coupling between thermal and micropolar effects, magnetic field and suspended particles number density may bring overstability in the system. It is also noted from the figures 3-10 that the Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave-number. However, the reverse may also occur for large wave-numbers, as has been depicted in figures 1 and 2 for $H=70, 100$ Gauss, respectively.

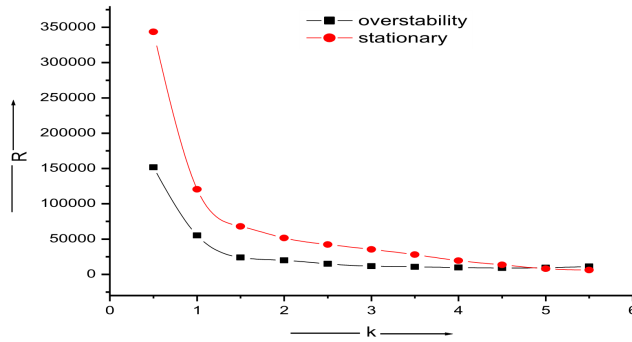


Figure 1. The variation of Rayleigh number (R) with wave number (k) for H=70 Gauss

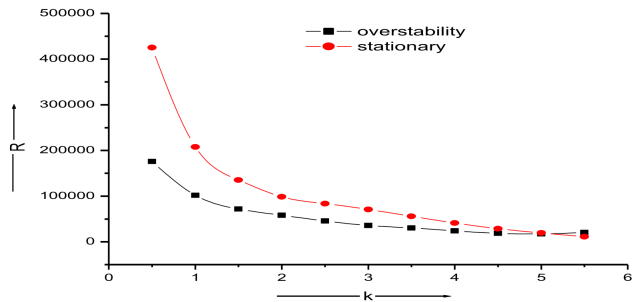


Figure 2. The variation of Rayleigh number (R) with wave number (k) for H=100 Gauss

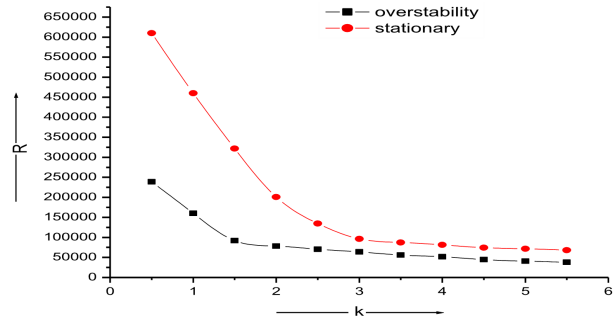


Figure 3. The variation of Rayleigh number (R) with wave number (k) for H=120 Gauss

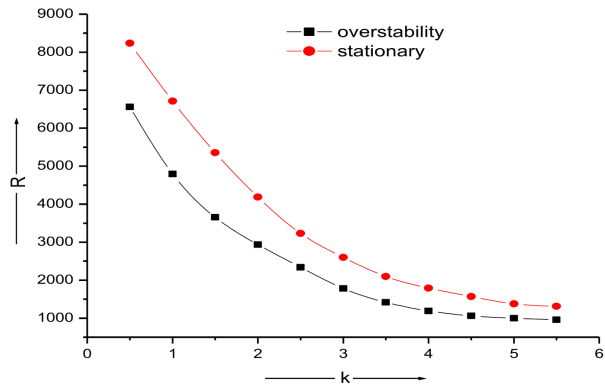


Figure 4. The variation of Rayleigh number (R) with wave number (k) for H=0 Gauss

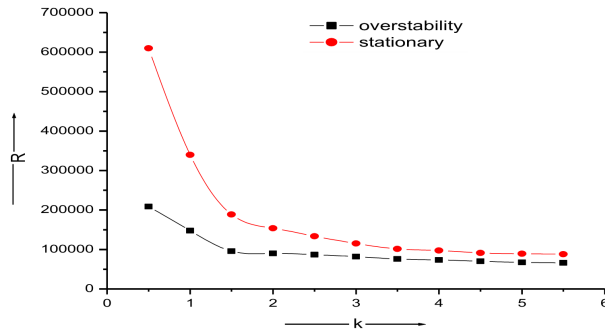


Figure 5. The variation of Rayleigh number (R) with wave number (k) for $\kappa = 0.5$ Gauss

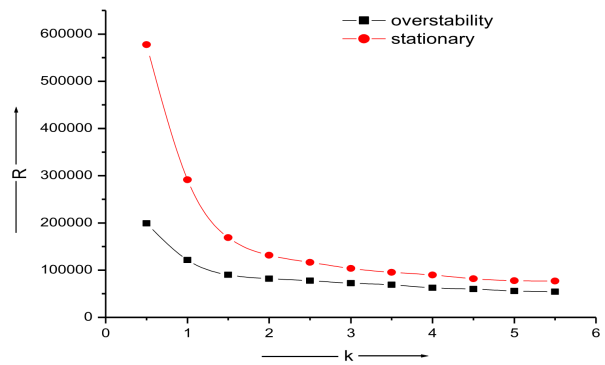


Figure 6. The variation of Rayleigh number (R) with wave number (k) for $\kappa = 0.7$ Gauss

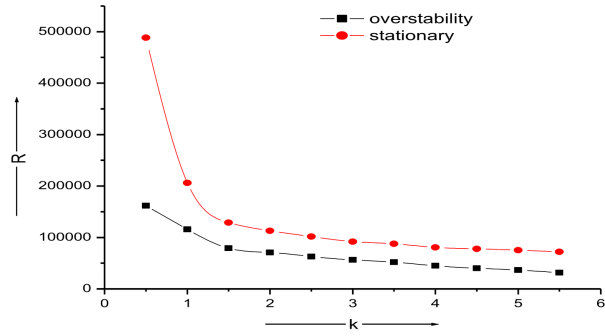


Figure 7. The variation of Rayleigh number (R) with wave number (k) for $\kappa = 1.0$ Gauss

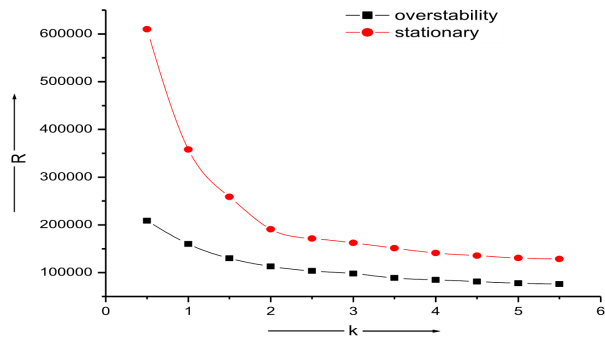


Figure 8. The variation of Rayleigh number (R) with wave number (k) for $\gamma' = 1.0$ Gauss

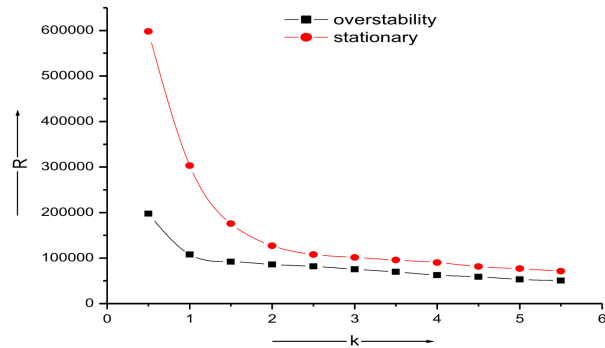


Figure 9. The variation of Rayleigh number (R) with wave number (k) for $\gamma' = 1.2$ Gauss

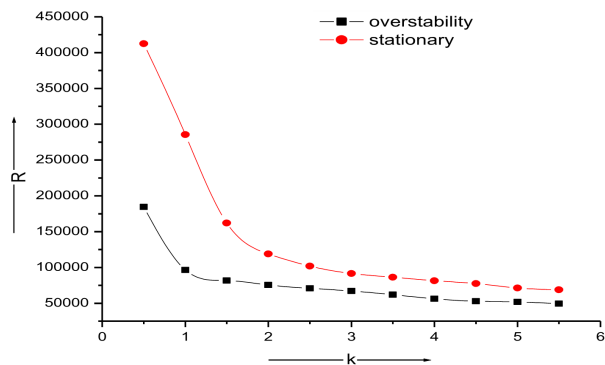


Figure 10. The variation of Rayleigh number (R) with wave number (k) for $\gamma' = 1.4$ Gauss

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