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Forbidden Subgraphs for Planar Blict and Blitact Graphs

Research Article

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Abstract: Many graphs which are encountered in the study of graph theory are characterized by a type of configuration or subgraphs they possess. However, there are occasions when such graphs are more easily defined or described by the kind of subgraphs they are not permitted to contain. Such subgraphs are called forbidden subgraphs. In this paper, we present characterizations of graphs whose blict and blitact graphs are planar, outerplanar, minimally nonouterplanar and 2-minimally nonouterplanar in terms of forbidden subgraphs.

Keywords: Forbidden, inner point number, planar, outerplanar, k-minimally nonouterplanar, blict graph, blitact graph.

1. Introduction

Normally a characterization of graphs having a given property by means of "forbidding" a certain family of subgraphs has a great interest due to its practical applications. Greenwell and Hemminger [1] characterized graphs with planar line graphs in terms of forbidden subgraphs. In this paper, we shall consider a graph as a nontrivial connected graph. We use the terminology of [5].

In [6], the idea of a minimally nonouterplanar graph is introduced. The inner point number $i(G)$ of a planar graph G is the minimum possible number of points not belonging to the boundary of the exterior region in any boundary of G in the plane. Obviously G is planar if and only if $i(G)=0$. A graph G is minimally nonouterplanar if $i(G)=1$, and G is k-minimally nonouterplanar ($k\geq 2$) if $i(G)=k$. A line of a plane graph G is called a boundary line if it is on the boundary of the exterior region, otherwise it is called a nonboundary line. If $B=\{u_1, u_2, ..., u_r; r \geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cutpoint, then they are adjacent blocks. The blocks, cutpoints and lines of a graph are called its members.

The blict graph $B_n(G)$ of a graph G as the graph whose set of points is the union of the set of blocks, cutpoints and lines of G and in which two points are adjacent if and only if the corresponding blocks and lines of G are adjacent or the corresponding members are incident.

The blitact graph $B_m(G)$ of a graph G as the graph whose point set is the union of the set of blocks, cutpoints and lines of G and in which two points are adjacent if and only if the corresponding members are adjacent or incident. These concepts were

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introduced by Kulli and Biradar [2]. Many other graph valued functions in graph theory were studied, for example, in [6–22].

The following will be useful in the proof of our results.

Remark 1.1 ([2]). *If $G = K_{1,p}$, $p \geq 2$ then $B_n(G)$ (or $B_m(G)$) $= K_{p+1}.K_{p+1}$.*

Theorem 1.2 ([4]). *A graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$.*

Theorem 1.3 ([2]). *The blict graph $B_n(G)$ of a graph G is planar if and only if G satisfies the following conditions;*

(1). G is planar.

(2). The degree of each point of G is at most three.

(3). A cutpoint is not adjacent to other three cutpoints.

(4). A cutpoint incident with a nonline block B is not adjacent to other two.

(5). Cutpoints either of one is not incident with B.

(6). If a block B has two non-adjacent cutpoints then either of one should not be adjacent to other cutpoint which is not incident with B.

Theorem 1.4 ([2]). *The blict graph $B_n(G)$ of a graph G is outerplanar if and only if G is a path of length at most three or G is a cycle.*

Theorem 1.5 ([2]). *The blict graph $B_n(G)$ of a graph G is minimally nonouterplanar if and only if G satisfies the following conditions;*

(1). deg $\nu \leq 3$ for every point ν of G and

(2). G is a block with exactly two points of degree 3 and these are adjacent or

(3). G is a cycle together with an endline adjoined to some point or

(4). G is a path of length 4.

Theorem 1.6 ([2]). *Let G be a connected graph. Then $B_n(G) = B_m(G)$ if and only if G has at most one cutpoint or no two cutpoints of G are adjacent.*

Theorem 1.7 ([3]). *The blict graph $B_n(G)$ of a graph G is 2-minimally nonouterplanar if and only if G satisfies the following conditions:*

(1). deg $\nu \leq 3$ for every point ν of G and

(2). G is a m-minimally nonouterplanar (p, q) block with $q - p + m = 2$ or

(3). G has exactly two noncutpoints of degree 3 and these are adjacent and a unique cutpoint ν of degree 3, ν lies on two blocks of G in which one block has an endpoint of G or

(4). G is a triangle together with a path of length 2 adjoined to some point or

(5). G is a (p, q) tree, $4 \leq P \leq 6$ except P_4 and P_5 and if $\Delta(G) = 3$ then it has exactly one point of degree 3 and at most one point of degree 2.

2. Main Results

In the following theorem we characterize those graphs whose blict graphs are planar in terms of forbidden subgraphs by using Theorem 1.3.

Theorem 2.1. *A graph G has a planar blict graph if and only if it has no subgraph homeomorphic to $K_{3,3}$ or $K_{1,4}$ and also to G_1 or G_2 of Fig. 1 with respect to the cutpoints.*

Proof. Let G be a graph with a planar blict graph. We now show that all graphs homeomorphic to $K_{3,3}$ or $K_{1,4}$ and also to G_1 or G_2 with respect to the cutpoints, have nonplanar blict graphs. It follows from Theorem 1.3, since graphs homeomorphic to $K_{3,3}$ are nonplanar, graphs homeomorphic to $K_{1,4}$ have a point of degree 4, graphs homeomorphic to G_1 with respect to the cutpoints have a cutpoint which is adjacent to other 3 outpoints, graphs homeomorphic to G_2 with respect to the cutpoints have a cutpoint incident with a nonline block B and is adjacent to other 2 cutpoints either of one is not incident with B.

Conversely, suppose that G contains no subgraph homeomorphic to $K_{3,3}$ or $K_{1,4}$ and also G_1 or G_2 with respect to the cutpoints. It implies that G has no subgraph homeomorphic to $K_{3,3}$ or K_5. Then by Theorem 1.2, G is planar.

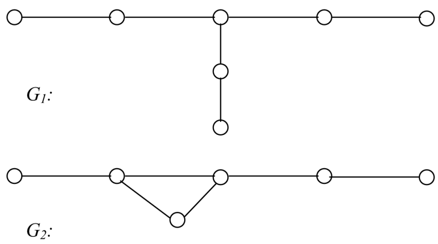


Figure 1.

Now assume $\Delta(G) = 4$. Then G has a point of degree 4. Then it has a subgraph homeomorphic to $K_{1,4}$, a contradiction. Thus degree of each point is at most three. Let ν be the cutpoint of degree 3. We consider the following cases.

Case 1. Suppose ν lies on 3 blocks such that G is $K_{1,3}$ together with the paths of length ≥ 1 adjoined to each endpoint. Then G has a subgraph homeomorphic to G_1 with respect to the cutpoints, a contradiction.

Case 2. Suppose ν lies on 2 blocks such that G is a cycle together with a path P_m $(m \geq 2)$ is adjoined at ν and suppose a path P_n $(n \geq 1)$ is adjoined at a point u $(u \neq \nu)$ on the cycle. Then G has a subgraph homeomorphic to G_2, again a contradiction.

From the above cases, we conclude that (i) a cutpoint is not adjacent to other three cutpoints (ii) a cutpoint incident with a nonline block B is not adjacent to other 2 cutpoints either of one is not incident with B and (iii) if a block B has non adjacent cutpoints then either of one is not adjacent to the other cutpoint which is not incident with B. Thus Theorem 1.3 implies that G has a planar blict graph. □

We now establish a characterization of graphs whose blict graphs are outerplanar in terms of forbidden subgraphs by using Theorem 1.4.

Theorem 2.2. *A graph G has an outerplanar blict graph if and only if it has no subgraph homeomorphic to $K_{1,3}$ and also to P_5 with respect to the cutpoints.*

Proof. Let G be a graph with an outerplanar blict graph. We now show that all graphs homeomorphic to $K_{1,3}$ and also to P_5 with respect to the cutpoints, have nonouterplanar blict graphs. It follows from Theorem 1.4, since graphs homeomorphic to $K_{1,3}$ have a point of degree 3 and therefore they are neither paths nor cycles, graphs homeomorphic to P_5 with respect to the cutpoints have a path of length at least 4.

Conversely, suppose that G contains no subgraph homeomorphic to $K_{1,3}$ and also P_5 with respect to the cutpoints. Now assume $\Delta(G) = 3$. Then G has a point of degree 3. Then it has a subgraph homeomorphic to $K_{1,3}$, a contradiction. Thus $\Delta(G) \leq 2$. Then G is either a cycle or a path. Assume G is a path of length 4. Then it has a subgraph homeomorphic to P_5, again a contradiction. Therefore, G is a path of length at most three. Thus Theorem 1.4 implies that G has an outerplanar blict graph. □

We now characterize those graphs whose blict graphs are minimally nonouterplanar in terms of forbidden subgraphs by using Theorem 1.5.

Theorem 2.3. *Let G be a graph for which the blict graph $B_n(G)$ is nonouterplanar. Then $B_n(G)$ is minimally nonouterplanar if and only if $G \neq K_{1,3}$ and it has no subgraph homeomorphic to $K_{1,4}$, $K_{2,3}$, K_4, G_1 or G_2 (see Fig. 2(a)) and also to G_3, G_4 or G_5 (see Fig. 2(b)) with respect to the cutpoints.*

Proof. Let G be a graph for which the blict graph $B_n(G)$ is nonouterplanar.

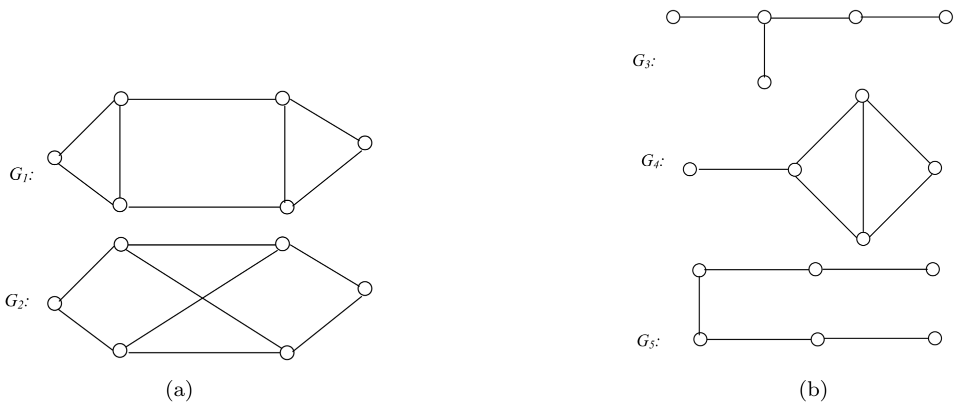


Figure 2.

Now suppose $B_n(G)$ is minimally nonouterplanar. We now show that all graphs homeomorphic to $K_{1,4}$, $K_{2,3}$, K_4, G_1 and G_2 and also to G_3, G_4 or G_5 with respect to the cutpoints, have no minimally nonouterplanar blict graphs. It follows from Theorem 1.5, since graphs homeomorphic to $K_{1,4}$ have a point of degree 4, graphs homeomorphic to $K_{2,3}$, K_4, G_1 or G_2 is a m-minimally nonouterplanar (p, q) block with $q - p + m > 1$, graphs homeomorphic to G_4 with respect to the outpoints have a cycle together with a path P_n $(n \geq 1)$ is adjoined at some point and a path P_m $(m \geq 1)$ is joined between the two non adjacent points on the cycle, graphs homeomorphic to G_3 have no cycle together with an endline adjoined to some point, graphs homeomorphic to G_5 have a path of length at least 5.

Conversely, suppose that G contains no subgraph homeomorphic to $K_{1,4}$, $K_{2,3}$, K_4, G_1 and G_2 and also G_3, G_4 or G_5 with respect to the cutpoints. If possible suppose $G = K_{1,3}$. Then by Remark 1.1, $B_n(G) = K_4.K_4$, which is 2-minimally nonouterplanar.

Now assume $\Delta(G) = 4$. Then G has a point of degree 4. Then it has a subgraph homeomorphic to $K_{1,4}$, a contradiction. Thus degree of each point is at most 3. Suppose the degree of each point is at most two. Then by Theorem 1.4, the blict graph $B_n(G)$ is outerplanar. Thus Theorem 1.4 implies that G has at least one cutpoint or two noncutpoints of degree 3. We consider the following cases.

Case 1. Suppose G is a nonseparable graph. We consider the following subcases.

Subcase 1.1. Suppose G is a cycle with exactly two points of degree 3 and are joined by a path P_n $(n \geq 3)$. Then G has a subgraph homeomorphic to $K_{2,3}$, a contradiction.

Subcase 1.2. Suppose G is a cycle with exactly two pairs of nonadjacent points of degree 3 and the corresponding pairs of points are joined by a path P_i $(i \geq 3)$ and P_j $(j \geq 3)$ respectively. Then G has a subgraph homeomorphic to G_1, again a contradiction.

Subcase 1.3. Suppose G is nonouterplanar. Then G has a nonouterplanar block B with more than 3 points. On embedding B on the plane, the maximum number of points lie on the exterior cycle C of B. Since B is nonouterplanar, there exists at least one point that lies in the interior of C. Let ν be the point interior to C and adjacent to two points of C, degree of ν must be three. Otherwise B contains two noncutpoints of degree 3. Hence there is a path from ν to some point of C. Thus a subgraph of B is homeomorphic to K_4. From the above cases, we conclude that G is a m-minimally nonouterplanar (p, q) block with $q - p + m = 1$.

Case 2. Suppose G is a separable graph. We consider the following subcases.

Subcase 2.1. Suppose there are two or more cutpoints of degree 3, each of which lies on either 2 or 3 blocks. Then G has a subgraph homeomorphic to G_3 with respect to the cutpoints, a contradiction.

Subcase 2.2. Suppose G has a unique cutpoint ν of degree 3 and ν lies on 2 blocks. Then we have the following subcases.

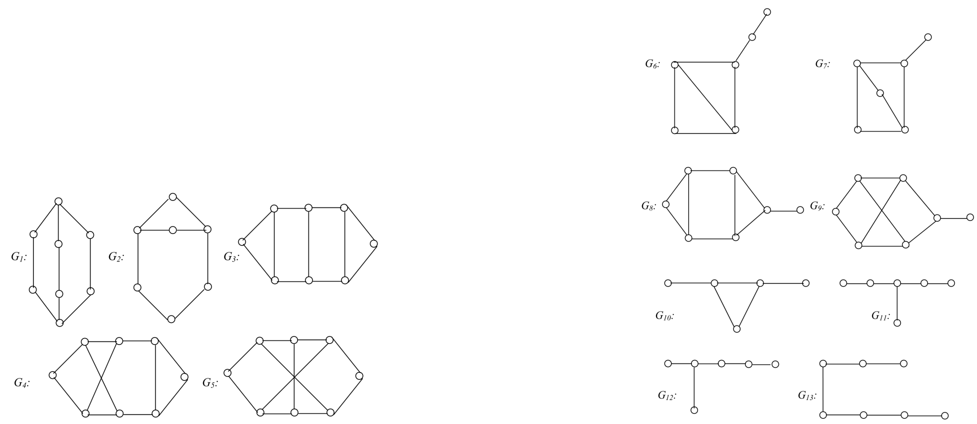
Subcase 2.2.1. Suppose one of the blocks contain a cycle C and the other is a line uv, where u lies on the cycle C and v is incident with an end line. Then G has a subgraph homeomorphic to G_3 with respect to the cutpoints, a contradiction.

Subcase 2.2.2. Suppose one of these blocks is a line and the other is a cyclic block having exactly two noncutpoints of degree 3. Then G has a subgraph homeomorphic to G_4 with respect to the cutpoints, again a contradiction. From the above cases we conclude that G is a cycle together with an endline adjoined to some point. Thus Theorem 1.5 implies that G has a minimally nonouterplanar blict graph. □

In the following theorem we characterize those graphs whose blict graphs are 2-minimally nonouterplanar in terms of forbidden subgraphs by using Theorem 1.7.

Theorem 2.4. *Let G be a graph for which $B_n(G)$ is not m $(m \leq 1)$-minimally nonouterplanar. Then $B_n(G)$ is 2-minimally nonouterplanar if and only if it has no subgraph homeomorphic to $K_{1,4}$, K_4, G_1, G_2, G_3, G_4 or G_5 (see Fig. 3(a)) and also to any one of the graphs of Fig. 3(b) with respect to the cutpoints.*

Proof. Let G be a graph for which $B_n(G)$ is not m $(m \leq 1)$-minimally nonouterplanar.



(a) (b)

Figure 3.

Suppose $B_n(G)$ is 2-minimally nonouterplanar. We now show that all graphs homeomorphic to $K_{1,4}$, K_4, G_1, G_2, G_3, G_4 or G_5 and also to any one of the graphs of Fig. 3(b) with respect to the cutpoints have no 2-minimally nonouterplanar blict graphs. It follows from Theorem 1.7, since graphs homeomorphic to $K_{1,4}$ have $\Delta(G) > 3$, graphs homeomorphic to K_4, G_1, G_2, G_3, G_4 or G_5 is a block with $q - p + m > 2$, graphs homeomorphic to G_6, G_7, G_8, G_9 or G_{10} with respect to the cutpoints have no two adjacent noncutpoints of degree 3 with a unique cutpoint ν of degree 3 which lies on two blocks of G in which one block has an endpoint of G, graphs homeomorphic to G_{12} with respect to the cutpoints, have no triangle together with a path of length 2 adjoined to some point, graphs homeomorphic to G_{13} have a path of length at least 6 with respect to the cutpoints, graphs homeomorphic to G_{11} or G_{12} are trees with $\Delta(G) = 3$ having one point of degree 3 and at least two points of degree 2.

Conversely, suppose that G contains no subgraph homeomorphic to $K_{1,4}$, K_4, G_1, G_2, G_3, G_4 or G_5 (see Fig. 3(a)) and also to any one of the graphs of Fig. 3(b) with respect to the cutpoints. Now assume $\Delta(G) = 4$. Then G has a point of degree 4. Then it has a subgraph homeomorphic to $K_{1,4}$, a contradiction. Thus degree of each point is at most 3. Suppose the degree of each point is at most 2. Then by Theorem 1.4, the blict graph $B_n(G)$ is outerplanar. Thus Theorem 1.4 implies that G has at least one cutpoint or two noncutpoints of degree 3. We consider the following cases.

Case 1. Suppose G is a nonseparable graph. Then no two points of degree 3 are adjacent. Otherwise $B_n(G)$ is minimally nonouterplanar. We consider the following subcases.

Subcase 1.1. Suppose G is a cycle with exactly two points of degree 3 and are joined by a path P_n $(n \geq 4)$. Then G has a subgraph homeomorphic to G_1, a contradiction.

Subcase 1.2. Suppose G is a cycle with exactly two pairs of non adjacent points of degree 3 and the corresponding pairs of points are joined by a path P_i $(i \geq 4)$ and P_j $(j \geq 3)$ respectively. Then G has a subgraph homeomorphic to G_2, a contradiction.

Subcase 1.3. Suppose G is a cycle with exactly three pairs of nonadjacent points of degree 3 and the corresponding pairs of points are joined by a path P_i $(i \geq 3)$, P_j $(j \geq 3)$ and P_k $(k \geq 3)$ respectively. Then G has a subgraph homeomorphic to G_3, again a contradiction.

Subcase 1.4. Suppose G is nonouterplanar. Then G has a nonouterplanar block B with more than 3 points. On embedding B on the plane, the maximum number of points lie on the exterior cycle C of B. Since B is nonouterplanar, there exists at least one point which lies in the interior of C. Let ν be the point interior to C and adjacent to two points of C, degree of ν must be three. Otherwise B contains two noncutpoints of degree 3. Hence there is a path from ν to some point of C. Thus a subgraph of B is homeomorphic to K_4. From the above cases, we conclude that G is a m-minimally nonouterplanar (p, q) block with $q - p + m = 2$.

Case 2. Suppose G is separable. We consider the following subcases.

Subcase 2.1. Suppose G has a unique cutpoint ν of degree 3 and ν lies on 2 blocks. Then we have the following subcases.

Subcase 2.1.1. Suppose one of the blocks is a cyclic block B having two adjacent points of degree 3 and a path P_n $(n \geq 3)$ is adjoined at the point ν. Then G has a subgraph homeomorphic to G_6 with respect to the cutpoints, a contradiction.

Subcase 2.1.2. Suppose the cyclic block B has two nonadjacent points of degree 3 and are joined by a path P_m $(m \geq 3)$ and a path P_n $(n \geq 2)$ is adjoined at the point ν. Then G has a subgraph homeomorphic to G_7 with respect to the cutpoints, a contradiction.

Subcase 2.1.3. Suppose the cyclic block B has two pairs of points of degree 3 and are joined by the paths P_m $(m \geq 2)$ and P_n $(n \geq 2)$ respectively and a path P_k $(k \geq 2)$ is adjoined at the point ν. Then G has a subgraph homeomorphic to G_8 or G_9, a contradiction.

Subcase 2.1.4. Suppose B is a triangle having two cutpoints u and ν together with the path P_i $(i \geq 2)$ and P_j $(j \geq 2)$

adjoined at the points u and ν respectively. Then G has a subgraph homeomorphic to G_{10} with respect to the cutpoints, a contradiction.

Subcase 2.1.5. Suppose B is a triangle together with a path P_n $(n \geq 4)$ is adjoined at the point ν. Then G has a subgraph homeomorphic to G_{12} with respect to the cutpoints, a contradiction.

Case 3. Suppose G is a tree. Assume G is a path P_n $(n \geq 5)$ and an endline is adjoined at the point of degree 2. Then G has a subgraph homeomorphic to either G_{11} or G_{12} with respect to the cutpoints, a contradiction. Assume G is a path of length at least 6. Then it has a subgraph homeomorphic to G_{13}, again a contradiction. Thus by Theorem 2.8, G has a 2-minimally nonouterplanar blict graph. This completes the proof of the theorem. □

The characterizations of planar, outerplanar, minimally nonouterplanar and 2-minimally nonouterplanar blitact graphs in terms of forbidden subgraphs are similar to the characterizations for blict graphs. Since the graphs satisfying the conditions of Theorem 2.1, 2.2, 2.3 and 2.4 are having at most one cutpoint and by Theorem 1.6, we have $B_n(G) = B_m(G)$ whenever G has at most one cutpoint. Also the adjacency of the points corresponding to the remaining cutpoints of G do not affect the 2-minimally nonouterplanarity of $B_n(G)$ and $B_m(G)$.

Theorem 2.5. *A graph G has a planar blitact graph if and only if it has no subgraph homeomorphic to $K_{3,3}$ or $K_{1,4}$ and also to G_1 or G_2 of Fig. 1 with respect to the cutpoints.*

Proof. The proof is similar to the proof of Theorem 1.1 and hence we omit the proof. □

Theorem 2.6. *A graph G has an outerplanar blitact graph if and only if it has no subgraph homeomorphic to $K_{1,3}$ and also to P_5 with respect to the cutpoints.*

Proof. The proof is similar to the proof of Theorem 2.2 and hence we omit the proof. □

Theorem 2.7. *Let G be a graph for which the blitact graph $B_m(G)$ is nonouterplanar. Then $B_m(G)$ is minimally nonouterplanar if and only if $G \neq K_{1,3}$ and it has no subgraph homeomorphic to $K_{1,4}$, $K_{2,3}$, K_4, G_1 or G_2 (see Fig. 2(a)) and also to G_3, G_4 or G_5 (see Fig. 2(b)) with respect to the cutpoints.*

Proof. The proof is similar to the proof of Theorem 2.3 and hence we omit the proof. □

Theorem 2.8. *Let G be a graph for which $B_m(G)$ is not m $(m \leq 1)$ minimally nonouterplanar. Then $B_m(G)$ is 2-minimally nonouterplanar if and only if it has no subgraph homeomorphic to $K_{1,4}$, K_4, G_1, G_2, G_3, G_4 or G_5 (See Fig. 3(a)) and also to any one of the graphs of Fig. 3 (b) with respect to the cutpoints.*

Proof. The proof is similar to the proof of Theorem 2.4 and hence we omit the proof. □

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