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Properties of Fuzzy HX Ideal of a HX Ring

Research Article

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Abstract: In this paper , we introduce the concept of fuzzy HX left ideal and fuzzy HX right ideal of a HX ring. Also we have discussed the properties of fuzzy HX right (left) ideal by establishing the relationship among them.

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Keywords: Fuzzy HX ring, fuzzy ideal, fuzzy HX left ideal, fuzzy HX right ideal, fuzzy HX ideal.

1. Introduction

In 1965, Zadeh [10] introduced the concept of fuzzy subset of a set X as a function from into the closed unit interval 0 and 1 and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [9] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [3] introduced the concept of HX group. In 1988, Professor Li Hong Xing [5] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [1, 2] gave the structures of HX ring on a class of ring. In this paper we define a new algebraic structure of a fuzzy HX left ideal and fuzzy HX right ideal of a HX ring and investigate some related properties.

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R, +, \cdot)$ is a ring, e is the additive identity element of R and xy, we mean $x \cdot y$.

3. Fuzzy HX Right Ideal

Definition 3.1. *Let R be a ring. Let μ be a fuzzy subset defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\Re$ is called a fuzzy HX right ideal on $\Re$ or a fuzzy right ideal induced by μ if the following conditions are satisfied. For all $A, B \in \Re$,*

(i). $\lambda^\mu(A - B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,

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(ii). $\lambda^{\mu}(AB) \geq \lambda^{\mu}(A)$,

where $\lambda^{\mu}(A) = \max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}$.

Theorem 3.2. *If μ is a fuzzy right ideal of a ring R then the fuzzy subset λ^{μ} is a fuzzy HX right ideal of a HX ring $\Re$.*

Proof. Let μ be a fuzzy right ideal of R.

$$
\begin{aligned}
(i) \qquad \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} &= \min\{\max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}, \max\{\mu(y)/ \text{ for all } y \in B \subseteq R\}\} \\
&= \min\{\mu(x_0), \mu(y_0)\} \\
&\leq \mu(x_0 - y_0), \text{since } \mu \text{ is a fuzzy ideal of R} \\
&\leq \max\{\mu(x-y)/ \text{ for all } x-y \in A-B \subseteq R\} \\
&\leq \lambda^{\mu}(A-B) \\
\lambda^{\mu}(A-B) &\geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \\
(ii) \qquad \lambda^{\mu}(AB) &= \max\{\mu(xy)/ \text{ for all } x \in A \subseteq R \text{ and } y \in B \subseteq R\} \\
&= \mu(x_0 y_0), \text{ , for } x_0 \in A \text{ and } y_0 \in B. \\
&\geq \mu(x_0), \mu \text{ is a fuzzy right ideal of R} \\
&= \max\{\mu(x)/ \text{ for all } x \in A \subseteq R\} \\
&\geq \lambda^{\mu}(A) \\
\lambda^{\mu}(AB) &\geq \lambda^{\mu}(A).
\end{aligned}
$$

Hence, λ^{μ} is a fuzzy HX right ideal of a HX ring $\Re$. □

Theorem 3.3. *Let μ and η be any two fuzzy sets on R. Let λ^{μ} and γ^{η} be any two fuzzy HX right ideals of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy HX right ideal of a HX ring $\Re$.*

Proof. Let $A, B \in \Re$.

$$
\begin{aligned}
(i) \qquad (\lambda^{\mu} \cap \gamma^{\eta}(A-B) &= \min\{\lambda^{\mu}(A-B), \gamma^{\eta}(A-B)\} \\
&\geq \min\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}, \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\} \\
&= \min\{\min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\}, \min\{\lambda^{\mu}(B), \gamma^{\eta}(B)\}\} \\
&\geq \min\{(\lambda^{\mu} \cap \gamma^{\eta})(A), (\lambda^{\mu} \cap \gamma^{\eta})(B)\}. \\
(\lambda^{\mu} \cap \gamma^{\eta})(A-B) &\geq \min\{(\lambda^{\mu} \cap \gamma^{\eta})(A), (\lambda^{\mu} \cap \gamma^{\eta})(B)\}. \\
(ii) \qquad (\lambda^{\mu} \cap \gamma^{\eta})(AB) &= \min\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\} \\
&\geq \min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\} \\
&\geq (\lambda^{\mu} \cap \gamma^{\eta})(A) \\
(\lambda^{\mu} \cap \gamma^{\eta})(AB) &\geq (\lambda^{\mu} \cap \gamma^{\eta})(A).
\end{aligned}
$$

Hence, the intersection of two fuzzy HX right ideals of a HX ring $\Re$ is also a fuzzy HX right ideal of $\Re$. □

Theorem 3.4. *Let μ and η be any two fuzzy sets of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy HX right ideals of $\Re$ then, their union $(\lambda^{\mu} \cup \gamma^{\eta})$ is also a fuzzy HX right ideal of $\Re$.*

Proof.

$$
\begin{aligned}
(i) \qquad (\lambda^{\mu} \cup \gamma^{\eta})(A - B) &= \max\{\lambda^{\mu}(A - B), \gamma^{\eta}(A - B)\} \\
&\geq \max\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}, \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\} \\
&= \min\{\max\{\lambda^{\mu}(A), \gamma^{\eta}(A)\}, \max\{\lambda^{\mu}(B), \gamma^{\eta}(B)\}\} \\
&\geq \min\{(\lambda^{\mu} \cup \gamma^{\eta})(A), (\lambda^{\mu} \cup \gamma^{\eta})(B)\}. \\
(\lambda^{\mu} \cup \gamma^{\eta})(A - B) &\geq \min\{(\lambda^{\mu} \cup \gamma^{\eta})(A), (\lambda^{\mu} \cup \gamma^{\eta})(B)\}. \\
(ii) \qquad (\lambda^{\mu} \cup \gamma^{\eta})(AB) &= \max\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\} \\
&\geq \max\{\lambda^{\mu}(A), \gamma^{\eta}(A)\} \\
&\geq (\lambda^{\mu} \cup \gamma^{\eta})(A). \\
(\lambda^{\mu} \cup \gamma^{\eta})(AB) &\geq (\lambda^{\mu} \cup \gamma^{\eta})(A).
\end{aligned}
$$

Hence, the union of two fuzzy HX right ideals of a HX ring $\Re$ is also a fuzzy HX right ideal of a HX ring $\Re$. □

Theorem 3.5. *Let μ and η be any two fuzzy right ideals of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy HX right ideals of $\Re$ then, $\lambda^{\mu} \times \gamma^{\eta}$ is also a fuzzy HX right ideal of a HX ring $\Re$.*

Proof. Let $A, B \in \Re_1 \times \Re_2$ where $A = (C, D), B = (E, F)$.

$$
\begin{aligned}
(i) \qquad (\lambda^{\mu} \times \gamma^{\eta})(A - B) &= (\lambda^{\mu} \times \gamma^{\eta})((C, D) - (E, F)) \\
&= (\lambda^{\mu} \times \gamma^{\eta})(C - E, D - F) \\
&\geq \min\{\lambda^{\mu}(C - E), \gamma^{\eta}(D - F)\} \\
&= \min\{\min\{\lambda^{\mu}(C), \lambda^{\mu}(E)\}, \min\{\gamma^{\eta}(D), \gamma^{\eta}(F)\}\} \\
&= \min\{\min\{\lambda^{\mu}(C), \gamma^{\eta}(D)\}, \min\{\lambda^{\mu}(E), \gamma^{\eta}(F)\}\} \\
&\geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(C, D), (\lambda^{\mu} \times \gamma^{\eta})(E, F)\} \\
&= \min\{(\lambda^{\mu} \times \gamma^{\eta})(A), (\lambda^{\mu} \times \gamma^{\eta})(B)\} \\
(\lambda^{\mu} \times \gamma^{\eta})(A - B) &\geq \min\{(\lambda^{\mu} \times \gamma^{\eta})(A), (\lambda^{\mu} \times \gamma^{\eta})(B)\}. \\
(ii) \qquad (\lambda^{\mu} \times \gamma^{\eta})(AB) &= (\lambda^{\mu} \times \gamma^{\eta})((C, D)(E, F)) \\
&= (\lambda^{\mu} \times \gamma^{\eta})(CE, DF) \\
&= \min\{\lambda^{\mu}(CE), \gamma^{\eta}(DF)\} \\
&\geq \min\{\lambda^{\mu}(C), \gamma^{\eta}(D)\} \\
&= (\lambda^{\mu} \times \gamma^{\eta})(C, D) \\
&= (\lambda^{\mu} \times \gamma^{\eta})(A). \\
(\lambda^{\mu} \times \gamma^{\eta})(AB) &\geq (\lambda^{\mu} \times \gamma^{\eta})(A).
\end{aligned}
$$

Hence, $\lambda^{\mu} \times \gamma^{\eta}$ is a fuzzy HX right ideal of the HX ring $\Re$. □

4. Fuzzy HX Left Ideal

Definition 4.1. *Let R be a ring. Let μ be a fuzzy subset defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^{μ} of $\Re$ is called a fuzzy HX left ideal on $\Re$ or a fuzzy left ideal induced by μ if the following conditions are satisfied. For all*

$A, B \in \Re$,

(i). $\lambda^{\mu}(A - B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

(ii). $\lambda^{\mu}(AB) \geq \lambda^{\mu}(B)$

where $\lambda^{\mu}(A) = \max\{\mu(x) /$ *for all* $x \in A \subseteq R\}$.

Theorem 4.2. *If μ is a fuzzy left ideal of a ring R then the fuzzy subset λ^{μ} is a fuzzy HX left ideal of a HX ring $\Re$.*

Proof. Let μ be a fuzzy left ideal of R.

$$
\begin{aligned}
(i) \qquad \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} &= \min\{\max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}, \max\{\mu(y)/ \text{ for all } y \in B \subseteq R\}\} \\
&= \min\{\mu(x_0), \mu(y_0)\} \\
&\leq \mu(x_0 - y_0), \quad \text{since } \mu \text{ is a fuzzy ideal of R} \\
&\leq \max\{\mu(x-y)/ \text{ for all } x - y \in A - B \subseteq R\} \\
&\leq \lambda^{\mu}(A - B) \\
\lambda^{\mu}(A - B) &\geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \\
(ii) \qquad \lambda^{\mu}(AB) &= \max\{\mu(xy)/ \text{ for all } x \in A \subseteq R \ \text{ and } y \in B \subseteq R\} \\
&= \mu(x_0 y_0), \text{ for } x_0 \in A \text{ and } y_0 \in B. \\
&\geq \mu(y_0), \mu \ \text{ is a fuzzy left ideal of R} \\
&= \max\{\mu(y)/ \text{ for all } y \in B \subseteq R\} \\
&\geq \lambda^{\mu}(B). \\
\lambda^{\mu}(AB) &\geq \lambda^{\mu}(B)
\end{aligned}
$$

Hence, λ^{μ} is a fuzzy HX left ideal of a HX ring $\Re$. □

Theorem 4.3. *Let μ and η be any two fuzzy sets on R. Let λ^{μ} and γ^{η} be any two fuzzy HX left ideals of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy HX left ideal of a HX ring $\Re$.*

Proof. Let $A, B \in \Re$.

$$
\begin{aligned}
(i) \qquad (\lambda^{\mu} \cap \gamma^{\eta})(A - B) &= \min\{\lambda^{\mu}(A - B), \gamma^{\eta}(A - -B)\} \\
&\geq \min\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}, \min\{\gamma^{\eta}(A), \gamma^{\eta}(B)\}\} \\
&= \min\{\min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\}, \min\{\lambda^{\mu}(B), \gamma^{\eta}(B)\}\} \\
&\geq \min\{(\lambda^{\mu} \cap \gamma^{\eta})(A), (\lambda^{\mu} \cap \gamma^{\eta})(B)\}. \\
(\lambda^{\mu} \cap \gamma^{\eta})(A - B) &\geq \min\{(\lambda^{\mu} \cap \gamma^{\eta})(A), (\lambda^{\mu} \cap \gamma^{\eta})(B)\}. \\
(ii) \qquad (\lambda^{\mu} \cap \gamma^{\eta})(AB) &= \min\{\lambda^{\mu}(AB), \gamma^{\eta}(AB)\} \\
&\geq \min\{\lambda^{\mu}(B), \gamma^{\eta}(B)\} \\
&\geq (\lambda^{\mu} \cap \gamma^{\eta})(B). \\
(\lambda^{\mu} \cap \gamma^{\eta})(AB) &\geq (\lambda^{\mu} \cap \gamma^{\eta})(B).
\end{aligned}
$$

Hence, the intersection of two fuzzy HX left ideals of a HX ring $\Re$ is also a fuzzy HX left ideal of $\Re$. □

Theorem 4.4. *Let μ and η be any two fuzzy left ideals of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ^μ and γ^η are any two fuzzy HX left ideals of $\Re$ then, their union $(\lambda^\mu \cup \gamma^\eta)$ is also a fuzzy HX left ideal of $\Re$.*

Proof.

$$
\begin{aligned}
(i) \qquad (\lambda^\mu \cup \gamma^\eta)(A-B) &= \max\{\lambda^\mu(A-B), \gamma^\eta(A-B)\} \\
&\geq \max\{\min\{\lambda^\mu(A), \lambda^\mu(B)\}, \min\{\gamma^\eta(A), \gamma^\eta(B)\}\} \\
&= \min\{\max\{\lambda^\mu(A), \gamma^\eta(A)\}, \max\{\lambda^\mu(B), \gamma^\eta(B)\}\} \\
&\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}. \\
(\lambda^\mu \cup \gamma^\eta)(A-B) &\geq \min\{(\lambda^\mu \cup \gamma^\eta)(A), (\lambda^\mu \cup \gamma^\eta)(B)\}. \\
(ii) \qquad (\lambda^\mu \cup \gamma^\eta)(AB) &= \max\{\lambda^\mu(AB), \gamma^\eta(AB)\} \\
&\geq \max\{\lambda^\mu(B), \gamma^\eta(B)\} \\
&\geq (\lambda^\mu \cup \gamma^\eta)(B). \\
(\lambda^\mu \cup \gamma^\eta)(AB) &\geq (\lambda^\mu \cup \gamma^\eta)(B).
\end{aligned}
$$

Hence, the union of two fuzzy HX left ideals of a HX ring $\Re$ is also a fuzzy HX left ideal of a HX ring $\Re$. □

Theorem 4.5. *Let μ and η be any two fuzzy left ideals of R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. If λ^μ and γ^η are any two fuzzy HX left ideals of $\Re$ then, $\lambda^\mu \times \gamma^\eta$ is also a fuzzy HX left ideal of a HX ring $\Re$.*

Proof. Let $A, B \in \Re_1 \times \Re_2$ where $A = (C, D), B = (E, F)$.

$$
\begin{aligned}
(i) \qquad (\lambda^\mu \times \gamma^\eta)(A-B) &= (\lambda^\mu \times \gamma^\eta)((C,D)-(E,F)) \\
&= (\lambda^\mu \times \gamma^\eta)(C-E, D-F) \\
&\geq \min\{\lambda^\mu(C-E), \gamma^\eta(D-F)\} \\
&= \min\{\min\{\lambda^\mu(C), \lambda^\mu(E)\}, \min\{\gamma^\eta(D), \gamma^\eta(F)\}\} \\
&= \min\{\min\{\lambda^\mu(C), \gamma^\eta(D)\}, \min\{\lambda^\mu(E), \gamma^\eta(F)\}\} \\
&\geq \min\{(\lambda^\mu \times \gamma^\eta)(C,D), (\lambda^\mu \times \gamma^\eta)(E,F)\} \\
&= \min\{(\lambda^\mu \times \gamma^\eta)(A), (\lambda^\mu \times \gamma^\eta)(B)\} \\
(\lambda^\mu \times \gamma^\eta)(A-B) &\geq \min\{(\lambda^\mu \times \gamma^\eta)(A), (\lambda^\mu \times \gamma^\eta)(B)\}. \\
(ii) \qquad (\lambda^\mu \times \gamma^\eta)(AB) &= (\lambda^\mu \times \gamma^\eta)((C,D)(E,F)) \\
&= (\lambda^\mu \times \gamma^\eta)(CE, DF) \\
&= \min\{\lambda^\mu(CE), \gamma^\eta(DF)\} \\
&\geq \min\{\lambda^\mu(E), \gamma^\eta(F)\} \\
&= (\lambda^\mu \times \gamma^\eta)(E,F) \\
&= (\lambda^\mu \times \gamma^\eta)(B). \\
(\lambda^\mu \times \gamma^\eta)(AB) &\geq (\lambda^\mu \times \gamma^\eta)(B).
\end{aligned}
$$

Hence, $\lambda^\mu \times \gamma^\eta$ is a fuzzy HX left ideal of the HX ring $\Re$. □

5. Fuzzy HX Ideal

Definition 5.1. *Let R be a ring. Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of $\Re$ is called a fuzzy HX ideal on $\Re$ or a fuzzy ideal induced by μ if it is both fuzzy HX right ideal and fuzzy HX left ideal on $\Re$. That is, For all $A,\ B \in \Re$,*

(i). $\lambda^\mu(A - B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$,

(ii). $\lambda^\mu(AB) \geq \max\{\lambda^\mu(A), \lambda^\mu(B)\}$

where $\lambda^\mu(A) = \max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}$.

Example 5.2. *Let $C^0 = C - \{0\}$ where C is the set of all complex numbers. For all $a, b \in C^0$, define the operators $\oplus$ and $\otimes$ on C^0 as $a \oplus b = ab$ and $a \otimes b = |a|^{\ln|b|}$. Clearly $(C^0, \oplus, \otimes)$ is a ring. Define, a fuzzy set μ on C^0 as,*

$$\mu(x) = \mu(a + ib) = \begin{cases} 0.9, & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.7, & \text{if } a < 0 \text{ and } b = 0 \\ 0.5, & \text{if } b \neq 0 \end{cases}$$

where, a is the real part of x lies in X-axis and b is the imaginary part of x lies in the Y-axis. Then, clearly, $\mu(x\Theta y) = \mu(x \oplus (-y)) \geq \min\{\mu(x), \mu(y)\}$, $\mu(x \otimes y) \geq \mu(x)$ and $\mu(x \otimes y) \geq \mu(y)$. Clearly, μ is both fuzzy right and left ideal on C^0. That is, μ is a fuzzy ideal on C^0.

Let $I = (1, \infty)$ and $H = \{1, -1, i, -i\}$. Define $\Re = \{a \oplus I \ / \ a \in H\}$. For,

$$1 \in H \Rightarrow 1 \oplus I = 1(1, \infty) = (1, \infty)$$

$$-1 \in H \Rightarrow -1 \oplus I = -1(1, \infty) = (-\infty, -1)$$

$$i \in H \Rightarrow i \oplus I = i(1, \infty) = (i, \infty)$$

$$-i \in H \Rightarrow -i \oplus I = -i(1, \infty) = (-\infty, -i).$$

Now, $\Re = \{a \oplus I \ / \ a \in H\} = \{(1, \infty), (-\infty, -1), (i, \infty), (-\infty, -i)\} = \{Q, A, B, C\}$. For any $X, Y \in \Re$, define the operations $\oplus$ and $\otimes$ on $\Re$ as,

$$X \oplus Y = XY = \{xy/x \in X \text{ and } y \in Y\}$$

$$X \otimes Y = \mid X \mid^{\ln|Y|} = \{\mid x \mid^{\ln|y|} /x \in X \text{ and } y \in Y\}$$

Then, Clearly, $(\Re, \oplus, \otimes)$ is a HX ring on $(P_0(C^0), \oplus, \otimes)$. Define a fuzzy set $\lambda^\mu : \Re \to [0, 1]$ as,

⊕	Q	A	B	C
Q	Q	A	B	C
A	A	Q	C	B
B	B	C	A	Q
C	C	B	Q	A

⊗	Q	A	B	C
Q	Q	Q	Q	Q
A	Q	Q	Q	Q
B	Q	Q	Q	Q
C	Q	Q	Q	Q

$$\lambda^\mu(Q) = \sup\{\mu(x)/x \in Q\} = 0.9$$

$$\lambda^\mu(A) = \sup\{\mu(x)/x \in A\} = 0.7$$

$$\lambda^\mu(B) = \sup\{\mu(x)/x \in B\} = 0.5$$

$$\lambda^\mu(C) = \sup\{\mu(x)/x \in C\} = 0.5$$

Now, $-Q = A$; $-A = Q$; $-B = C$; $-C = B$.

$$Q\Theta Q = Q \oplus A = A$$
$$Q\Theta A = Q \oplus Q = Q$$
$$Q\Theta B = Q \oplus C = C$$
$$Q\Theta C = Q \oplus B = B$$

$$A\Theta Q = A \oplus A = Q$$
$$A\Theta A = A \oplus Q = A$$
$$A\Theta B = A \oplus C = B$$
$$A\Theta C = A \oplus B = C$$

$$B\Theta Q = B \oplus A = C$$
$$B\Theta A = B \oplus Q = B$$
$$B\Theta B = B \oplus C = Q$$
$$B\Theta C = B \oplus B = A$$

$$C\Theta Q = C \oplus A = B$$
$$C\Theta A = C \oplus Q = C$$
$$C\Theta B = C \oplus C = A$$
$$C\Theta C = C \oplus B = Q$$

For any $X, Y \in \Re$, we have, $\lambda^{\mu}(X\Theta Y) = \lambda^{\mu}(X \oplus (-Y)) \geq \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$, $\lambda^{\mu}(X \otimes Y) \geq \lambda^{\mu}(X)$ and $\lambda^{\mu}(X \otimes Y) \geq \lambda^{\mu}(Y)$. Clearly, λ^{μ} is both fuzzy HX right and left ideal on $\Re$. That is, λ^{μ} is a fuzzy HX ideal on $\Re$.

Theorem 5.3. *If μ is a fuzzy ideal of a ring R then the fuzzy subset λ^{μ} is a fuzzy HX ideal of a HX ring $\Re$.*

Remark 5.4.

(i). If μ is not a fuzzy ideal of R then the fuzzy subset λ^{μ} of $\Re$ is a fuzzy HX ideal of $\Re$, provided $|X| \geq 2$ for all $X \in \Re$.

(ii). If μ is a fuzzy subset of a ring R and λ^{μ} be a fuzzy HX ideal on $\Re$, such that $\lambda^{\mu}(A) = \max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}$, then μ may or may not be a fuzzy ideal of R, which can be illustrated by the following example.

Example 5.5. *Let $C^0 = C - \{0\}$, where C is the set of all complex numbers. For all $a, b \in C^0$, define the operators $\oplus$ and $\otimes$ on C^0 as $a \oplus b = ab$ and $a \otimes b = |a|^{\ln|b|}$. Clearly, $(C^0, \oplus, \otimes)$ is a ring. Define, a fuzzy set μ on C^0 as,*

$$\mu(x) = \mu(a + ib) = \begin{cases} 0.6, & \text{if } a \geq 0 \text{ and } b = 0 \\ 0.5, & \text{if } a < 0 \text{ and } b = 0 \\ 0.3, & \text{if } b > 0 \\ 0.2, & \text{if } b < 0 \end{cases}$$

where, a is the real part of x lies in X-axis and b is the imaginary part of x lies in the Y-axis. Let $x = 1 + 2i$ *and* $y = -2$*. Then,* $\mu(x\Theta y) = \mu(x \oplus (-y)) = \mu(-2 - 4i) = 0.2$.

$$\min\{\mu(x), \mu(y)\} = \min\{\mu(1 + 2i), \mu(-2)\} = \min\{0.3, 0.5\} = 0.3.$$

Hence, $\mu(x\Theta y) \geq \min\{\mu(x), \mu(y)\}$*. Clearly,* μ *is a not a both fuzzy right and left ideal on* C^0*.*

Let $I = (1, \infty)$ and $H = \{1, -1, i, -i\}$. Define $\Re = \{a \oplus I/a \in H\}$. For,

$$1 \in H \Rightarrow 1 \oplus I = 1.(1, \infty) = (1, \infty)$$
$$-1 \in H \Rightarrow -1 \oplus I = -1.(1, \infty) = (-\infty, -1)$$
$$i \in H \Rightarrow i \oplus I = i.(1, \infty) = (i, \infty)$$
$$-i \in H \Rightarrow -i \oplus I = -i.(1, \infty) = (-\infty, -i).$$

Now, $\Re = \{a \oplus I/a \in H\} = \{(1, \infty), (-\infty, -1), (i, \infty), (-\infty, -i)\}$. Let $\Re = \{\{(1, \infty), (-\infty, -1)\}, \{(i, \infty), (-\infty, -i)\}\} = \{Q, A\}$. For any $X, Y \in \Re$, define the operations $\oplus$ and $\otimes$ on $\Re$ as,

$$X \oplus Y = XY = \{xy/x \in X \text{ and } y \in Y\}$$
$$X \otimes Y = \mid X \mid^{\ln|Y|} = \{\mid x \mid^{\ln|y|} /x \in X \ \text{ and } y \in Y\}$$

Then, Clearly, $(\Re, \oplus, \otimes)$ is a HX ring on $(P_0(C^0), \oplus, \otimes)$. Define a fuzzy set $\lambda^\mu : \Re \to [0, 1]$ as,

⊕	Q	A
Q	Q	A
A	A	Q

⊗	Q	A
Q	Q	Q
A	Q	Q

$$\lambda^\mu(Q) = \max\{\mu(x)/x \in Q\} = 0.6$$
$$\lambda^\mu(A) = \max\{\mu(x)/x \in A\} = 0.3$$

Now, $-Q = Q$; $-A = A$.

$$Q\Theta Q = Q \oplus Q = Q$$
$$Q\Theta A = Q \oplus A = A$$
$$A\Theta Q = A \oplus Q = A$$
$$A\Theta A = A \oplus A = Q$$

For any $X, Y \in \Re$, we have, $\lambda^\mu(X\Theta Y) = \lambda^\mu(X \oplus (-Y)) \geq \min\{\lambda^\mu(X), \lambda^\mu(Y)\}$, $\lambda^\mu(X \otimes Y) \geq \lambda^\mu(X)$ and $\lambda^\mu(X \otimes Y) \geq \lambda^\mu(Y)$. Clearly, λ^μ is a both fuzzy HX right and left ideal on $\Re$. Hence, λ^μ is a fuzzy HX ideal on $\Re$.

Theorem 5.6. *Let* μ *be fuzzy set on R. Let* λ^μ *be fuzzy HX ideal of a HX ring* $\Re$ *then* λ^μ *is a fuzzy HX subring of a HX ring* $\Re$*.*

Proof. Let λ^μ be fuzzy HX ideal of a HX ring $\Re$. Then, for all $A, B \in \Re$,

(i). $\lambda^\mu(A - B) \geq \min\{\lambda^\mu(A), \lambda^\mu(B)\}$, (ii). $\lambda^{\mu}(AB) \geq \max\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

where $\lambda^{\mu}(A) = \max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}$.

$$\begin{aligned} \lambda^{\mu}(AB) &\geq \max\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \\ &\geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}. \\ \text{That is, } \lambda^{\mu}(AB) &\geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}. \end{aligned}$$

Hence, λ^{μ} is a fuzzy HX subring of a HX ring $\Re$. □

Remark 5.7. *The converse of the above theorem is not true.*

Theorem 5.8. *Let μ and η be any two sets on R. Let λ^{μ} and γ^{η} be any two fuzzy HX ideals of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy HX ideal of a HX ring $\Re$.*

Remark 5.9.

(i). The intersection of family of fuzzy HX ideals of a HX ring $\Re$ is also fuzzy HX ideal of $\Re$.

(ii). Let R be a ring. Let μ and η be fuzzy ideals of R and $\mu \cap \eta$ is also a fuzzy ideal of R then $\varphi^{\mu\cap\eta}$ is a fuzzy HX ideal of $\Re$ induced by $\mu \cap \eta$ of R.

Theorem 5.10. *If λ^{μ}, γ^{η} , $\varphi^{\mu\cap\eta}$ are fuzzy HX ideals of a HX ring $\Re$ induced by μ , η , $\mu \cap \eta$ of R then $\varphi^{\mu\cap\eta} = \lambda^{\mu} \cap \gamma^{\eta}$.*

Proof. Let λ^{μ} and γ^{η} be fuzzy HX ideals of $\Re$. By Theorem 5.7, $\lambda^{\mu} \cap \gamma^{\eta}$ is a fuzzy HX ideal of a HX ring $\Re$. $\varphi^{\mu\cap\eta}$ is a fuzzy HX ideal of $\Re$ induced by $\mu \cap \eta$ of R.

$$\begin{aligned} \varphi^{\mu\cap\eta}(A) &= \max\{(\mu \cap \eta)(x)/ \text{ for all } x \in A \subseteq R\} \\ &= \max\{\min\{\mu(x), \eta(x)\}/ \text{ for all } x \in A \subseteq R\} \\ &= \min\{\max\{\mu(x)/ \text{ for all } x \in A \subseteq R\}, \max\{\eta(x)/ \text{ for all } x \in A \subseteq R\}\} \\ &= \min\{\lambda^{\mu}(A), \gamma^{\eta}(A)\} \\ \varphi^{\mu\cap\eta}(A) &= (\lambda^{\mu} \cap \gamma^{\eta})(A), \text{ for any } A \in \Re. \end{aligned}$$

Hence, $\varphi^{\mu\cap\eta} = \lambda^{\mu} \cap \gamma^{\eta}$. □

Theorem 5.11. *Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy HX ring and γ^{η} be a fuzzy HX right (left) ideal of a HX ring $\Re$ then their intersection, $\lambda^{\mu} \cap \gamma^{\eta}$ is also a fuzzy HX right (left) ideal of a HX ring $\Re$.*

Theorem 5.12. *Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy HX ring and γ^{η} be a fuzzy HX ideal of a HX ring $\Re$ then their intersection, $\lambda\mu \cap \gamma^{\eta}$ is also a fuzzy HX ideal of a HX ring $\Re$.*

Theorem 5.13. *Let μ and η be any two fuzzy ideals of R. Let $\Re \subset 2^{R} - \{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy HX ideals of $\Re$ then, their union $(\lambda^{\mu} \cup \gamma^{\eta})$ is also a fuzzy HX ideal of $\Re$.*

Remark 5.14.

(i). Union of family of fuzzy HX ideals of a HX ring $\Re$ is also fuzzy HX ideal of $\Re$.

(ii). Let R be a ring. Let μ and η be fuzzy ideals of R then $\varphi^{\mu\cup\eta}$ is a fuzzy HX ideal of $\Re$ induced by $\mu \cup \eta$ of R.

Theorem 5.15. *Let R be a ring. Let μ and η be fuzzy ideals of R. If λ^{μ}, γ^{η}, $\varphi^{\mu\cup\eta}$ are fuzzy HX ideals of a HX ring $\Re$ induced by μ, η, $\mu\cup\eta$ of R then $\varphi^{\mu\cup\eta} = \lambda^{\mu}\cup\gamma^{\eta}$.*

Proof. Let λ^{μ} and γ^{η} be fuzzy HX ideals of $\Re$. By theorem 5.12, $\lambda^{\mu}\cup\gamma^{\eta}$ is a fuzzy HX ideal of a HX ring $\Re$. $\varphi^{\mu\cup\eta}$ is a fuzzy HX ideal of $\Re$ induced by $\mu\cup\eta$ of R.

$$\begin{aligned}
\varphi^{\mu\cup\eta}(A) &= \max\{(\mu\cup\eta)(x)/ \text{ for all } x\in A\subseteq R\}\\
&= \max\{\max\{\mu(x),\eta(x)\}/ \text{ for all } x\in A\subseteq R\}\\
&= \max\{\max\{\mu(x)/ \text{ for all } x\in A\subseteq R\},\max\{\eta(x)/ \text{ for all } x\in A\subseteq R\}\}\\
&= \max\{\lambda^{\mu}(A),\gamma^{\eta}(A)\} = (\lambda^{\mu}\cup\gamma^{\eta})(A).\\
\therefore \varphi^{\mu\cup\eta} &= \lambda^{\mu}\cup\gamma^{\eta}.
\end{aligned}$$

□

Theorem 5.16. *Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy HX ring and γ^{η} be a fuzzy HX right (left) ideal of a HX ring $\Re$ then their union, $\lambda^{\mu}\cup\gamma^{\eta}$ is also a fuzzy HX right (left) ideal of a HX ring $\Re$.*

Theorem 5.17. *Let μ and η be any two fuzzy sets on R. Let λ^{μ} be a fuzzy HX ring and γ^{η} be a fuzzy HX ideal of a HX ring $\Re$ then their union, $\lambda^{\mu}\cup\gamma^{\eta}$ is also a fuzzy HX ideal of a HX ring $\Re$.*

Theorem 5.18. *Let μ and η be any two fuzzy ideals of R. Let $\Re\subset 2^{R}-\{\phi\}$ be a HX ring. If λ^{μ} and γ^{η} are any two fuzzy HX ideals of $\Re$ then, $\lambda^{\mu}\times\gamma^{\eta}$ is also a fuzzy HX ideal of a HX ring $\Re$.*

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