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A Study on Properties of Intuitionistic Fuzzy Sets of Third Type

Research Article

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Abstract: In this paper, we define the new operators on Intuitionistic Fuzzy Sets of Third Type and study some of their properties.

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1. Introduction

To overcome the uncertainty and vagueness, the inherent in the real world in 1965, L. A. Zadeh introduced the notion of fuzzy sets. K. T. Atanassov introduced the extended notion of Intuitionistic Fuzzy Sets in the year 1983. The authors further extended the Intuitionistic Fuzzy Sets namely, Intuitionistic Fuzzy Sets of Third Type and studied some of their properties. In this paper, we introduced the new operators on Intuitionistic Fuzzy Sets of Third Type and establish some of their properties. In section 2, we recollect some basic definitions and in section 3, we define the new operators $G_{\alpha,\beta}, H_{\alpha,\beta}$ and $H^*_{\alpha,\beta}$ and establish some of their properties. The paper is concluded in section 4.

2. Preliminaries

In this section, we give some definitions of IFS and its extensions.

Definition 2.1 ([1]). *Let X be a non-empty set. An IFS A in X is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1,$$

for each $x \in X$.

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Definition 2.2 ([1]). *Let X be a non-empty set. An Intuitionistic Fuzzy Set of Second Type (IFSST) A in X is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership and non-membership functions of A, respectively, and

$$0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1,$$

for each $x \in X$.

Definition 2.3 ([2]). *Let X be the non-empty set. An Intuitionistic Fuzzy Set of Root Type (IFSRT) A in X is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership and non-membership functions of A, respectively, and

$$0 \leq \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\nu_A(x)}}{2} \leq 1,$$

for each $x \in X$.

Definition 2.4 ([3]). *Let X be the non-empty set. An Intuitionistic fuzzy set of third type (IFSTT) A in X is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and

$$0 \leq \mu_A^3(x) + \nu_A^3(x) \leq 1,$$

for each $x \in X$.

Definition 2.5 ([3]). *Let A and B be two IFSTTs of the non-empty set X such that*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$
$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

We define the following operations on A and B :

(i). $A \subset B$ *iff* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x),\ \forall x \in X$

(ii). $A \supset B$ *iff* $\mu_A(x) \geq \mu_B(x)$ *and* $\nu_A(x) \leq \nu_B(x), \forall x \in X$

(iii). $A = B$ *iff* $\mu_A(x) = \mu_B(x)$ *and* $\nu_A(x) = \nu_B(x), \forall x \in X$

(iv). $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)),\ \min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$

(v). $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)),\ \max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$

(vi). $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$, *where $\bar{A}$ is the complement of A.*

Definition 2.6 ([4]). *The degree of non-determinacy (uncertainty) of an element $x \in X$ in IFSTT A is defined by*

$$\pi_A(x) = \sqrt[3]{1 - \mu_A^3(x) - \nu_A^3(x)}.$$

Remark 2.7. *In case of ordinary fuzzy sets, $\pi_A(x) = 0$, for every $x \in X$.*

Definition 2.8 ([4]). *For every IFSTT A, we define the following operators:*

(i). The Necessity operator $\Box A = \{\langle x, \mu_A(x), \sqrt[3]{1 - \mu_A^3(x)}\,\rangle : x \in X\}$

(ii). The Possibility operator $\Diamond A = \{\langle x, \sqrt[3]{1 - \nu_A^3(x)}, \nu_A(x)\rangle : x \in X\}$

Definition 2.9 ([5]). *Let $\alpha \in [0, 1]$ be a fixed number. Given an IFSTT A, an operator D_α is defined as*

$$D_\alpha(A) = \{\langle x, \sqrt[3]{\mu_A^3(x) + \alpha\pi_A^3(x)}\ , \sqrt[3]{\nu_A^3(x) + (1 - \alpha)\pi_A^3(x)}\rangle : x \in X\}.$$

Remark 2.10. *It is easy to prove that $D_\alpha(A)$ is an IFSTT.*

Definition 2.11 ([5]). *Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. The operator $F_{\alpha,\beta}$, on an IFSTT A, is defined as*

$$F_{\alpha,\beta}(A) = \{\langle x, \sqrt[3]{\mu_A^3(x) + \alpha\ \pi_A^3(x)}\ , \sqrt[3]{\nu_A^3(x) + \beta\ \pi_A^3(x)}\ \rangle : x \in X\}.$$

Remark 2.12. *It is obvious that, $F_{\alpha,\beta}(A)$ is an IFSTT.*

3. Operators on IFSTT

Definition 3.1. *Let $\alpha, \beta \in [0, 1]$. Given an IFSTT A, an operator $G_{\alpha,\beta}$ is defined as*

$$G_{\alpha,\beta}(A) = \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\beta}\nu_A(x)\ \rangle : x \in X\}.$$

Remark 3.2. *It is clear that, $G_{\alpha,\beta}(A)$ is an IFSTT.*

Definition 3.3. *Let $\alpha, \beta \in [0, 1]$. Given an IFSTT A, an operator $H_{\alpha,\beta}$ is defined as*

$$H_{\alpha,\beta}(A) = \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta\pi_A^3(x)}\ \rangle : x \in X\}.$$

Remark 3.4. *Clearly, $H_{\alpha,\beta}(A)$ is an IFSTT.*

Definition 3.5. *Let $\alpha, \beta \in [0, 1]$. Given an IFSTT A, an operator $H^*_{\alpha,\beta}$ is defined as*

$$H^*_{\alpha,\beta}(A) = \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta(1 - \alpha\mu_A^3(x) - \nu_A^3(x))}\ \rangle : x \in X\}.$$

Remark 3.6. *Clearly, $H^*_{\alpha,\beta}(A)$ is an IFSTT.*

Proposition 3.7. *For every three real numbers $\alpha,\ \beta, \gamma \in [0, 1]$ and for every IFSTT A, we have*

(i). if $\alpha \leq \gamma$, $G_{\alpha,\beta}(A) \subset G_{\gamma,\beta}(A)$

(ii). if $\beta \leq \gamma$, $G_{\alpha,\beta}(A) \supset G_{\alpha,\gamma}(A)$

(iii). $\overline{G_{\alpha,\beta}(\bar{A})} = G_{\beta,\alpha}(A)$

(iv). if $\gamma, \delta \in [0,1]$*, then* $G_{\alpha,\beta}(G_{\gamma,\delta}(A)) = G_{\alpha.\gamma,\beta.\delta}(A) = G_{\gamma,\delta}(G_{\alpha,\beta}(A))$.

Proof.

$$(i) \qquad \begin{aligned} G_{\alpha,\beta}(A) &= \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\beta}\nu_A(x)\rangle : x \in X\} \\ &\subset \{\langle x, \sqrt[3]{\gamma}\mu_A(x), \sqrt[3]{\beta}\nu_A(x)\rangle : x \in X\} \\ &= G_{\alpha,\beta}(A) \end{aligned}$$

$$(ii) \qquad \begin{aligned} G_{\alpha,\beta}(A) &= \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\beta}\nu_A(x) \ \rangle : x \in X\} \\ &\supset \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\gamma}\nu_A(x) \ \rangle : x \in X\} \\ &= G_{\alpha,\gamma}(A) \end{aligned}$$

$$(iii) \qquad \begin{aligned} \overline{G_{\alpha,\beta} \ (\bar{A})} &= \overline{G_{\alpha,\beta}\{ \ \langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}} \\ &= \overline{\ \{\langle x, \sqrt[3]{\alpha}\nu_A(x), \sqrt[3]{\beta}\mu_A(x)\rangle : x \in X\}} \\ &= \overline{\ \{\langle x, \sqrt[3]{\beta}\mu_A(x), \sqrt[3]{\alpha}\nu_A(x)\rangle : x \in X\}} \\ &= G_{\beta,\alpha}(A). \end{aligned}$$

$$(iv) \qquad \begin{aligned} G_{\alpha,\beta}(G_{\gamma,\delta}(A)) &= G_{\alpha,\beta}(\{\langle x, \sqrt[3]{\gamma}\mu_A(x), \sqrt[3]{\delta}\nu_A(x) \ \rangle : x \in X\}) \\ &= \{\langle x, \sqrt[3]{\alpha.\gamma}\mu_A(x), \sqrt[3]{\beta.\delta}\nu_A(x) \ \rangle : x \in X\} \\ &= G_{\alpha.\gamma,\beta.\delta}(A) \\ &= \{\langle x, \sqrt[3]{\gamma.\alpha}\mu_A(x), \sqrt[3]{\delta.\beta}\nu_A(x) \ \rangle : x \in X\} \\ &= G_{\gamma,\delta}(G_{\alpha,\beta}(A)) \end{aligned}$$

□

Proposition 3.8. *For every three real numbers* $\alpha, \ \beta, \gamma \in [0,1]$ *and for every IFSTT A, we have*

(i). $G_{\alpha,\beta}(A \cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$

(ii). $G_{\alpha,\beta}(A \cup B) = G_{\alpha,\beta}(A) \cup G_{\alpha,\beta}(B)$.

Proof.

$$(i) \qquad \begin{aligned} G_{\alpha,\beta}(A \cap B) &= G_{\alpha,\beta}(\{\langle x, \min(\mu_A(x), \mu_B(x)), \ \max(\nu_A(x), \nu_B(x))\rangle : x \in X\} \\ &= \{\langle x, \sqrt[3]{\alpha}\min(\mu_A(x), \mu_B(x)), \ \sqrt[3]{\beta}\max(\nu_A(x), \nu_B(x))\rangle : x \in X\} \\ &= \{\langle x, \ \min(\sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\alpha}\mu_B(x)), \ \max(\sqrt[3]{\beta}\nu_A(x), \sqrt[3]{\beta}\nu_B(x))\rangle : x \in X\} \\ &= G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B) \end{aligned}$$

$$(ii) \qquad \begin{aligned} G_{\alpha,\beta}(A \cup B) &= G_{\alpha,\beta}(\{\langle x, \max(\mu_A(x), \mu_B(x)), \ \min(\nu_A(x), \nu_B(x))\rangle : x \in X\} \\ &= \{\langle x, \sqrt[3]{\alpha}\max(\mu_A(x), \mu_B(x)), \ \sqrt[3]{\beta}\min(\nu_A(x), \nu_B(x))\rangle : x \in X\} \\ &= \{\langle x, \ \max(\sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\alpha}\mu_B(x)), \ \min(\sqrt[3]{\beta}\nu_A(x), \sqrt[3]{\beta}\nu_B(x))\rangle : x \in X\} \\ &= G_{\alpha,\beta}(A) \cup G_{\alpha,\beta}(B) \end{aligned}$$

□

Theorem 3.9. *For all real numbers* $\alpha, \beta, \gamma, \delta \in [0,1]$ *and for every IFSTT A, we have*

(i). $\Box H_{\alpha,\beta}(A) = H_{\alpha,\beta}(\Box A)$

(ii). $H_{\alpha,\beta}(G_{\gamma,\delta}(A)) \subset G_{\alpha,\beta}(H_{\gamma,\delta}(A))$

(iii). $H^*_{\alpha,\beta}(G_{\gamma,\delta}(A)) \subset G_{\alpha,\beta}(H^*_{\gamma,\delta}(A))$.

Proof. Since,

$$\begin{aligned}
(i) \qquad \Box H_{\alpha,\beta}(A) &= \Box(\{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta\pi_A^3(x)}\ \rangle : x \in X\}) \\
&= \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{1 - \alpha\mu_A^3(x)}\ \rangle : x \in X\} \text{ and} \\
H_{\alpha,\beta}(\Box A) &= H_{\alpha,\beta}(\{\langle x, \mu_A(x), \sqrt[3]{1-\mu_A^3}\rangle : x \in X\}) \\
&= \{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{1 - \mu_A^3(x) + \beta\pi_A^3(x)}\rangle : x \in X\} \text{ and hence,} \\
\Box H_{\alpha,\beta}(A) &= H_{\alpha,\beta}(\Box A) \\
(ii) \qquad H_{\alpha,\beta}(G_{\gamma,\delta}(A)) &= H_{\alpha,\beta}(\{\langle x, \sqrt[3]{\gamma}\mu_A(x), \sqrt[3]{\delta}\nu_A(x)\rangle : x \in X\}) \\
&= \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta\nu_A^3(x) + \beta\pi_A^3(x)}\rangle : x \in X\} \\
&\subset \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta\nu_A^3(x) + \beta\delta\pi_A^3(x)}\rangle : x \in X\} \\
&= \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta(\nu_A^3(x) + \beta\pi_A^3(x))}\rangle : x \in X\} \\
&= \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta}\sqrt[3]{\nu_A^3(x) + \beta\pi_A^3(x)}\rangle : x \in X\} \\
&= \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta}\sqrt[3]{\nu_A^3(x) + \beta\pi_A^3(x)}\rangle : x \in X\} \\
&= G_{\gamma,\delta}(\{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta\pi_A^3(x)}\rangle : x \in X\}) \\
&= G_{\gamma,\delta}(H_{\alpha,\beta}(A)) \\
(iii) \qquad H^*_{\alpha,\beta}(G_{\gamma,\delta}(A)) &= H^*_{\alpha,\beta}(\{\langle x, \sqrt[3]{\gamma}\mu_A(x), \sqrt[3]{\delta}\nu_A(x)\rangle : x \in X\}) \\
&= \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta\nu_A^3(x) + \beta(1 - \alpha\gamma\mu_A^3(x) - \delta\nu_A^3(x))}\rangle : x \in X\} \\
&\subset \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta\nu_A^3(x) + \beta(\delta - \alpha\delta\mu_A^3(x) - \delta\nu_A^3(x))}\rangle : x \in X\} \\
&= \{\langle x, \sqrt[3]{\alpha\gamma}\mu_A(x), \sqrt[3]{\delta(\nu_A^3(x) + \beta(1 - \alpha\mu_A^3(x) - \nu_A^3(x)))}\rangle : x \in X\} \\
&= G_{\gamma,\delta}(\{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta(1 - \alpha\mu_A^3(x) - \nu_A^3(x))}\rangle : x \in X\}) \\
&= G_{\alpha,\beta}(H^*_{\gamma,\delta}(A))
\end{aligned}$$

□

4. Conclusion

In this paper, we have introduced the new operators on IFSTT and studied some of their properties. It is still open to check the operators already defined on IFS in case of IFSTT.

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