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Properties of Intuitionistic L-Fuzzy Sets of Second Type

Research Article

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Abstract: In this paper, we introduce the Intuitionistic L-Fuzzy Sets of second type and study some of their properties.

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1. Introduction

Fuzzy sets were introduced by Lotfi.A.Zadeh in 1965 as a generalisation of classical(crisp)sets. Further the fuzzy sets are generalised by Krassimir.T.Atanassov in which he has taken non-membership values also into consideration and introduced Intuitionistic Fuzzy sets [IFS] and their extensions like Intuitionistic Fuzzy sets of second type, Intuitionistic L-Fuzzy sets, Temporal Intuitionistic Fuzzy sets. In section 2, we give some basic definitions and in section 3, we define the Intuitionistic L-Fuzzy sets of second type [ILFSST] and some basic Operations. Also we establish some of their properties. The paper is concluded in section 4 .

2. Preliminaries

In this section, we give some basic definitions.

Definition 2.1 ([6]). *A Fuzzy set [FS] A in a universal set E is defined by,* $A = \{\langle x, \mu_A(x)\rangle / x \in E\}$*, where* $\mu_A : E \to [0, 1]$ *is the membership function representing the membership degree of element x in the FS A such that* $0 \le \mu_A(x) \le 1$.

Definition 2.2 ([6]). *The support of a Fuzzy Set A in a universal set E is denoted by Supp(A) and is defined as, Supp* $(A) = \{x : \mu_A(x) > 0, x \in E\}$.

Example 2.3. *Let* $X = \{1, 2, 3\}$ *and* $A = \{\langle 1, 0.2\rangle, \langle 2, 0\rangle, \langle 3, 0.4\rangle\}$ *then, Supp* $(A) = \{1, 3\}$.

Definition 2.4 ([1]). *An Intuitionistic Fuzzy set[IFS] A in a universal set E is defined as an object of the form,*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

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where $\mu_A : E \to [0,1]$ *and* $\nu_A : E \to [0,1]$ *denote the degree of membership and the degree of non-membership of the element* $x \in E$ *respectively, satisfying* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

The value $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the degree of uncertainty of the element $x \in E$ to the IFS A.

Definition 2.5 ([1])**.** *The support of an Intuitionistic Fuzzy Set A in a universal set E is denoted by Supp(A) and is defined as,* $Supp\ (A) = \{x : \mu_A(x) > 0, \nu_A(x) > 0, x \in E\}$.

Example 2.6. *Let* $X = \{1, 2, 3\}$ *and* $A = \{\langle 1, 0.2, 0.8\rangle, \langle 2, 0.1, 0.7\rangle, \langle 3, 0, 0\rangle\}$ *then,* $Supp\ (A) = \{1, 2\}$.

Definition 2.7 ([1])**.** *An Intuitionistic Fuzzy sets of second type[IFSST] A in a universal set E is defined as an object of the form,*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

where $\mu_A : E \to [0,1]$ *and* $\nu_A : E \to [0,1]$ *denote the degree of membership and the degree of non-membership of the element* $x \in E$ *respectively, satisfying* $0 \leq \mu_A(x)^2 + \nu_A(x)^2 \leq 1$.

The value $\pi_A(x) = \sqrt{1 - \mu_A(x)^2 - \nu_A(x)^2}$ is the degree of uncertainty of the element $x \in E$ to the IFSST A.

Definition 2.8 ([1])**.** *The support of an Intuitionistic Fuzzy Sets of second type A is denoted by Supp(A) and defined as,* $Supp\ (A) = \{x : \mu_A(x)^2 > 0, \nu_A(x)^2 > 0, x \in E\}$.

Example 2.9. *Let* $X = \{1, 2, 3\}$ *and* $A = \{\langle 1, 0, 0\rangle, \langle 2, 0.1, 0.7\rangle, \langle 3, 0.6, 0.2\rangle\}$ *then,* $Supp\ (A) = \{2, 3\}$.

Definition 2.10 ([1])**.** *An Intuitionistic L-Fuzzy set[ILFS] A in a universal set E is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

where $\mu_A : E \to L$ *and* $\nu_A : E \to L$ *denote the degree of membership and the degree of non-membership of the element* $x \in E$ *respectively, satisfying* $\mu_A(x) \leq N(\nu_A(x))$, $N : L \to L$ *is an unary involute order reversing operation and E be fixed.*

The value $\pi_A(x) = N(sup(\mu_A(x), \nu_A(x)))$ is the degree of uncertainty of the element $x \in E$ to the ILFS A.

Definition 2.11. *The support of an Intuitionistic L-Fuzzy Set A, is denoted by Supp(A) and defined as,*

$$Supp\ (A) = \{x : \mu_A(x) > 0, N(\nu_A(x)) > 0, x \in E\}.$$

Example 2.12. *Let* $X = \{1, 2, 3, 4\}$ *and* $A = \{\langle 1, 0.4, 0.3\rangle, \langle 2, 0, 0\rangle, \langle 3, 0, 0\rangle, \langle 4, 0.1, 0.7\rangle\}$ *then,* $Supp\ (A) = \{1, 4\}$.

3. Operations on Intuitionistic L-Fuzzy Sets of Second Type

In this section, we define the new Intuitionistic L-Fuzzy sets of second type[ILFSST] and establish some of their properties.

Definition 3.1. *An Intuitionistic L-Fuzzy sets of second type[ILFSST] A in a universal set E is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\},$$

where $\mu_A : E \to L$ *and* $\nu_A : E \to L$ *denote the degree of membership and the degree of non-membership of the element* $x \in E$ *respectively, satisfying* $\mu_A(x)^2 \leq N(\nu_A(x))^2$, $N : L \to L$ *is an unary involute order reversing operation and E be fixed.*

The value $\pi_A(x) = \sqrt{N(sup(\mu_A(x)^2, \nu_A(x)^2))}$ is the degree of uncertainty of the element $x \in E$ to the ILFSST A.

Definition 3.2. *The support of an Intuitionistic L-Fuzzy Sets of second type A is denoted by $Supp(A)$ and defined as,*

$$Supp\ (A) = \{x : \mu_A(x)^2 > 0, N(\nu_A(x))^2 > 0, x \in E\}.$$

Example 3.3. *Let $X = \{1, 2, 3, 4\}$ and $A = \{\langle 1, 0, 0\rangle, \langle 2, 0, 0\rangle, \langle 3, 0.6, 0.3\rangle, \langle 4, 0.1, 0.7\rangle\}$ then, $Supp\ (A) = \{3, 4\}$.*

Definition 3.4. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in E\}$. For every two ILFSSTs, A and B, we define the following operations and Relations.*

(a). $A \cup B = \{\langle x, sup(\mu_A(x), \mu_B(x)), inf(\nu_A(x), \nu_B(x))\rangle / x \in E\}$

(b). $A \cap B = \{\langle x, inf(\mu_A(x), \mu_B(x)), sup(\nu_A(x), \nu_B(x))\rangle / x \in E\}$

(c). $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in E\}$.

Theorem 3.5. *For every two ILFSSTs, A and B, we have*

(a). $A \cup A = A$

(b). $A \cap A = A$

(c). $A \cup B = B \cup A$

(d). $A \cap B = B \cap A$.

Proof.

(a). $A \cup A = \{\langle x, sup(\mu_A(x), \mu_A(x)), inf(\nu_A(x), \nu_A(x))\rangle / x \in E\}$
$A \cup A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$
$A \cup A = A$.

(b). $A \cap A = \{\langle x, inf(\mu_A(x), \mu_A(x)), sup(\nu_A(x), \nu_A(x))\rangle / x \in E\}$
$A \cap A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\}$
$A \cap A = A$.

(c). $A \cup B = \{\langle x, sup(\mu_A(x), \mu_B(x)), inf(\nu_A(x), \nu_B(x))\rangle / x \in E\}$
$A \cup B = \{\langle x, sup(\mu_B(x), \mu_A(x)), inf(\nu_B(x), \nu_A(x))\rangle / x \in E\}$
$A \cup B = B \cup A$.

(d). $A \cap B = \{\langle x, inf(\mu_A(x), \mu_B(x)), sup(\nu_A(x), \nu_B(x))\rangle / x \in E\}$
$A \cap B = \{\langle x, inf(\mu_B(x), \mu_A(x)), sup(\nu_B(x), \nu_A(x))\rangle / x \in E\}$
$A \cap B = B \cap A$. □

Theorem 3.6. *For every three ILFSSTs, A, B and C, we have*

(a). $(A \cup B) \cup C = A \cup (B \cup C)$

(b). $(A \cap B) \cap C = A \cap (B \cap C)$.

Proof.

(a). $(A \cup B) \cup C = \{\langle x, sup(\mu_A(x), \mu_B(x)), inf(\nu_A(x), \nu_B(x))\rangle / x \in E\} \cup \{\langle x, \mu_C(x), \nu_C(x)\rangle / x \in E\}$
$(A \cup B) \cup C = \{\langle x, sup(sup(\mu_A(x), \mu_B(x)), \mu_C(x)), inf(inf(\nu_A(x), \nu_B(x)), \nu_C(x))\rangle / x \in E\}$

$(A \cup B) \cup C = \{\langle x, sup(\mu_A(x), sup(\mu_B(x), \mu_C(x))), inf(\nu_A(x), inf(\nu_B(x), \nu_C(x)))\rangle / x \in E\}$

$(A \cup B) \cup C = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\} \cup \{\langle x, sup(\mu_B(x), \mu_C(x)), inf(\nu_B(x), \nu_C(x))\rangle / x \in E\}$

$(A \cup B) \cup C = A \cup (B \cup C)$

(b). $(A \cap B) \cap C = \{\langle x, inf(\mu_A(x), \mu_B(x)), sup(\nu_A(x), \nu_B(x))\rangle / x \in E\} \cap \{\langle x, \mu_C(x), \nu_C(x)\rangle / x \in E\}$

$(A \cap B) \cap C = \{\langle x, inf(inf(\mu_A(x), \mu_B(x)), \mu_C(x)), sup(sup(\nu_A(x), \nu_B(x)), \nu_C(x))\rangle / x \in E\}$

$(A \cap B) \cap C = \{\langle x, inf(\mu_A(x), inf(\mu_B(x), \mu_C(x))), sup(\nu_A(x), sup(\nu_B(x), \nu_C(x)))\rangle / x \in E\}$

$(A \cap B) \cap C = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in E\} \cap \{\langle x, inf(\mu_B(x), \mu_C(x)), sup(\nu_B(x), \nu_C(x))\rangle / x \in E\}$

$(A \cap B) \cap C = A \cap (B \cap C)$. □

Theorem 3.7. *For every two ILFSSTs, A and B, we have*

(a). $\overline{\overline{A} \cup \overline{B}} = A \cap B$

(b). $\overline{\overline{A} \cap \overline{B}} = A \cup B$.

Proof.

(a). $\overline{\overline{A} \cup \overline{B}} = \overline{\{\langle x, sup(\nu_A(x), \nu_B(x)), inf(\mu_A(x), \mu_B(x))\rangle / x \in E\}}$

$\overline{\overline{A} \cup \overline{B}} = \{\langle x, inf(\mu_A(x), \mu_B(x)), sup(\nu_A(x), \nu_B(x))\rangle / x \in E\}$

$\overline{\overline{A} \cup \overline{B}} = A \cap B$

(b). $\overline{\overline{A} \cap \overline{B}} = \overline{\{\langle x, inf(\nu_A(x), \nu_B(x)), sup(\mu_A(x), \mu_B(x))\rangle / x \in E\}}$

$\overline{\overline{A} \cap \overline{B}} = \{\langle x, sup(\mu_A(x), \mu_B(x)), inf(\nu_A(x), \nu_B(x))\rangle / x \in E\}$

$\overline{\overline{A} \cap \overline{B}} = A \cup B$. □

4. Conclusion

In this paper, we have introduced the new extension of Intuitionistic Fuzzy set[IFS] namely Intuitionistic L-Fuzzy sets of second type[ILFSST] and studied some of their properties. In future we will study some more properties and applications of ILFSST.

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