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A Right Inverse Function for Collatz Function

Research Article

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Abstract: A right inverse function $S(x)$ from the set of natural numbers N into itself for classical Collatz function is defined by

$$S(x) = \begin{cases} 2m+1, & \text{if } x = 3m+2 \\ 6m, & \text{if } x = 3m \\ 6m+2, & \text{if } x = 3m+1. \end{cases}$$

This study assumes that the positive integers x, satisfying $S^k(x) > x$, for some k as integers favourable to Collatz conjecture. The integers of the form $x = 3m+2$ poses difficulty for Collatz conjecture. Hence $N_3 = \{3m+2 : m \in N\}$ is spilitted into many sets till it happens that $y = 3^{k+1}p + x$ with $x \in N_3$ are favourable to Collatz conjecture (with $p = 0, 1, 2, \dots$).

MSC: 11B05, 11A07, 11A99, 11Y55.

Keywords: Right inverse, Collatz conjecture.

1. Introduction

Let $T : N \to N$, on the set of natural numbers N be defined by

$$T(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even,} \\ \frac{3x+1}{2}, & \text{if } x \text{ is odd.} \end{cases}$$

The Collatz conjecture claims that repeated iteration of $T(x)$, starting from any positive integer x, eventually reaches the value 1. Let $S : N \to N$, be defined by

$$S(x) = \begin{cases} 2m+1, & \text{if } x = 3m+2 \\ 6m, & \text{if } x = 3m \\ 6m+2, & \text{if } x = 3m+1, \end{cases}$$

when $m = 0, 1, 2, 3, \dots$ for the cases $x = 3m+1$ and $x = 3m+2$, and when $m = 1, 2, 3, 4, \dots$ for the case $x = 3m$. Then $T \circ S : N \to N$ is the identity mapping, so that S becomes a right inverse of T. To each $x \in N$, $T^k(x)$ reaches eventually 1,

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by varying k, if and only if $\{ x \in N : T^k(x) < x$, for some k, depending on $x \} = N - \{1\}$. This result shows that the positive integers x, satisfying $T^k(x) < x$, for some k may be considered as integers favourable to Collatz conjecture. So the positive integers x, satisfying $S^k(x) > x$, for some k are considered as integers favourable to Collatz conjecture in this article. This article is to find this type of favourable integers. But this excludes the case $x = 1$ or 2, because $S(1) = 2, S^2(1) = 1$ and $S(2) = 1, S^2(2) = 2$. There are articles [1, 4, 8–10, 12, 13] of this type which find particular integers "favourable" to Collatz conjecture, apart from a type of articles [2, 3, 5–7, 11, 14–17] proving theoretical results.

2. First Approach

Case 1: Suppose $x = 3m$ or $x = 3m + 1$, for some $m = 1, 2, 3, 4, \ldots$. Then $S(x) = 2x > x$. Therefore the numbers of the form $x = 3m$ or $x = 3m + 1$, for $m = 1, 2, 3, 4, \ldots$ are integers favourable to Collatz conjecture.

Case 2: Suppose $x = 3m+2$ and $m = 3n$, with $n = 1, 2, 3, 4, \ldots$. Then $x = 9n+2 = 3(3n)+2$, $S(x) = 2(3n)+1 = 3(2n)+1$, $S^2(x) = 6(2n) + 2 > 9n + 2 = x$. Hence $S^2(x) > x$. Therefore $x = 9n + 2$, for $n = 1, 2, 3, 4, \ldots$ are integers favourable to Collatz conjecture.

Case 3: Suppose $x = 3m + 2$ and $m = 3n + 1$, with $n = 0, 1, 2, 3, 4, \ldots$. Then $x = 9n + 5 = 3(3n + 1) + 2$, $S(x) = 2(3n + 1) + 1 = 3(2n + 1)$= a multiple of 3, $S^2(x) = 6(2n + 1) = 3(4n + 2) > 9n + 5 = x$. Hence $S^2(x) > x$. Therefore $x = 9n + 5$, for $n = 0, 1, 2, 3, 4....$, are integers favourable to Collatz conjecture.

Case 4: Suppose $x = 9m+8$ and $m = 3n$, with $n = 0, 1, 2, 3, 4, \ldots$. Then $x = 27n+8 = 3(9n+2)+2$, $S(x) = 2(9n+2)+1 = 3(6n + 1) + 2$, $S^2(x) = 2(6n + 1) + 1 = 3(4n + 1)$= a multiple of 3, $S^3(x) = 6(4n + 1) = 3(8n + 2)$= a multiple of 3, $S^4(x) = 6(8n + 2) = 48n + 12 > 27n + 8 = x$. Hence $S^4(x) > x$. Therefore $x = 9n + 5$, for $n = 0, 1, 2, 3, 4, \ldots$ are integers favourable to Collatz conjecture.

Case 5: Suppose $x = 9m + 8$, $m = 3n + 1$, and $n = 3p$ with $p = 0, 1, 2, 3, 4, \ldots$. Then $x = 81p + 17 = 3(27p + 5) + 2$, $S(x) = 2(27p + 5) + 1 = 3(18p + 5) + 1$, $S^2(x) = 6(18p + 5) + 2 = 108p + 32 > 81p + 17 = x$. Hence $S^2(x) > x$. Therefore $x = 81p + 17$, for $p = 0, 1, 2, 3, 4, \ldots$ are integers favourable to Collatz conjecture.

Case 6: Suppose $x = 9m + 8$, $m = 3n + 1$, and $n = 3p + 1$ with $p = 0, 1, 2, 3, 4, \ldots$. Then $x = 81p + 44 = 3(27p + 13) + 2$, $S(x) = 2(27p + 13) + 1 = 3(18p + 9)$= a multiple of 3, $S^2(x) = 6(18p + 9) = 108p + 54 > 81p + 44 = x$. Hence $S^2(x) > x$. Therefore $x = 81p + 44$, for $p = 0, 1, 2, 3, 4, \ldots$ are integers favourable to Collatz conjecture.

Case 7: Suppose $x = 9m+8$, $m = 3n+2$, $n = 3p$, and $p = 3r$ with $r = 0, 1, 2, 3, 4.....$ Then $x = 243r+26 = 3(81r+8)+2$, $S(x) = 2(81r+8)+1 = 3(54r+5)+2$, $S^2(x) = 2(54r+5)+2 = 3(36r+3)+2$, $S^3(x) = 2(36r+3)+1 = 3(24r+2)+1$, $S^4(x) = 6(24r + 2) + 2 = 3(48r + 4)$, $S^5(x) = 2(48r + 4) + 1 = 3(32r + 3)$= a multiple of 3, $S^6(x) = 6(32r + 3) = 3(64r + 6)$ = a multiple of 3, $S^7(x) = 6(64r + 6) = 384r + 36 > 243r + 26 = x$. Hence $S^7(x) > x$. Therefore $x = 243r + 26$, for $r = 0, 1, 2, 3, 4....$, are integers favourable to Collatz conjecture.

Case 8: Suppose $x = 9m + 8$, $m = 3n + 2$, and $n = 3p + 1$, with $p = 0, 1, 2, 3, 4.....$ Then $x = 81p + 53 = 3(27p + 17) + 2$, $S(x) = 2(27p+17)+1 = 3(18p+11)+2$, $S^2(x) = 2(18p+11)+1 = 3(12p+7)+2$, $S^3(x) = 2(12p+7)+1 = 3(8p+5)$= a multiple of 3, $S^4(x) = 6(8p + 5) = 3(16p + 10)$= a multiple of 3, $S^5(x) = 6(16p + 10) = 96p + 60 > 81p + 53 = x$. Hence $S^5(x) > x$. Therefore $x = 81p + 53$, with $p = 0, 1, 2, 3, 4....$, are integers favourable to Collatz conjecture.

So we have the following conclusion.

Theorem 2.1. *The following types of integers are favourable to Collatz conjecture.*

(i). $x = 3m$, *with* $m = 1, 2, 3, 4......$

(ii). $x = 3n+1$, $x = 9n+2$, $x = 9n+5$, $x = 27n+8$, $x = 81n+17$, $x = 81n+44$, $x = 81n+53$, $x = 243n+26$ *with* $n = 0,1,2,3.....$

3. Second Approach

If we write $N - \{1,2\} = N_1 \cup N_2 \cup N_3$, where $N_1 = \{3m : m \in N\}$, $N_2 = \{3m+1 : m \in N\}$, $N_3 = \{3m+2 : m \in N\}$, then the integers in N_1 and N_2 are integers favourable to Collatz conjecture. But it is being difficult to find, whether the integers of N_3 are integers favourable to Collatz conjecture or not. For this difficult case, here N_3 is splitted in to many sets like, $B_1 = \{3m+2$ with $m = 3n : n = 1,2,3.....\}$, $B_2 = \{3m+2$ with $m = 3n+1 : n = 1,2,3.....\}$, $B_3 = \{9m+8$ with $m = 3n$: $n = 1,2,3.....\}$, $B_4 = \{9m+8$ with $m = 3n+1$ and $n = 3p : p = 0,1,2,3.....\}$, $B_5 = \{9m+8$ with $m = 3n+1$ and $n = 3p+1$: $p = 0,1,2,3.....\}$, $B_6 = \{9m+8$ with $m = 3n+2$, $n = 3p$ and $p = 3r : r = 0,1,2,3.....\}$, $B_7 = \{9m+8$ with $m = 3n+2$ and $n = 3p+1 : p = 0,1,2,3.....\}$ etc., to find integers favourable to Collatz conjecture. B_1, B_2, B_3, B_4, B_5, B_6, B_7 are sets of integers favourable to Collatz conjecture in view of Theorem 2.1.

Let us consider the set $N_3 = \{5,8,11,14......\}$ again for discussion. Let us apply S succesively on these integers x in N_3 till $S^l(x) > (x)$ or $S^l(x)$ is a multiple of 3, for some smallest l. For the second case let k be the smallest integer such that divisibility of 3^k is done till it becomes a multiple of 3. That is there are l iterations in which integers of the type $3q+2$ are encountered k times, before $S^l(x)$ is a multiple of 3. Similarly, let same k be the smallest integer such that divisibility by 3^{k+1} is done till it satisfies the relation $S^l(x) > (x)$, for some smallest l for the first case. That is, there are l iterations in which integers of the type $3q+2$ are encountered $(k+1)$ times. Then it happens that $3^{k+1}p + x$ is favourable in both cases for Collatz conjecture (with $p = 0,1,2,3,.....$). One may understand the validity of this reason through the following illustrations.

Case 1: Consider $x = 5 = 3(1)+2$. Then $S(x) = 2(1)+1 = 3=$ a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0,1,2,3,.....$ Then $y = 3^2p+5$, with $p = 0,1,2,3,...$ and $y = 3(3p+1)+2$, $S(y) = 2(3p+1)+1 = 3(2p+1)=$ a multiple of 3, $S^2(y) = 6(2p+1) = 12p+6 > 3^2p+5 = y$. Hence $S^2(y) > y$, for all y in A_1, where $A_1 = \{3^2p+5 : p = 0,1,2....\}$ and A_1 has integers favourable to Collatz conjecture.

Case 2: Consider $x = 8 = 3(2)+2$. Then $S(x) = 2(2)+1 = 3(1)+2$, $S^2(x) = 2(1)+1 = 3 =$ a multiple of 3. Therefore the value of k is 2. Take $y = 3^{k+1}p + x$, with $p = 0,1,2,3,.....$ Then $y = 3^3p+5$, with $p = 0,1,2,3,...$ and $y = 3(9p+2)+2$, $S(y) = 2(9p+2)+1 = 3(6p+1)+2$, $S^2(y) = 2(6p+1)+1 = 3(4p+1) =$ a multiple of 3, $S^4(y) = 6(8p+2) = 48p+16 > 27p+8 = y$. Hence $S^4(y) > y$, for all y in A_2, where $A_2 = \{3^3p+8 : p = 0,1,2....\}$ and A_2 has integers favourable to Collatz conjecture.

Case 3: Consider $x = 11 = 3(3)+2$. Then $S(x) = 2(3)+1 = 3(2)+1$, $S^2(x) = 6(2)+2 = 14 > 11 = x$. Therefore the value of $(k+1)$ is 2. Take $y = 3^{k+1}p + x$, with $p = 0,1,2,3,.....$ Then $y = 3^2p+11$, with $p = 0,1,2,3,...$ and $y = 3(3p+3)+2$, $S(y) = 2(3p+3)+1 = 3(2p+2)+1$, $S^2(y) = 6(2p+2)+2 = 12p+14 > 9p+11 = y$. Hence $S^2(y) > y$, for all y in A_3, where $A_3 = \{3^2p+11 : p = 0,1,2....\}$ and A_3 has integers favourable to Collatz conjecture.

Case 4: Consider $x = 14 = 3(4)+2$. Then $S(x) = 2(4)+1 = 9=$ a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0,1,2,3,.....$ Then $y = 3^2p+14$, with $p = 0,1,2,3,...$ and $y = 3(3p+4)+2$, $S(y) = 2(3p+4)+1 = 3(2p+3)=$ a multiple of 3, $S^2(y) = 6(2p+3) = 12p+18 > 9p+14 = y$. Hence $S^2(y) > y$, for all y in A_4, where $A_1 = \{3^2p+14 : p = 0,1,2....\}$ and A_4 has integers favourable to Collatz conjecture.

Case 5: Consider $x = 17 = 3(5)+2$. Then $S(x) = 2(5)+1 = 3(3)+2$, $S^2(x) = 2(3)+1 = 3(2)+1$, $S^3(x) = 6(2)+2 = 3(4)+2$, $S^4(x) = 2(4)+1 = 9=$ a multiple of 3. Therefore the value of k is 3. Take $y = 3^{k+1}p + x$, with $p = 0,1,2,3,.....$

Then $y = 3^4p + 17$, with $p = 0, 1, 2, 3, ...$ and $y = 3(27p + 5) + 2$, $S(y) = 2(27p + 5) + 1 = 3(18p + 3) + 2$, $S^2(y) = 2(18p+3)+1 = 3(12p+2)+1$, $S^3(y) = 6(12p+2)+2 = 3(24p+4)+2$, $S^4(y) = 2(24p+4)+1 = 3(16p+3)$, a multiple of 3, $S^5(y) = 6(16p+3) = 96p+18 > 81p+17 = y$. Hence $S^5(y) > y$, for all y in A_5, where $A_5 = \{3^4p+17 : p = 0, 1, 2....\}$ and A_5 has integers favourable to Collatz conjecture.

Case 6: Consider $x = 20 = 3(6) + 2$. Then $S(x) = 2(6) + 1 = 3(4) + 1$, $S^2(x) = 6(4) + 2 = 26 > 20 = x$. Therefore the value of $(k + 1)$ is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, ...$ Then $y = 3^2p + 20$, with $p = 0, 1, 2, 3, ...$ and $y = 3(3p + 6) + 2$, $S(y) = 2(3p + 6) + 1 = 3(2p + 4) + 1$, $S^2(y) = 6(2p + 4) + 2 = 12p + 26 > 9p + 26 = y$. Hence $S^2(y) > y$, for all y in A_6, where $A_6 = \{3^2p+20 : p = 0, 1, 2....\}$ and A_6 has integers favourable to Collatz conjecture.

Case 7: Consider $x = 23 = 3(7) + 2$. Then $S(x) = 2(7) + 1 = 15$= a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^2p + 23$, with $p = 0, 1, 2, 3, \ldots$ and $y = 3(3p + 7) + 2$, $S(y) = 2(3p+7)+1 = 3(2p+5)$= a multiple of 3, $S^2(y) = 6(2p+5) = 12p+30 > 9p+23 = y$. Hence $S^2(y) > y$, for all y in A_7, where $A_7 = \{3^2p + 23 : p = 0, 1, 2, \ldots\}$ and A_7 has integers favourable to Collatz conjecture.

Case 8: Consider $x = 26 = 3(8)+2$. Then $S(x) = 2(8)+1 = 3(5)+2$, $S^2(x) = 2(5)+1 = 3(3)+2$, $S^3(x) = 2(3)+1 = 3(2)+1$, $S^4(x) = 6(2)+2 = 3(4)+2$, $S^5(x) = 2(4)+1 = 9$= a multiple of 3. Therefore the value of k is 4. Take $y = 3^{k+1}p+x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^5p+26$, with $p = 0, 1, 2, 3, ...$ and $y = 3(81p+8)+2$, $S(y) = 2(81p+8)+1 = 3(54p+5)+2$, $S^2(y) = 2(54p+5)+1 = 3(36p+3)+2$, $S^3(y) = 2(36p+3)+1 = 3(24p+2)+1$, $S^4(y) = 6(24p+2)+2 = 3(48p+4)+2$, $S^5(y) = 2(48p + 4) + 1 = 3(32p + 3)$= a multiple of 3 , $S^6(y) = 6(32p + 3) = 3(64p + 6)$= a multiple of 3, , $S^7(y) = 6(64p + 6) = 384p + 36 > 243p + 26 = y$. Hence $S^7(y) > y$, for all y in A_8, where $A_8 = \{3^5p + 26 : p = 0, 1, 2, \ldots\}$ and A_8 has integers favourable to Collatz conjecture.

Case 9: Consider $x = 29 = 3(9) + 2$. Then $S(x) = 2(9) + 1 = 3(6) + 1$, $S^2(x) = 6(6) + 2 = 38 > 29 = x$. Therefore the value of $(k + 1)$ is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^2p + 29$, with $p = 0, 1, 2, 3, \ldots$ and $y = 3(3p + 9) + 2$, $S(y) = 2(3p + 9) + 1 = 3(2p + 6) + 1$, $S^2(y) = 6(2p + 6) + 2 = 12p + 38 > 9p + 29 = y$. Hence $S^2(y) > y$, for all y in A_9, where $A_9 = \{3^2p+29 : p = 0, 1, 2, \ldots\}$ and A_9 has integers favourable to Collatz conjecture.

Case 10: Consider $x = 32 = 3(10) + 2$. Then $S(x) = 2(10) + 1 = 21$= a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^2p + 32$, with $p = 0, 1, 2, 3, \ldots$ and $y = 3(3p + 10) + 2$, $S(y) = 2(3p + 10) + 1 = 3(2p + 7)$= a multiple of 3, $S^2(y) = 6(2p + 7) = 12p + 42 > 9p + 32 = y$. Hence $S^2(y) > y$, for all y in A_{10}, where $A_{10} = \{3^2p + 32 : p = 0, 1, 2, \ldots\}$ and A_{10} has integers favourable to Collatz conjecture.

Case 11: Consider $x = 35 = 3(11) + 2$. Then $S(x) = 2(11) + 1 = 3(7) + 2$, $S^2(x) = 2(7) + 1 = 15$= a multiple of 3. Therefore the value of k is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^3p + 35$, with $p = 0, 1, 2, 3, \ldots$ and $y = 3(9p + 11) + 2$, $S(y) = 2(9p + 11) + 1 = 3(6p + 7) + 2$, $S^2(y) = 2(6p + 7) + 1 = 3(4p + 5)$= a multiple of 3, $S^3(y) = 6(4p + 5) = 3(8p + 10)$= a multiple of 3, $S^4(y) = 6(8p + 10) = 48p + 60 > 27p + 35 = y$. Hence $S^4(y) > y$, for all y in A_{11}, where $A_{11} = \{3^3p + 35 : p = 0, 1, 2, \ldots\}$ and A_{11} has integers favourable to Collatz conjecture.

Case 12: Consider $x = 38 = 3(12) + 2$. Then $S(x) = 2(12) + 1 = 3(8) + 1$, $S^2(x) = 6(8) + 2 = 50 > 38 = x$. Therefore the value of $(k + 1)$ is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^2p + 38$, with $p = 0, 1, 2, 3, \ldots$ and $y = 3(3p + 12) + 2$, $S(y) = 2(3p + 12) + 1 = 3(2p + 8) + 1$, $S^2(y) = 6(2p + 8) + 2 = 12p + 50 > 9p + 38 = y$. Hence $S^2(y) > y$, for all y in A_{12}, where $A_{12} = \{3^2p + 38 : p = 0, 1, 2, \ldots\}$ and A_{12} has integers favourable to Collatz conjecture.

Case 13: Consider $x = 41 = 3(13) + 2$. Then $S(x) = 2(13) + 1 = 27$= a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \ldots$. Then $y = 3^2p + 41$, with $p = 0, 1, 2, 3, \ldots$ and $y = 3(3p + 13) + 2$,

$S(y) = 2(3p + 13) + 1 = 3(2p + 9)$= a multiple of 3, $S^2(y) = 6(2p + 9) = 12p + 54 > 9p + 41 = y$. Hence $S^2(y) > y$, for all y in A_{13}, where $A_{13} = \{3^2p + 41 : p = 0, 1, 2, \dots\}$ and A_{13} has integers favourable to Collatz conjecture.

Case 14: Consider $x = 44 = 3(14) + 2$. Then $S(x) = 2(14) + 1 = 3(9) + 2$, $S^2(x) = 2(9) + 1 = 3(6) + 1$, $S^3(x) = 6(6) + 2 = 3(12) + 2$, $S^4(x) = 2(12) + 1 = 3(8) + 1$, $S^5(x) = 6(8) + 2 = 50 > 44$. Therefore the value of $(k + 1)$ is 4. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$. Then $y = 3^4p + 44$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(27p + 14) + 2$, $S(y) = 2(27p+14)+1 = 3(18p+9)+2$, $S^2(y) = 2(18p+9)+1 = 3(12p+6)+1$, $S^3(y) = 6(12p+6)+2 = 3(24p+12)+2$, $S^4(y) = 2(24p + 12) + 1 = 3(16p + 8) + 1$, $S^5(y) = 6(16p + 8) + 2 = 96p + 49 > 81p + 44 = y$. Hence $S^5(y) > y$, for all y in A_{14}, where $A_{14} = \{3^4p + 44 : p = 0, 1, 2, \dots\}$ and A_{14} has integers favourable to Collatz conjecture.

Case 15: Consider $x = 47 = 3(15) + 2$. Then $S(x) = 2(15) + 1 = 3(10) + 1$, $S^2(x) = 6(10) + 2 = 62 > 47 = x$. Therefore the value of $(k + 1)$ is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$. Then $y = 3^2p + 47$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(3p + 15) + 2$, $S(y) = 2(3p + 15) + 1 = 3(2p + 10) + 1$, $S^2(y) = 6(2p + 10) + 2 = 12p + 62 > 9p + 47 = y$. Hence $S^2(y) > y$, for all y in A_{15}, where $A_{15} = \{3^2p + 47 : p = 0, 1, 2, \dots\}$ and A_{15} has integers favourable to Collatz conjecture.

Case 16: Consider $x = 50 = 3(16) + 2$. Then $S(x) = 2(16) + 1 = 33$= a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$.. Then $y = 3^2p + 50$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(3p + 16) + 2$, $S(y) = 2(3p + 16) + 1 = 3(2p + 11)$= a multiple of 3, $S^2(y) = 6(2p + 16) = 12p + 66 > 9p + 50 = y$. Hence $S^2(y) > y$, for all y in A_{16}, where $A_{16} = \{3^2p + 50 : p = 0, 1, 2, \dots\}$ and A_{16} has integers favourable to Collatz conjecture.

Case 17: Consider $x = 53 = 3(17)+2$. Then $S(x) = 2(17)+1 = 3(11)+2$, $S^2(x) = 2(11)+1 = 3(7)+2$, $S^3(x) = 2(7)+1 = 15$= a multiple of 3. Therefore the value of k is 3. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$. Then $y = 3^4p + 53$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(27p + 17) + 2$, $S(y) = 2(27p + 17) + 1 = 3(18p + 11) + 2$, $S^2(y) = 2(18p + 11) + 1 = 3(12p + 7) + 2$, $S^3(y) = 2(12p + 7) + 1 = 3(8p + 5)$= a multiple of 3, $S^4(y) = 6(8p + 5) = 3(16p + 10)$= a multiple of 3, $S^5(y) = 6(16p + 10) = 96p + 60 > 81p + 53 = y$. Hence $S^5(y) > y$, for all y in A_{17}, where $A_{17} = \{3^4p + 53 : p = 0, 1, 2, \dots\}$ and A_{17} has integers favourable to Collatz conjecture.

Case 18: Consider $x = 56 = 3(18) + 2$. Then $S(x) = 2(18) + 1 = 3(12) + 1$, $S^2(x) = 6(12) + 2 = 72 > 56$. Therefore the value of $(k + 1)$ is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$. Then $y = 3^2p + 56$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(3p + 18) + 2$, $S(y) = 2(3p + 18) + 1 = 3(2p + 12) + 1$, $S^2(y) = 6(2p + 12) + 2 = 12p + 74 > 9p + 56 = y$. Hence $S^2(y) > y$, for all y in A_{18}, where $A_{18} = \{3^2p + 56 : p = 0, 1, 2, \dots\}$ and A_{18} has integers favourable to Collatz conjecture.

Case 19: Consider $x = 59 = 3(19) + 2$. Then $S(x) = 2(19) + 1 = 39$= a multiple of 3. Therefore the value of k is 1. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$.. Then $y = 3^2p + 59$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(3p + 19) + 2$, $S(y) = 2(3p + 19) + 1 = 3(2p + 13)$= a multiple of 3, $S^2(y) = 6(2p + 13) = 12p + 72) > 9p + 59 = y$. Hence $S^2(y) > y$, for all y in A_{19}, where $A_{19} = \{3^2p + 59 : p = 0, 1, 2, \dots\}$ and A_{19} has integers favourable to Collatz conjecture.

Case 20: Consider $x = 62 = 3(20) + 2$. Then $S(x) = 2(20) + 1 = 3(13) + 2$, $S^2(x) = 2(13) + 1 = 27$= a multiple of 3. Therefore the value of k is 2. Take $y = 3^{k+1}p + x$, with $p = 0, 1, 2, 3, \dots$. Then $y = 3^3p + 62$, with $p = 0, 1, 2, 3, \dots$ and $y = 3(9p + 20) + 2$, $S(y) = 2(9p + 20) + 1 = 3(6p + 13) + 2$, $S^2(y) = 2(6p + 13) + 1 = 3(4p + 9)$=a multiple of 3. $S^3(y) = 6(4p + 9) = 3(8p + 18)$ = a multiple of 3, $S^4(y) = 6(8p + 18) = 48p + 108 > 27p + 62 = y$.

Hence $S^4(y) > y$, for all y in A_{20}, where $A_{20} = \{3^3p + 62 : p = 0, 1, 2, \dots\}$ and A_{20} has integers favourable to Collatz conjecture. This procedure of finding k leads to the following conclusion.

Theorem 3.1. *The following integers of the form are favourable to Collatz conjecture, in addition to integers of the form* $3m$ *and* $3m+1$ *and the integers* 1 *and* 2.
(i) $x = 3^2p + 5$ *(ii)* $x = 3^3p + 8$ *(iii)* $x = 3^2p + 11$ *(iv)* $x = 3^2p + 14$ *(v)* $x = 3^4p + 17$ *(vi)* $x = 3^2p + 20$ *(vii)* $x = 3^2p + 23$ *(viii)* $x = 3^5p + 26$ *(xi)* $x = 3^2p + 29$ *(x)* $x = 3^2p + 32$ *(xi)* $x = 3^3p + 35$ *(xii)* $x = 3^2p + 38$ *(xiii)* $x = 3^2p + 41$ *(xiv)* $x = 3^4p + 44$ *(xv)* $x = 3^2p + 47$ *(xvi)* $x = 3^2p + 50$ *(xvii)* $x = 3^4p + 53$ *(xviii)* $x = 3^2p + 56$ *(xix)* $x = 3^2p + 59$ *(xx)* $x = 3^2p + 62$, *with* $p = 0, 1, 2, \ldots$.

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