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Thermal Radiation and Viscous Dissipation Effects on MHD Heat and Mass Difussion Flow Past a Surface Embedded in a Porous Medium With Chemical Reaction

Research Article

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Abstract: The present paper deals with thermal radiation effects on the transient hydro-magnetic natural convection flow past a vertical plate embedded in a porous medium with mass diffusion and fluctuating temperature about time at the plate, by taking account the heat due to viscous dissipation in the presence of chemical reaction. The governing equations are solved numerically by Ritz finite element method. The effects of various parameters entered into the equations of momentum, energy and concentration are evaluated numerically and plotted graphically, while numerical values of variations in skin-friction, rate of heat transfer in terms of Nusselt number at the plate are presented in tabular form. The results of the study are discussed.

Keywords: Natural convection, Thermal radiation, MHD, chemical reaction, porous medium, viscous dissipation.

1. Introduction

Natural convection flows are frequently encountered in physical and engineering problems such as chemical catalytic reactors, nuclear waste materials etc. Transient free convection is important in many practical applications, such as furnaces electronic components, solar collectors, thermal regulation process, security of energy systems etc. when a conductive fluid moves through a magnetic field and an ionized gas is electrically conductive, the fluid may be influenced by the magnetic field. Magneto-hydrodynamic free convection heat transfer flow is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, liquid metal fluids and MHD power generation systems etc. Transport processes in porous media are encountered in a broad range of scientific and engineering problems associated with the fiber and granular insulation materials, packed-bed chemical reactors and transpiration cooling. Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation and underground energy transport. The change in wall temperature causing the free convection flow could be a sudden or a periodic one, leading to a variation in the flow. In nuclear engineering, cooling of medium is more important safety point of view and during this cooling process the plate temperature starts oscillating about a non-zero constant mean temperature. Further, oscillatory flow has application in industrial and aerospace engineering. Thermal radiation in fluid dynamics has become a significant branch of the engineering

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sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power, and hazards engineering. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Bestman [1] studied free convection heat transfer to steady radiating non-Newtonian MHD flow past a vertical porous plate. Pop and Herwig [2] studied transient mass transfer from an isothermal vertical flat plate embedded in porous medium. Jha [3] studied MHD free convection and mass transfer flow through a porous medium but did not consider the effect of radiation which is of great relevance to astrophysical and cosmic studies. Takhar et al. [4] presented radiation effects on MHD free convection flow past a semi infinite vertical plate. Hossain and Takhar [5] presented the radiation effects on mixed convection along a vertical plate with uniform surface temperature. Bestman and Adjepong [6] studied unsteady hydro-magnetic free convection flow with radiative heat transfer in a rotating fluid. Chamkha [7] investigated unsteady convective heat and mass transfer past a semi-infinite permeable moving plate with heat absorption where it was found that increase in solutal Grashof number enhanced the concentration buoyancy effects leading to an increase in the velocity. Kim [8] studied unsteady MHD convective heat transfer past a semi infinite vertical porous moving plate with variable suction. Abd-El-Nay et al. [9] presented the radiation effects on MHD free convection flow over a vertical plate with variable surface temperature by finite difference solution. Ibrahim et al. [10] investigated unsteady magneto-hydrodynamic micro-polar fluid flow and heat transfer over a vertical porous medium in the presence of thermal and mass diffusion with constant heat source. Mbeledogu and Ogulu [11] studied heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. Chaudary and Jain [12] presented the combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium by Laplace transform technique.

The study of MHD transient heat and mass transfer with chemical reaction is of practical importance to engineers and scientists, due to its applications in several areas in many branches of science or engineering or technology. Muthucumaraswamy and Ganesan [13] presented the effects of chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Kandasamy et al. [14] studied the effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. The effect of chemical reaction on an electrically conducting viscous fluid flow over a non-linearly semi infinite stretching heat in the presence of a magnetic field which is normal to the sheet was studied by Raptis and Pedikis [15]. Chaudary and Jha [16] presented the effect of chemical reaction on MHD micro-polar fluid flow past a vertical plate in slip-flow regime. Ibrahim et al. [17] presented the chemical reaction and radiation absorption effects on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction.

In most of the studies mentioned above, viscous dissipation effect is neglected. The viscous dissipation effect is expected to be relevant for fluids with high values of dynamic viscosity as for high velocity flows. The viscous dissipation heat is important in the natural convective flows, when the field is of extreme size or at extremely low temperature or in high gravitational field. Soundalgekar [18] analyzed the viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Cookey et al. [19] studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Zueco Jordan [20] used network simulation method to study the effects radiation and dissipation on MHD unsteady free convection flow over vertical porous plate. Suneetha et al. [21] studied the effects of thermal radiation on natural conductive heat and mass transfer of a viscous incompressible gray-absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Hitesh [22] studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

Hence, based on the above mentioned investigations and applications, the object of this paper is to study the radiative magneto-hydrodynamic transient heat and mass transfer flow by free convection past a vertical plate with viscous dissipation in the presence of chemical reaction, when the temperature of the plate oscillates periodically about a constant mean temperature and the plate being embedded in a porous medium. The equations of momentum, energy and concentration are solved by using Ritz FEM. The behavior of the velocity, temperature, concentration, skin-friction and Nusselt number has been discussed for variations in the governing parameters.

2. Mathematical Analysis

We consider a two-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical plate that is embedded in a porous medium. The $x'-$axis is taken along the infinite plate and $y'-$ axis normal to it. Initially, the plate and the fluid are at same temperature T'_∞ with concentration level C'_∞ at all points. At time $t' > 0$, the plate temperature is raised to T'_w and a periodic temperature variation is assumed to be superimposed on this mean constant temperature of the plate and the concentration level at the plate raised to C'_w. A magnetic field of uniform strength is applied perpendicular to the plate and the magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. There is no applied electric field. Viscous and Darcy term is taken into account with the constant permeability of porous medium. The MHD term is derived from an order-of-magnitude analysis of the full Navier-Stokes equations. We regard the porous medium as an assembly of small identical spherical particles fixed in space. Under these conditions and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + \left[g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty)\right] - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{k'} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} + \frac{\nu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} - k'_r C' \tag{3}$$

with the following initial and boundary conditions:

$$\begin{aligned}
&u' = 0, T' = T'_\infty, C' = C'_\infty \ \text{ for all } \ y', t' \leq 0\\
&u' = 0, T' = T'_w + \in (T'_w - T'_\infty)\cos\omega' t', C' = C'_w \ \text{ at } \ y' = 0, t' > 0\\
&u' \to 0, T' \to T'_\infty, C' \to C'_\infty \ \text{ as } \ y' \to \infty, t' > 0
\end{aligned} \tag{4}$$

where u' is the velocity components in x' direction, $t'-$the time, $\rho-$the fluid density, $g-$the acceleration due to gravity, β and $\beta_c-$the thermal and concentration expansion coefficients respectively, $k'-$the permeability of the porous medium, $T'-$the temperature of the fluid, $T'_w-$the temperature of the fluid near the plate, $T'_\infty-$the temperature of the fluid far away from the plate, $C'-$the concentration at any point of the field, $C'_w-$the concentration the plate, $C'_\infty-$the concentration at the free stream, $\nu-$the kinematic viscosity, $\sigma-$the electrical conductivity of the fluid, $C_p-$the specific heat at constant pressure, $\mu-$the viscosity of the fluid, $k-$thermal conductivity of the fluid, $B_0-$the magnetic field component along y'-axis, induction, $D_M-$the mass diffusivity, $\in-$the amplitude (constant), $q_r-$the radiative heat flux.

The second term of R.H.S of the momentum Equation (1) denotes buoyancy effects, the third term is the MHD term, the forth term is bulk matrix linear resistance, that is Darcy term. The second term of R.H.S of the energy Equation (2) denotes the radiation term; the third term is viscous dissipation. The heat due to viscous dissipation is taken into account. Also, Darcy dissipation term is neglected for small velocities in Equation (2).

The temperature distribution is independent of the flow and heat transfer is by conduction alone. That is true for fluids in initial stage due to the absence of convective heat transfer or at small Grashof number ($G_r \leq 1$). Thermal radiation is assumed to be present in the form of a unidirectional flux in the $y-$direction i.e., q_r (Transverse to the vertical surface). By using the Rossel and approximation, the radiative heat flux is given by

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T'}{\partial y'} \tag{5}$$

where $\sigma_s-$is the Stefan-Boltzmann constant and $k_e-$is the mean absorption coefficient. It should be noted that by using the Rossel and approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then equation (5) can be linearzed by expanding T'^4in Taylor series about T'_∞ which after neglecting higher order terms takes the form:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

In view of Equation (5) and Equation (6), Equation (2) reduces to

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma_s T'^3_\infty}{3k_e\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{7}$$

We introduce the following non-dimensional variables:

$$\begin{gathered} y = \frac{y'}{L_R},\ t = \frac{t'}{t_R},\ u = \frac{u'}{U_R},\ \omega = \omega' t_R,\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ K = \frac{U_R^2 k'}{\nu^2},\ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},\ N = \frac{k_e k}{4\sigma_s T'^3_\infty} \\ S_c = \frac{\nu}{D_M},\ P_r = \frac{\mu C_p}{k},\ G_m = \frac{g\beta_c \nu (C'_w - C'_\infty)}{U_0^2},\ U_R = (\nu g\beta\Delta T)^{1/3},\ L_R = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3},\ E_c = \frac{U_R^2}{C_p\Delta T}, \\ t_R = (g\beta\Delta T)^{-1/2}\nu^{1/3} \end{gathered} \tag{8}$$

Substituting Equations (8) into Equations (2), (7), and (3), the following governing equations are obtained in non-dimensional form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta + G_m C - \left(M + \frac{1}{K}\right)u \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(1 + \frac{4}{3N}\right)\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - k_r C \tag{11}$$

where G_m is modified Grashof number, M is magnetic parameter, K is permeability parameter, P_r is Prandtl number, S_c is Schmidt number, N is radiation parameter, E_c is Eckert number, L_R is reference length, t_R is reference time, u is velocity component, U_R is reference velocity, θ is dimensionless temperature, C is dimensionless concentration, and ω is frequency of oscillation. The corresponding initial and boundary conditions:

$$\begin{aligned} &u = 0, \theta = 0, C = 0 \text{ for all } y, t \leq 0 \\ &u = 0, \theta = 1 + \in \cos\omega t, C = 1 \text{ at } y = 0, t > 0 \\ &u \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty, t > 0 \end{aligned} \tag{12}$$

where ωt is phase angle.

Skin-friction: In non-dimensional form, the skin-friction is given by $\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$.

Nusselt Number: From temperature field. The rate of heat transfer in non-dimensional form is expressed as $Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$.

3. Method of Solution

Equations (9)-(11) are coupled non-linear systems of partial differential equations, and are to be solved by using the physical boundary conditions given in equation (12). However, exact or approximate solutions are not possible for this set of equations and hence we can solve these equations by Ritz FEM. The linear functional for the Equation (9) over the two nodded linear element $(e), (y_j \leq y \leq y_k)$ is

$$J^{(e)}(u) = \frac{1}{2}\int_{y_j}^{y_k}\left[\left(\frac{\partial u^{(e)}}{\partial y}\right)^2 + \left(M + \frac{1}{K}\right)u^{(e)^2} + 2u^{(e)}\frac{\partial u^{(e)}}{\partial t} - 2u^{(e)}\left(\theta + G_m C\right)\right]dy = \text{Minimum} \tag{13}$$

Finite element model may be obtained from Equation (13) by substituting finite element approximation over the two-nodded linear element $(e), (y_j \leq y \leq y_k)$ of the form:

$$u^{(e)} = \psi^{(e)}\chi^{(e)}, \text{ here } \psi^{(e)} = [\psi_j, \psi_k] \text{ and } \chi^{(e)} = [u_j, u_k]^T \tag{14}$$

where u_j, u_k are the velocity components at j^{th} and k^{th} nodes of the typical element (e), $(y_j \leq y \leq y_k)$ and ψ_j, ψ_k are the basis functions defined as:

$$\psi_j = \frac{y_k - y}{y_k - y_j} \text{ and } \psi_k = \frac{y - y_j}{y_k - y_j}$$

Substituting Equation (14) into Equation (13), assembling the element equations by inter-element connectivity for two consecutive elements $y_{i-1} \leq y \leq y_i \text{and} y_i \leq y \leq y_{i+1}$, put row corresponding to the node i to zero, we obtain

$$\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1} = \frac{1}{h^2}\left(6 - Ph^2\right)\left(u_{i-1} + u_{i+1}\right) + \frac{1}{h^2}\left(12 + 4Ph^2\right)u_i + 6\left(\theta + G_m C\right) \tag{15}$$

where $P = M + \frac{1}{K}$. Applying the trapezoidal rule to Equation (15), the following system of equations in Crank-Nicholson method are obtained:

$$\left(1 - 3r + \frac{1}{2}rPh^2\right)u_{i-1}^{j+1} + (4 + 6r + 2rPh^2)u_i^{j+1} + \left(1 - 3r + \frac{1}{2}rPh^2\right)u_{i+1}^{j+1} = B_i^j \tag{16}$$

where

$$B_i^j = \left(1 + 3r - \frac{1}{2}rPh^2\right)u_{i-1}^j + (4 - 6r - 2rPh^2)u_i^j + \left(1 + 3r - \frac{1}{2}rPh^2\right)u_{i+1}^j$$

Here $r = k/h^2$ and h, k are the mesh sizes along the $y-$direction and time t, respectively and indices i and j refers to the space and time. Analogous equations can be obtained from equations (10) and (11). In the three obtained equations, taking $i = 1(1)n$ and using physical boundary conditions (12), we obtain the following tri-diagonal systems of equations:

$$AU = B$$

$$D\theta = E$$

$$FC = G$$

where A, D and F are tri-diagonal matrices of order n and whose elements are given by

$$a_{i,i} = 4 + 6r + 2rPh^2, d_{i,i} = 4NP_r + (6Nr + 8r), f_{i.i} = 4S_c + 6r + \frac{1}{2}rk_rS_ch^2; i = 1(1)n$$
$$a_{i-1,j} = 1 - 3r + \frac{1}{2}rPh^2, d_{i-1,j} = NP_r + (3Nr + 4r), f_{i-1.j} = S_c - 3r + \frac{1}{2}rk_rS_ch^2; i = 2(1)n$$

Here U, θ, C and B, E, G are column matrices having n components $u_i^{j+1}, \theta_i^{j+1}, C_i^{j+1}$ and B_i^j, B'^j_i, B''^j_i respectively. The solutions of the above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Computations are carried out until the steady state is reached. The Ritz FEM is shown to be convergent and stable.

4. Numerical Results and Discussion

In order to study the thermal radiation effects on the transient hydro-magnetic natural convection flow past a vertical plate embedded in a porous medium with mass diffusion and fluctuating temperature about time at the plate, by taking account the heat due to viscous dissipation with chemical reaction, the Ritz FEM has been adopted to solve the system of equations (9)-(11) subjected to the appropriate boundary conditions (12). In order to achieve the numerical solution, it is necessary to assign values of the dimensionless parameters involved in the analysis of the problem under consideration. Computational study has been carried out with an aim to investigate the variation of different quantities of interest involving the following values of the parameters. $P_r = 0.71, 7.0,\ S_c = 0.22, 0.78,\ N = 2.0, 4.0, 6.0,\ E_c = 0.1, 0.3, 0.5,\ M = 0.5, 1.0,\ K = 1.0, 2.0,\ G_m = 5.0, 7.0,\ k_r = 0.5, 1.0,\ \omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}$ and the amplitude parameter $\in$ is chosen 1.0. The computed results are presented through figures and tables.

Figure 1 depicts the transient temperature profiles against y (distance from the plate). The magnitude of temperature is maximum at the plate and then decays to zero asymptotically. The magnitude of temperature for air ($P_r = 0.71$) is greater than that of water ($P_r = 7.0$). This is due to the fact that thermal conductivity of fluid decreases with increasing P_r, resulting a decrease in thermal boundary layer thickness. Also, the temperature falls with an increase in the phase angle ωt for both air and water. Figure 2 shows the effect of the Eckert number E_c on the temperature field. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. The effect of viscous dissipation on flow field is to increase the energy, yielding a greater fluid temperature and as a consequence greater buoyancy force. The increase in the buoyancy force due to an increase in the dissipation parameter enhances the temperature. It is also observed that the magnitude of temperature for air ($P_r = 0.71$) is greater than that of water ($P_r = 7.0$). The effect of radiation parameter N on the temperature variations is depicted in Figure 3. The radiation parameter N (i.e., Stark number) defines the relative contribution of conduction heat transfer to thermal radiation transfer. As N increases, considerable reduction is observed in the temperature profiles from the peak value at the plate ($y = 0$) across the boundary layer regime to free stream ($y \to \infty$), at which the temperature is negligible for any value of N. It is also observed that the magnitude of temperature for air ($P_r = 0.71$) is greater than that of water ($P_r = 7.0$). Figure 4 shows the effect of time parameter t on the temperature profiles. It is seen that the temperature increasing with increasing value of time parameter for both air and water. It is also observed that the magnitude of temperature for air ($P_r = 0.71$) is greater than that of water($P_r = 7.0$).

Figure 5 depicts the effects of Schmidt number S_c, chemical reaction parameter k_r and time parameter t. We take the Schmidt number $S_c = 0.22$ and 0.78 as would corresponds to hydrogen and ammonia, respectively and observe that as the Schmidt number is increased the concentration and concentration boundary are both seen to decrease, whereas increase in the chemical reaction rate constant k_r leads to decrease in the concentration of the fluid. However, we observe an increase in the temperature and temperature boundary layer as the time parameter increases.

The effects of Schmidt number S_c and chemical reaction parameter k_r on the velocity profiles are presented in Figure 6. It is clear that the velocity increases and attains its maximum value in the vicinity of the plate then tends to zero as $y \to \infty$. The velocity decreases owing to an increase in the value of S_c when the plate is isothermal for both $P_r = 0.71$ and $P_r = 7.0$. The velocity for air is higher than that of water. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly. Further, the velocity decreases with increasing value of k_r for both air ($P_r = 0.71$) and water ($P_r = 7.0$) when hydrogen gas is presented in the flow. Figure 7 depicts the effects of Eckert number E_c and radiation parameter N on the velocity profiles for both air ($P_r = 0.71$) and water ($P_r = 7.0$). It can be seen that an increase in the Eckert number leads to increase in the velocity whereas an increase in the radiation parameter decreases the velocity. Moreover, velocity is marginally affected by the variations in the Eckert number. The effects of magnetic parameter M and porosity parameter K on the velocity profiles are presented in Figure 8. It can be seen that the velocity near the plate exceeds at the plate i.e., the velocity overshoot occurs. It is observed that an increase in the magnetic parameter leads to fall in the velocity. It is due to the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the flow and thus reducing its velocity. Further, the velocity increases with increasing value of porosity parameter for both air and water. The magnitude of velocity for air is higher than that of water. Figure 9 depicts the effects of modified Grashof number G_m and phase angle ωt on the velocity profiles for air and water. It is obvious from the figure that the maximum velocity attains in the vicinity of the plate then decreases to zero as $y \to \infty$. It is noted that the velocity increases with increasing G_m for both air and water. It is due to the fact that an increase in the modified Grashof number has the tendency to increase the mass buoyancy effect. Further, an increase in the phase angle leads to decrease in the velocity for both air and water. The effect of time parameter t on the velocity profiles for both air and water are presented in Figure 10. It is seen that the velocity increases with increasing value of time parameter for both air and water. It is also found that the effect of time on the velocity is more dominant than other parameters.

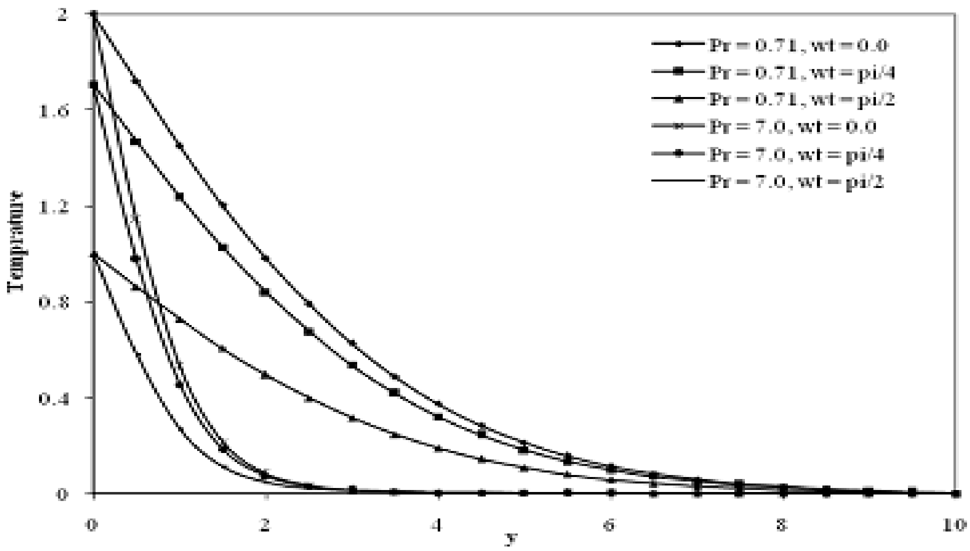

Figure 1. **Temperature profiles showing the effect of phase angle ωt at $t = 1.0$**

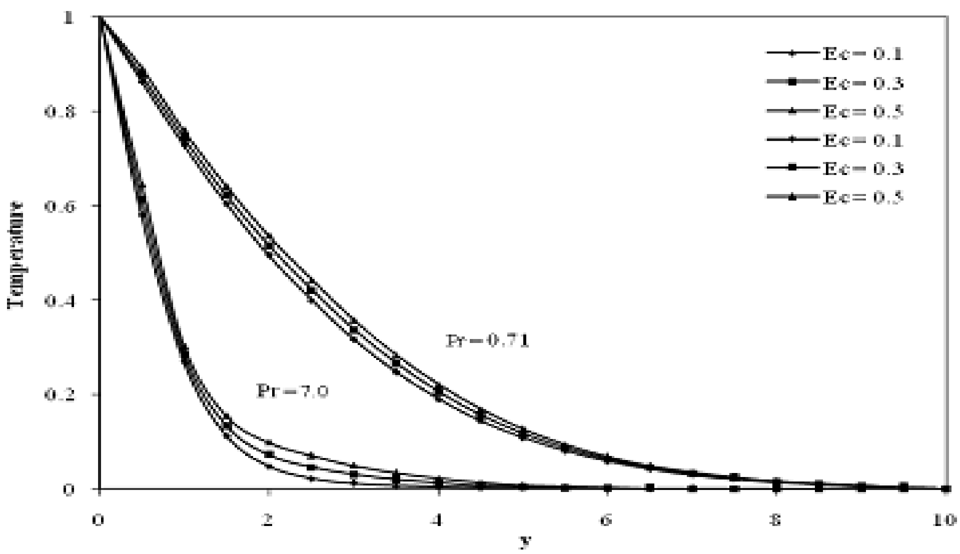

Figure 2. **Temperature profiles showing the effect of Eckert number E_c at $t = 1.0$**

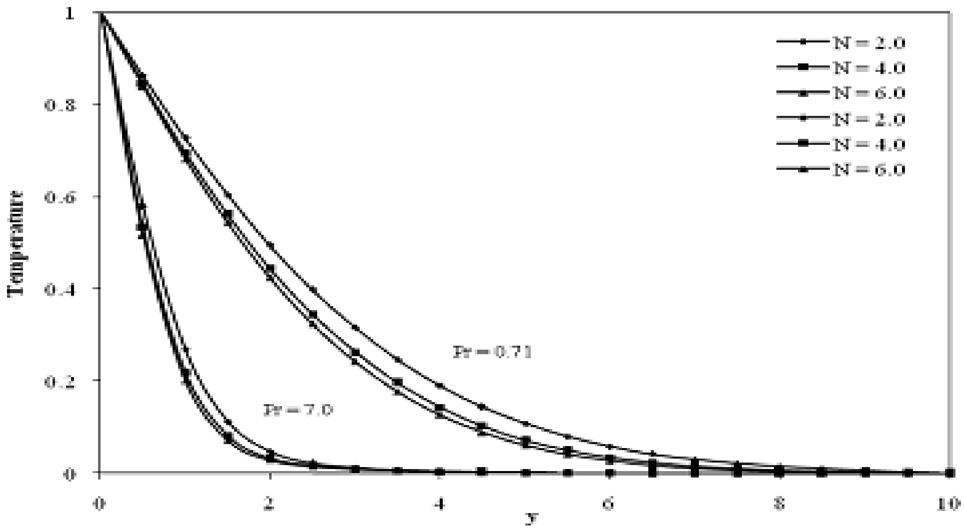

Figure 3. **Temperature profiles showing the effect of radiarion parameter** N **at**$t = 1.0$

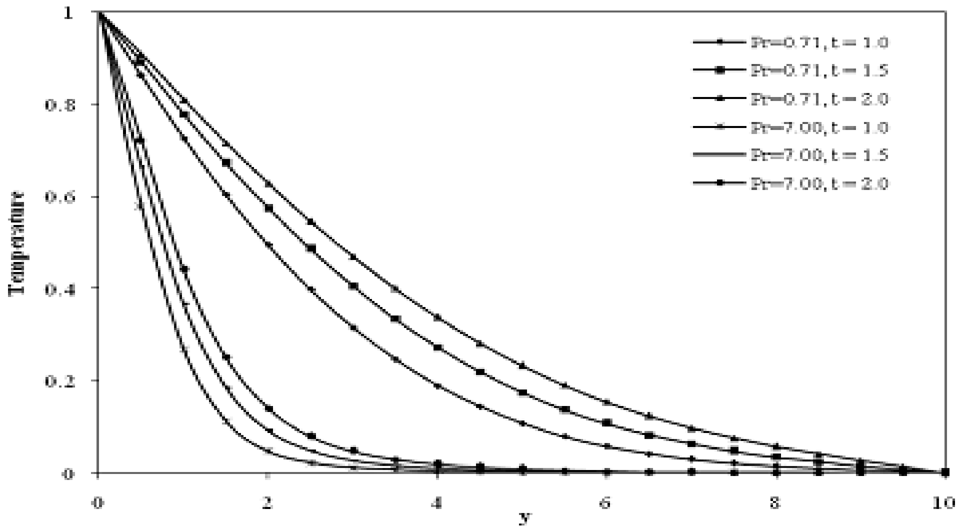

Figure 4. **Temperature profiles showing the effect of time parameter** t

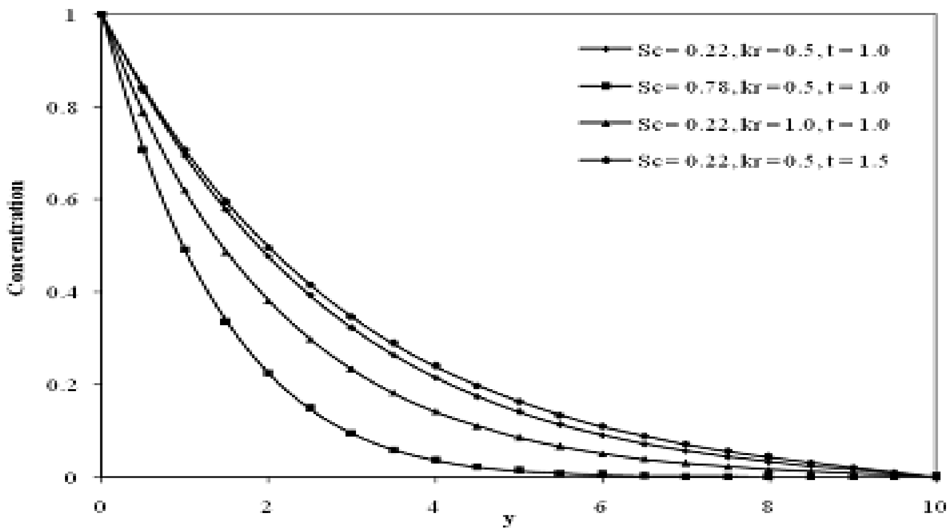

Figure 5. **Concentration profiles**

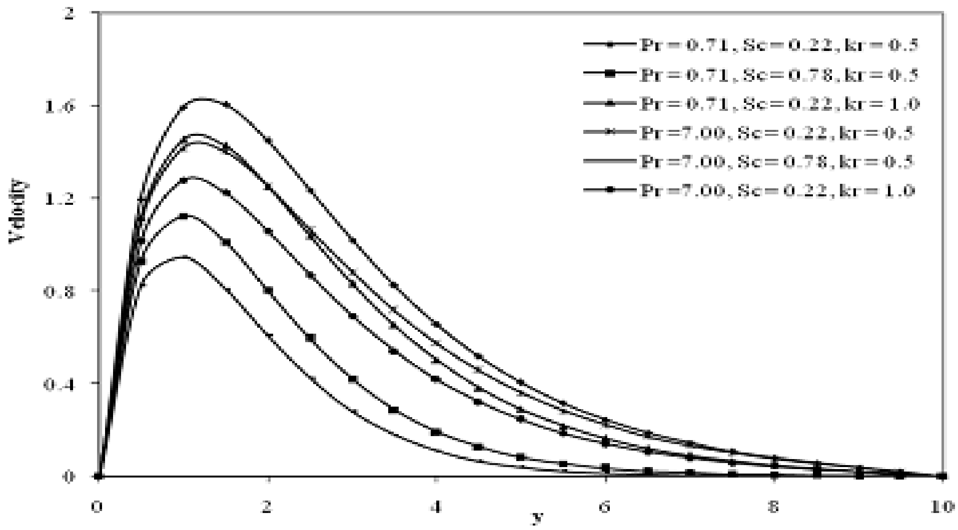

Figure 6. **Velocity profiles showing the effect of S_c and k_r at $t = 1.0$**

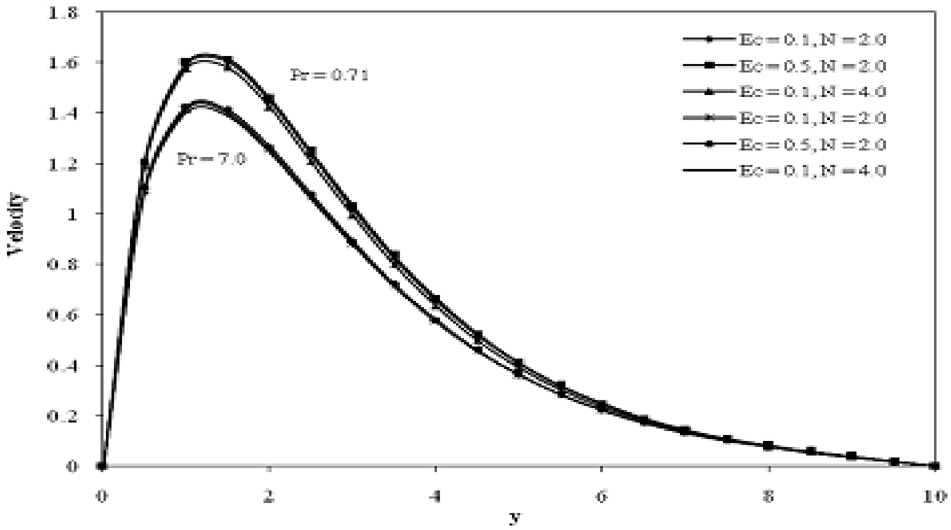

Figure 7. **Velocity profiles showing the effect of E_c and N at $t = 1.0$**

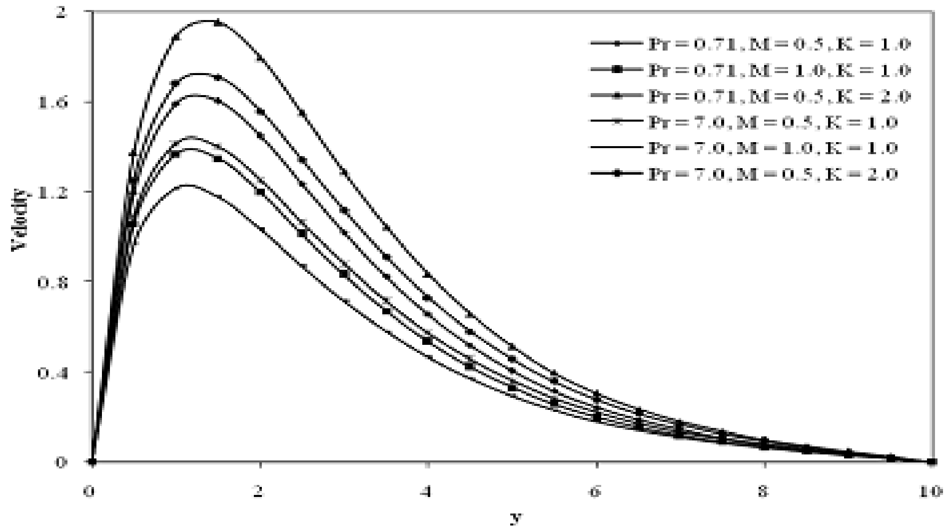

Figure 8. **Velocity profiles showing the effect of M and K at $t = 1.0$**

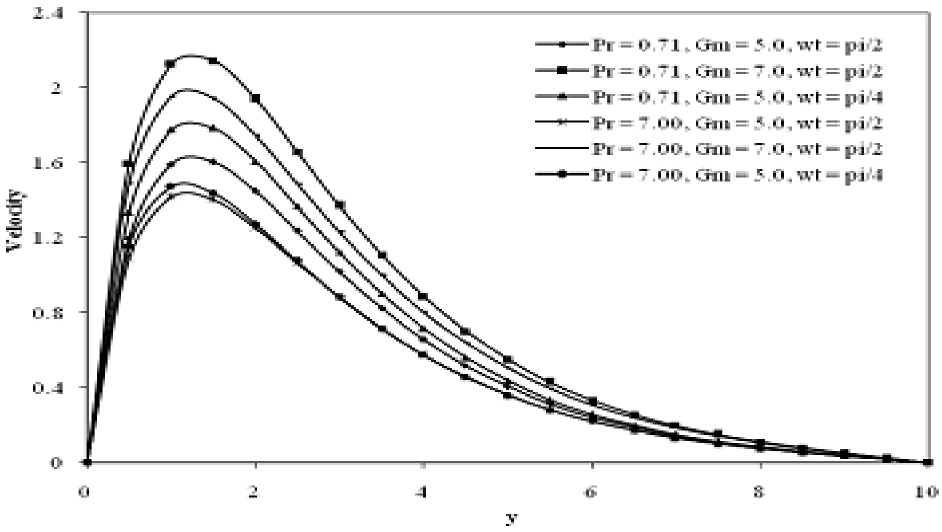

Figure 9. **Velocity profiles showing the effect of G_m and ωt at $t = 1.0$**

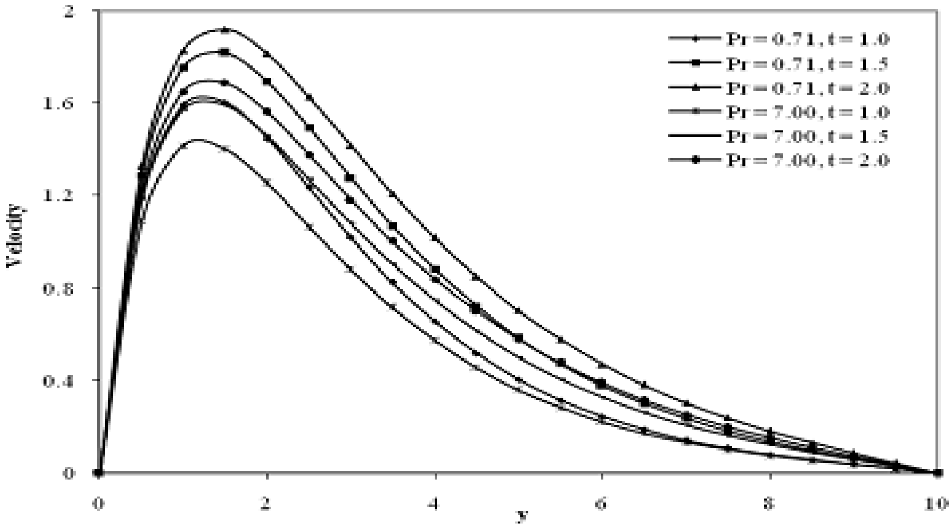

Figure 10. **Velocity profiles showing the effect of time parameter t**

S_c	N	E_c	M	K	k_r	G_m	ωt	t	τ $P_r = 0.71$	τ $P_r = 7.00$
0.22	2	0.1	0.5	1	0.5	5	Pi/2	1	2.39109	2.182508
0.78	2	0.1	0.5	1	0.5	5	Pi/2	1	1.857578	1.647782
0.22	4	0.1	0.5	1	0.5	5	Pi/2	1	2.373566	2.161982
0.22	2	0.5	0.5	1	0.5	5	Pi/2	1	2.40215	2.200878
0.22	2	0.1	1	1	0.5	5	Pi/2	1	2.117538	1.936982
0.22	2	0.1	0.5	2	0.5	5	Pi/2	1	2.748616	2.503318
0.22	2	0.1	0.5	1	1	5	Pi/2	1	2.233724	2.024586
0.22	2	0.1	0.5	1	0.5	7	Pi/2	1	3.191738	2.985466
0.22	2	0.1	0.5	1	0.5	5	Pi/4	1	2.668598	2.31073
0.22	2	0.1	0.5	1	0.5	5	Pi/2	1.5	2.571298	2.367058

Table 1. **Effects of $P_r, S_c, N, E_c, M, K, k_r, \omega t, G_m$ and t on skin-friction(τ) for air ($P_r = 0.71$) and water ($P_r = 7.00$)**

N	E_c	ωt	t	Nu $P_r = 0.71$	Nu $P_r = 7.00$
2	0.1	Pi/2	1	0.271002	0.62015
4	0.1	Pi/2	1	0.30096	0.630432
2	0.5	Pi/2	1	0.264974	0.61295
2	0.1	Pi/4	1	0.46297	1.044722
2	0.1	Pi/2	1.5	0.226632	0.597466

Table 2. **Effects of $N, E_c, \omega t$ and t on Nusselt number (Nu) for air $(P_r = 0.71)$ and water $(P_r = 7.00)$**

The effects of the material parameters $S_c, N, E_c, M, K, k_r, G_m, \omega t$ and t on the skin-friction coefficient (τ) for both air $(P_r = 0.71)$ and water $(P_r = 7.00)$ are presented in the Table-1, respectively. It is clear that an increase in S_c, N, M, k_r and ωt leads to decrease in the skin-friction coefficient whereas an increase in E_c, K, G_m and t leads to increase in the skin-friction coefficient. It also observed that skin-friction for air $P_r = 0.71$ is higher than that of water $P_r = 7.00$.

The effects of the material parameters $N, E_c, \omega t$ and t on the Nusselt number (Nu) are presented in Table-2 for both air $(P_r = 0.71)$ and water $(P_r = 7.00)$, respectively. It is observed that an increase in the radiation parameter N leads to increase in the Nusselt number whereas an increase in Eckert number E_c, phase angle ωt and time parameter t leads to decrease in the Nusselt number. Also, observe that the Nusselt number for water $P_r = 7.00$ is higher than that of air $P_r = 0.71$. The reason is that smaller values of Prandtl number are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Prandtl number. Hence, the rate of heat transfer is reduced.

5. Conclusion

In this study the governing equations have been examined for the radiative magneto-hydrodynamic transient heat and mass transfer flow by free convection past a vertical plate with viscous dissipation in the presence of chemical reaction, when the temperature of the plate oscillates periodically about a constant mean temperature and the plate being embedded in a porous medium. The resulting partial differential equations have been solved numerically by Ritz FEM.

We present results to illustrate the flow characteristics for the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number and show how the flow fields are influenced by the material parameters of the flow problem. We can conclude from these results that the radiation indeed affects the temperature and therefore the velocity hence the skin-friction.

Increase in the Eckert number leads to increase in the temperature and velocity. Also, we observe that Eckert number has a little effect on the velocity. As Schmidt number increased results in a decrease in the concentration and velocity fields. An increase in the modified Grashof number increases the velocity field. It is also found that the effect of time parameter on the velocity is more dominant than other parameters. Further, this investigation may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, underground water in river beds, filtration and water purification processes.

This study of flow past a vertical surface can be utilized as the basis of many scientific and engineering applications; it finds its applications for the design of the blanket of liquid metal around a thermonuclear fusion fission hybrid reactor.

In metallurgy, it can be applied during the solidification process. The results of the problem are also great interest in geophysics, in the study of interaction of geomagnetic field with the fluid in the geothermal region.

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