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Some Fixed Point Theorems in Cone Banach Spaces Using Φ_p Operator

Research Article

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Abstract: In this paper, some contraction principles of Cone Banach Space(CBS) are stated and proved with the help of Φ_p operator and also some fixed point theorems related to the above concepts are studied.

MSC: 47H10, 54H25.

Keywords: Cone Banach Space, Fixed point,Φ_p operator.

1. Introduction

The notion of cone metric space is initiated by Huang and Zhang [4] and also they discussed some properties of the convergence of sequences and proved the fixed point theorems of a contraction mapping for cone metric spaces; Any mapping T of a complete cone metric space X into itself that satisfies, for some $0 \leq k < 1$, the inequality $d(Tx, Ty) \leq kd(x, y)$, for all $x, y \in X$ has a unique fixed point. Some fixed theorems in cone Banach space are proved by Karapinar[3].

In this paper, some new contraction principles of CBSs are proved and investigated some fixed point theorems of CBSs using p-Laplacian operator.

2. Preliminaries

Throughout this paper, E means a Banach algebra, $E := (E, \|.\|)$ stands for real Banach space.

Definition 2.1. *A subset P of E is called a cone if and only if:*

(i). P is closed, nonempty and $P \neq 0$

(ii). $ax + by \in P$ for all $x, y \in P$ and nonnegative real numbers a, b

(iii). $P \cap (P) = \{0\}$.

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Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We will write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while x, y will stand for $y - x \in intP$, where $intP$ denotes the interior of P.

The cone P is called normal if there is a number $K > 0$ such that $0 \leq x \leq y$ implies $\|x\| \leq K\|y\|$ for all $x, y \in E$. The least positive number satisfying the above is called the normal constant.

Example 2.2 ([7]). *Let $K > 1$. be given. Consider the real vector space with $E = \{ax + b : a, b \in R; x \in R[1 - \frac{1}{k}, 1]\}$ with supremum norm and the cone $P = \{ax + b : a \geq 0, b \leq 0\}$ in E. The cone P is regular and so normal.*

Definition 2.3 ([5]). *A Banach algebra is an algebra E that has a norm relative to E is a Banach space and such that for all $x, y \in E$*

(i). $\|xy\| \leq \|x\|\|y\|$

(ii). $\|e\| \leq 1$

Where e is the multiplicative identity in E.

Definition 2.4. *Suppose that E is a real Banach space, then P is a cone in E with $intP \neq \emptyset$, and $\leq$ is partial ordering with respect to P. Let X be a nonempty set, a function $d : X \times X \to E$. is called a cone metric on X if it satisfies the following conditions with*

(i). $d(x, y) \geq 0$, *and* $d(x, y) = 0$ *if and only if* $x = y \ \forall x, y \in X$,

(ii). $d(x, y) = d(y, x)$, $\forall x, y \in X$,

(iii). $d(x, y) \leq d(x, z) + d(z, y)$, $\forall x, y, z \in X$,

Then (X, d) is called a cone metric space (CMS).

Example 2.5. *Let $E = R^2$; $P = \{(x, y) : x, y \geq 0\}$; $X = R$ and $d : X \times X \to E$ such that $d(x, y) = (|x - y|, \alpha|x - y|)$, where $\alpha \geq 0$ is a constant. Then (X, d) is a cone metric space.*

Definition 2.6. *Let X be a vector space over R. Suppose the mapping $\|.\|_C : X \to E$ satisfies*

(i). $\|x\|_C \geq 0$ *for all* $x \in X$,

(ii). $\|x\|_C = 0$ *if and only if* $x = 0$,

(iii). $\|x + y\|_C \leq \|x\|_C + \|y\|_C$ *for all* $x, y \in X$,

(iv). $\|kx\|_C = |k|\|x\|_C$ *for all* $k \in R$ *and for all* $x \in X$,

then $\|.\|_C$ is called a cone norm on X, and the pair $(X, \|.\|_C)$ is called a cone normed space (CNS).

Definition 2.7. *Let $(X, \|.\|_C)$ be a CNS, $x \in X$ and $\{x_n\}_{n\geq 0}$ be a sequence in X. Then $\{x_n\}_{n\geq 0}$ converges to x whenever for every $c \in E$ with $0 \ll E$, there is a natural number $N \in N$ such that $\|x_n - x\|_C \ll c$ for all $n \geq N$. It is denoted by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$*

Definition 2.8. *Let $(X, \|.\|_C)$ be a CNS, $x \in X$ and $\{x_n\}_{n\geq 0}$ be a sequence in X. $\{x_n\}_{n\geq 0}$ is a Cauchy sequence whenever for every $c \in E$ with $0 \ll c$, there is a natural number $N \in N$, such that $\|x_n - x_m\|_C \ll c$ for all $n, m \geq N$*

Definition 2.9. *Let $(X, \|.\|_C)$ be a CNS, $x \in X$ and $\{x_n\}_{n\geq 0}$ be a sequence in X. $(X, \|.\|_C)$ is a complete cone normed space if every Cauchy sequence is convergent. Complete cone normed spaces will be called cone Banach spaces.*

Definition 2.10. *Let E be Banach algebra and $(E, \|.\|_C)$ be a Banach space $\Phi_p : E \to E$ is an increasing and positive mapping.*

$$(ie)\ \Phi_p(x) = \|x\|^{p-2}x, \ \ where\ \frac{1}{p} + \frac{1}{q} = 1.$$

If $E = R$, then $\Phi_p : R \to R$ is a $p-$Laplacian operator,

$$(ie)\ \Phi_p(x) = |x|^{p-2}x, \ \ for\ some\ p > 1.$$

Lemma 2.11. *Show that the operator $\Phi_p : E \to E$ holds the following properties:*

(i). If $x \leq y$, then $\Phi_p(x) \leq \Phi_p(y)$, $\forall x, y \in E$

(ii). Φ_p is a continuous bijection and its inverse mapping is also continuous.(That is, Φ_p is homeomorphism)

(iii). $\Phi_p(xy) = \Phi_p(x)\Phi_p(y)$ $\forall x, y \in E$.

(iv). $\Phi_p(x + y) \leq \Phi_p(x) + \Phi_p(y)$ $\forall x, y \in E$

Definition 2.12. *Let C be a closed and convex subset of a cone Banach space with the norm $\|.\|_C$ and $T : C \to C$ be a mapping. Consider the condition $\|Tx - Ty\|_C \leq \|x - y\|_C$ for all $x, y \in C$, then T is called non expansive.*

3. Main Result

Theorem 3.1. *Let C be a closed and convex subset of a Banach space X with the norm $\|.\|_C$. Let E be a Banach algebra and $\Phi_p : E \to E$ and $T : C \to C$ be mappings and T satisfy the following condition:*

$$\Phi_p(d(x, Ty)) + \Phi_p(d(y, Tx)) \leq k\Phi_p(d(x, y)) \tag{1}$$

for all $x, y \in C$, where $2^{p-1} \leq k < 4^{p-1}$ in E. Then T has at least one fixed point.

Proof. Let $x_0 \in C$ be arbitrary. Define a sequence $\{x_n\}$ in the following way: $x_{n+1} = \frac{x_n + Tx_n}{2}, n = 0, 1, 2, 3, \cdots$. Then $x_n - Tx_n = 2(x_n - x_{n+1})$. Which yields that $d(x_n, Tx_n) = \|x_n - Tx_n\|_C = 2\|x_n - x_{n+1}\|_C = 2d(x_n, x_{n+1})$. Substitute $x = x_{n-1}$ and $y = x_n$ in (1). Then we have

$$\begin{aligned} \Phi_p(d(x_{n-1}, Tx_n)) + \Phi_p(d(x_n, Tx_{n-1})) &\leq k\Phi_p(d(x_n, x_{n-1})) \\ \Phi_p(2d(x_{n-1}, x_{n+1})) + \Phi_p(2d(x_n, x_n)) &\leq k\Phi_p(d(x_n, x_{n-1})) \\ \Phi_p(2d(x_{n-1}, x_{n+1})) &\leq k\Phi_p(d(x_n, x_{n-1})) \end{aligned}$$

From the property of Φ_p operator,

$$\Phi_p(2(d(x_{n-1}, x_n) + d(x_n, x_{n+1}))) \leq k\Phi_p(d(x_n, x_{n-1}))$$

from (2.10) we get,

$$\begin{aligned} d(x_n, x_{n+1}) &\leq \left(\frac{\Phi_q(k)}{2} - 1\right)(d(x_{n-1}, x_n)) \\ \text{similarly}\ \ d(x_{n-1}, x_n) &\leq \left(\frac{\Phi_q(k)}{2} - 1\right)(d(x_{n-2}, x_{n-1})) \\ \Rightarrow\ \ d(x_n, x_{n+1}) &\leq \left(\frac{\Phi_q(k)}{2} - 1\right)^2(d(x_{n-2}, x_{n-1})) \\ &\vdots \end{aligned}$$

$$d(x_n, x_{n+1}) \leq \left(\frac{\Phi_q(k)}{2} - 1\right)^n (d(x_0, x_1)) \tag{2}$$

Let $m > n$, them from above equation (2), we get

$$\begin{aligned} d(x_m, x_n) &\leq d(x_m, x_{m-1}) + \cdots + d(x_{n+1}, x_n) \\ &\leq \left[\left(\frac{\Phi_q(k)}{2} - 1\right)^{m-1} + \left(\frac{\Phi_q(k)}{2} - 1\right)^{m-2} + \cdots + \left(\frac{\Phi_q(k)}{2} - 1\right)^n\right] d(x_1, x_0) \\ &\leq \frac{(\frac{\Phi_q(k)}{2} - 1)^n}{2 - \frac{\Phi_q(k)}{2}} d(x_1, x_0). \end{aligned}$$

Since $2^{p-1} \leq k < 4^{p-1}$, $\{x_n\}$ is a Cauchy sequence in C. Because C is a closed and convex subset of a cone Banach space, thus $\{x_n\}$ sequence converges to some $z \in C$. That is, $x_n \to z,\ z \in C$. Regarding the inequality,

$$d(z, Tx_n) \leq d(z, x_n) + d(x_n, Tx_n)$$

$$d(z, Tx_n) \leq d(z, x_n) + 2d(x_n, x_{n+1})$$

as $n \to \infty$, then $d(z, Tx_n) \leq 0$. Thus $Tx_n \to z$. Finally, we substitute $x = z$ and $y = x_n$ in (1). Then we can get

$$\Phi_p(d(z, Tx_n)) + \Phi_p(d(x_n, Tz)) \leq k\Phi(d(z, x_n))$$

from the property of Φ_p mapping,

$$\Phi_p(d(z, Tx_n) + d(x_n, Tz)) \leq k\Phi(d(z, x_n))$$

when $n \to \infty$, $d(z, Tz) = 0$. Then $Tz = z$. □

Theorem 3.2. *Let C be a closed and convex subset of a Banach space X with the norm $\|.\|_C$. Let E be a Banach algebra and $\Phi_p : E \to E$ and $T : C \to C$ be mappings and T satisfy the following condition:*

$$\Phi_p(d(x, Tx)) + \Phi_p(d(y, Ty)) + \Phi_p(d(x, Ty)) + \Phi_p(d(y, Tx)) \leq k\Phi_p(d(x, y)) \tag{3}$$

for all $x, y \in C$, where $2^{p-1} \leq k < 4^{p-1}$ in E. Then T has at least one fixed point.

Proof. Let $x_0 \in C$ be arbitrary. Define a sequence $\{x_n\}$ in the following way:

$$x_{n+1} = \frac{x_n + Tx_n}{2}, n = 0, 1, 2, 3, \cdots$$

Then

$$x_n - Tx_n = 2(x_n - x_{n+1})$$

Which yields that

$$d(x_n, Tx_n) = \|x_n - Tx_n\|_C = 2\|x_n - x_{n+1}\|_C = 2d(x_n, x_{n+1})$$

Substitute $x = x_{n-1}$ and $y = x_n$ in (3). Then we have

$$\Phi_p(d(x_{n-1}, Tx_{n-1})) + \Phi_p(d(x_n, Tx_n)) + \Phi_p(d(x_{n-1}, Tx_n)) + \Phi_p(d(x_n, Tx_{n-1})) \leq k\Phi_p(d(x_n, x_{n-1}))$$

From the property of Φ_p operator,

$$\Phi_p(2d(x_{n-1},x_n)) + \Phi_p(2d(x_n,x_{n+1})) + \Phi_p(2d(x_{n-1},x_{n+1})) + \Phi_p(2d(x_n,x_n)) \leq k\Phi_p(d(x_n,x_{n-1}))$$
$$\Phi_p(2d(x_{n-1},x_n)) + \Phi_p(2d(x_n,x_{n+1})) + \Phi_p(2(d(x_{n-1},x_n) + d(x_n,x_{n+1}))) \leq k\Phi_p(d(x_n,x_{n-1}))$$
$$\Phi_p(4(d(x_{n-1},x_n) + d(x_n,x_{n+1}))) \leq k\Phi_p(d(x_n,x_{n-1}))$$

from (2.10) we get,

$$d(x_n,x_{n+1}) \leq \left(\frac{\Phi_q(k)}{4} - 1\right)(d(x_{n-1},x_n))$$

similarly

$$d(x_{n-1},x_n) \leq \left(\frac{\Phi_q(k)}{4} - 1\right)(d(x_{n-2},x_{n-1}))$$
$$\Rightarrow \quad d(x_n,x_{n+1}) \leq \left(\frac{\Phi_q(k)}{4} - 1\right)^2(d(x_{n-2},x_{n-1}))$$
$$\vdots$$
$$d(x_n,x_{n+1}) \leq \left(\frac{\Phi_q(k)}{4} - 1\right)^n(d(x_0,x_1)) \tag{4}$$

Let $m > n$, them from above equation (4) we get,

$$\begin{aligned} d(x_m,x_n) &\leq d(x_m,x_{m-1}) + \cdots + d(x_{n+1},x_n) \\ &\leq \left[\left(\frac{\Phi_q(k)}{4} - 1\right)^{m-1} + \left(\frac{\Phi_q(k)}{4} - 1\right)^{m-2} + \cdots + \left(\frac{\Phi_q(k)}{4} - 1\right)^{n}\right] d(x_1,x_0) \\ &\leq \frac{(\frac{\Phi_q(k)}{4} - 1)^n}{4 - \frac{\Phi_q(k)}{4}} d(x_1,x_0). \end{aligned}$$

Since $2^{p-1} \leq k < 4^{p-1}$, $\{x_n\}$ is a Cauchy sequence in C. Because C is a closed and convex subset of a cone Banach space, thus $\{x_n\}$ sequence converges to some $z \in C$. That is, $x_n \to z,\ z \in C$. Regarding the inequality,

$$d(z,Tx_n) \leq d(z,x_n) + d(x_n,Tx_n)$$
$$d(z,Tx_n) \leq d(z,x_n) + 2d(x_n,x_{n+1})$$

as $n \to \infty$, then $d(z,Tx_n) \leq 0$. Thus $Tx_n \to z$. Finally, we substitute $x = z$ and $y = x_n$ in (3). Then we can get

$$\Phi_p(d(z,Tz)) + \Phi_p(d(x_n,Tx_n)) + \Phi_p(d(z,Tx_n)) + \Phi_p(d(x_n,Tz)) \leq k\Phi(d(z,x_n))$$

from the property of Φ_p mapping,

$$\Phi_p(d(z,Tz) + d(x_n,Tx_n) + d(z,Tx_n) + d(x_n,Tz)) \leq k\Phi_p(d(z,x_n))$$

when $n \to \infty$, $d(z,Tz) = 0$. Then $Tz = z$. □

Theorem 3.3. *Let C be a closed and convex subset of a Banach space X with the norm $\|.\|_C$. Let E be a Banach algebra and $\Phi_p : E \to E$ and $T : C \to C$ be mappings and T satisfy the following condition:*

$$\alpha\Phi_p(d(Tx,Ty)) + \beta\Phi_p(d(x,Tx)) + \gamma\Phi_p(d(y,Ty)) + \delta\Phi_p(d(x,Ty)) + \omega\Phi_p(d(y,Tx)) \leq k\Phi_p(d(x,y)) \tag{5}$$

for all $x, y \in C$, where $0 \leq \Phi_q(k) < \Phi_q(\alpha) + 2(\Phi_q(\beta) + \Phi_q(\gamma) + \Phi_q(\delta) + \Phi_q(\omega))$. Then T has at least one fixed point.

Proof. Let $x_0 \in C$ be arbitrary. Define a sequence $\{x_n\}$ in the following way:

$$x_{n+1} = \frac{x_n + Tx_n}{2}, n = 0, 1, 2, 3, \cdots$$

Then

$$x_n - Tx_n = 2(x_n - x_{n+1})$$

Which yields that

$$d(x_n, Tx_n) = \|x_n - Tx_n\|_C = 2\|x_n - x_{n+1}\|_C = 2d(x_n, x_{n+1})$$

Thus the triangle inequality implies

$$d(x_n, Tx_n) - d(x_n, Tx_{n-1}) \leq d(Tx_{n-1}, Tx_n).$$

$$2d(x_n, x_{n+1}) - d(x_{n-1}, x_n) \leq d(Tx_{n-1}, Tx_n).$$

By substituting $x = x_{n-1}$ and $y = x_n$ in (5). Then we have

$$\alpha\Phi_p(d(Tx_{n-1}, Tx_n)) + \beta\Phi_p(d(x_{n-1}, Tx_{n-1})) + \gamma\Phi_p(d(x_n, Tx_n)) + \delta\Phi_p(d(x_{n-1}, Tx_n)) + \omega\Phi_p(d(x_n, Tx_{n-1})) \leq k\Phi_p(d(x_{n-1}, x_n))$$

$$\alpha\Phi_p(2d(x_n, x_{n+1}) - d(x_{n-1}, x_n)) + \beta\Phi_p(d(x_{n-1}, Tx_{n-1})) + \gamma\Phi_p(d(x_n, Tx_n)) + \delta\Phi_p(d(x_{n-1}, Tx_n)) + \omega\Phi_p(d(x_n, Tx_{n-1})) \leq k\Phi_p(d(x_{n-1}, x_n))$$

From the property of Φ_p operator,

$$\alpha\Phi_p(2d(x_n, x_{n+1}) - d(x_{n-1}, x_n)) + \beta\Phi_p(2d(x_{n-1}, x_n)) + \gamma\Phi_p(2d(x_n, Tx_{n+1})) + \delta\Phi_p(d(x_{n-1}, x_{n+1})) + \omega\Phi_p(d(x_n, x_n)) \leq k\Phi_p(d(x_{n-1}, x_n))$$

$$2\alpha\Phi_p d(x_n, x_{n+1}) - \alpha\Phi_p d(x_{n-1}, x_n) + 2\beta\Phi_p(d(x_{n-1}, x_n)) + 2\gamma\Phi_p(d(x_n, Tx_{n+1})) + 2\delta\Phi_p(d(x_n, x_{n+1}) + d(x_{n-1}, x_n)) + \omega\Phi_p(d(x_n, x_n)) \leq k\Phi_p(d(x_{n-1}, x_n))$$

$$2\alpha\Phi_p d(x_n, x_{n+1}) - \alpha\Phi_p d(x_{n-1}, x_n) + 2\beta\Phi_p(d(x_{n-1}, x_n)) + 2\gamma\Phi_p(d(x_n, Tx_{n+1})) + 2\delta\Phi_p(d(x_n, x_{n+1}) + d(x_{n-1}, x_n)) + \leq k\Phi_p(d(x_{n-1}, x_n))$$

from (2.10) we get,

$$d(x_n, x_{n+1}) \leq \left(\frac{\Phi_q(r) + \Phi_q(\alpha) - 2(\Phi_q(\beta) + \Phi_q(\delta))}{2\Phi_q(\alpha) + 2\Phi_q(\gamma) + 2\Phi_q(\delta)}\right) d(x_{n-1}, x_n)$$

for all $n \geq 1$. Repeating this relation, we get $d(x_n, x_{n+1}) \leq h^n d(x_0, x_1)$, where $h = \left(\frac{\Phi_q(r)+\Phi_q(\alpha)-2(\Phi_q(\beta)+\Phi_q(\delta))}{2\Phi_q(\alpha)+2\Phi_q(\gamma)+2\Phi_q(\delta)}\right) < 1$. Let $m > n$ then from above equation, we have

$$\begin{aligned} d(x_m, x_n) &\leq d(x_m, x_{m-1}) + \cdots + d(x_{n+1}, x_n) \\ &\leq [h^{m-1} + \cdots + h^n] d(x_1, x_0) \\ &\leq \frac{h^n}{1-h} d(x_1, x_0) \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence in C and thus it converges to some $z \in C$. Since $0 \leq \Phi_q(k) < \Phi_q(\alpha) + 2(\Phi_q(\beta) + \Phi_q(\gamma) + \Phi_q(\delta) + \Phi_q(\omega))$, $\{x_n\}$ is a Cauchy sequence in C. Because C is a closed and convex subset of a cone Banach space, thus $\{x_n\}$ sequence converges to some $z \in C$. That is, $x_n \to z,\ z \in C$. Regarding the inequality,

$$d(z, Tx_n) \leq d(z, x_n) + d(x_n, Tx_n)$$

$$d(z, Tx_n) \leq d(z, x_n) + 2d(x_n, x_{n+1})$$

as $n \to \infty$, then $d(z, Tx_n) \leq 0$. Thus $Tx_n \to z$. Finally, we substitute $x = z$ and $y = x_n$ in (5) . Then we can get

$$\alpha\Phi_p(d(z, Tx_n)) + \beta\Phi_p(d(z, Tz)) + \gamma\Phi_p(d(x_n, Tx_n)) + \delta\Phi_p(d(z, Tx_n)) + \omega\Phi_p(d(x_n, Tz)) \leq k\Phi_p(d(z, x_n))$$

from the property of Φ_p mapping,

$$\Phi_p(\alpha(d(Tz, Tx_n)) + \beta(d(z, Tz)) + \gamma(d(x_n, Tx_n)) + \delta(d(z, Tx_n)) + \omega(d(x_n, Tz))) \leq k\Phi_p(d(z, x_n))$$

$$\Phi_p(\alpha(d(Tz, z)) + \beta(d(z, Tz)) + \gamma(d(z, z)) + \delta(d(z, z)) + \omega(d(z, Tz))) \leq k\Phi_p(d(z, z))$$

$$\Phi_p(\alpha(d(Tz, z)) + \beta(d(z, Tz)) + \omega(d(z, Tz))) \leq k\Phi_p(d(z, z))$$

$$\Phi_p((\alpha + \beta + \omega)d(z, Tz)) \leq k\Phi_p(d(z, z))$$

when $n \to \infty,\ d(z, Tz) = 0$. Then $Tz = z$. □

Corollary 3.4. *Let C be a closed and convex subset of a Banach space X with the norm $\|.\|_C$. Let E be a Banach algebra and $\Phi_p : E \to E$ and $T : C \to C$ be mappings and T satisfy the following condition:*

$$\alpha\Phi_p(d(Tx, Ty)) + \beta\Phi_p(d(x, Tx)) + \gamma\Phi_p(d(y, Ty)) + \delta\Phi_p(d(x, Ty)) \leq k\Phi_p(d(x, y)) \tag{6}$$

for all $x, y \in C$, where $0 \leq \Phi_q(k) < \Phi_q(\alpha) + 2(\Phi_q(\beta) + \Phi_q(\gamma) + \Phi_q(\delta))$. Then T has at least one fixed point.

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