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Current Updates in Fluid Dynamics with Special Reference to Europe

Research Article

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Abstract: Fluid dynamics is study of motion of liquid and gases. One of the fundamental equation in fluid dynamics is the Navier-Stokes equation. It is equation of motion of fluid or gases in IR^3. There is considerable work done on Navier-Stokes equation and Euler equation in decade. Existence and smoothness of solution for the Navier-Stokes equation in IR^2 have been obtained. Leraythe French mathematician has shown that the Navier stokes equations is three dimensions have a weak solution. Ladyzhenskya studied the behavior of weak solutions of the Euler and Navier-Stokes equation. Solutions of the Navier-Stokes equations often include turbulence, which remains one of the greatest unsolved problems in physics, despite its immense importance in science and engineering. In present paper researcher is comparing all the existing solution of Navier-Stokes equations and also reviewing finding obtained by European mathematicians on Solution of Navier-Stokes equations.

Keywords: Navier-Stokes equations, Fluid dynamics, Leray.

1. Introduction

Fluid is either gas or liquid. Dynamics deals with motion of object. Fluid dynamics is largely developed in 17^{th}, 18^{th} and 19^{th} century by great physicist and mathematicians like Newton, Lagrange, Cauchy, Navier, Stokes. All the equations in fluid dynamics can be studied using derivative and it's allied properties.

The equations of motion of gas molecules or liquid are known as Navier-Stokes equations studied in branch of physics called as fluid dynamics. These are highly complicated 2^{nd} ordered non-linear partial differential equations. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the stress in the fluid is the sum of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term-hence describing viscous flow. They may be used to study the weather, ocean currents, various water flow in a pipe and air flow around a wing. The Navier-Stokes equations help with the design of aircraft and cars, the analysis of pollution, the study of blood flow, the design of power stations. Solution of the Navier-Stokes and Euler equations with initial conditions (Cauchy Problems) for 2D or 3D cases were obtained covering series form by analytical iterative method using Fourier and Laplace transform. The Navier-Stokes equations in IR^N where $N = 3$ is given by

$$\frac{\partial uk}{\partial t} + \sum_{n=1}^{N} un \frac{\partial uk}{\partial xn} = v\Delta uk - \frac{\partial p}{\partial xk} + f_t(x,t),\ t \geq 0,\ 1 \leq k \leq N$$

$$Divu = \sum_{n=1}^{N} \frac{\partial uk}{\partial xn} = 0(x \in IR^N\ t \geq 0).$$

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This is non linear partial differential equation with initial conditions for velocity and applied force.

Since January 1992, lattice gases have attracted much attention since models were proposed to simulate fluid flows: the FHP for 2D and the FCHC for 3D have been shown to satisfy the incompressible Navier-Stokes equation. Y.H.Qian, D.D'humieres and P.Lallemand proposed the lattice BGK models, as an alternative to lattice gases or the lattice Boltzmann equation, to obtain an efficient numerical scheme for the simulation of fluid dynamics. With a properly chosen equilibrium distribution, the Navier-Stokes equation is obtained from the kinetic BGK equation at the second-order of approximation. Compared to lattice gases, the present model is noise-free, has Galileian invariance and a velocity independent pressure. It occupies a relaxation parameter that influences the stability of the new scheme. Numerical simulations are shown to confirm the speed of sound and the shear viscosity.

S.Bochner studied application of Fourier and Laplace transformation for system of linear partial differential equations with constant coefficients and initial velocity, applied force are smooth decreasing functions towards minus infinity then solution of such system is also smooth. I.M.Gelf and has introduced perfect space of functions and vector function of velocity.

The Reynolds number can be defined for several different situations where a fluid is in relative motion to a surface. These definitions generally include the fluid properties of density and viscosity, plus a velocity and a characteristic length or characteristic dimension. The concept was introduced by George Gabriel Stokes in 1851.

R.Mikulevicius has used Markov process in perturbation equation to solve the deterministic Navier-stokes equations. The mild solution of Navier-stokes equation is obtained very recently by Kato. All these solutions are in L^p is linear space with p-norm. The following are major out comes in recent years:

2. Main Result

1. Exact solutions : Some exact solutions to the Navier-Stokes equations exist. Examples of degenerate cases-with the non-linear terms in the Navier-Stokes equations equal to zero-are Poiseuille flow, Couette flow and the oscillatory Stokes boundary layer. But also more interesting examples, solutions to the full non-linear equations, exist; for example the Taylor-Green vortex.

2. A three-dimensional steady-state vortex solution: One set of solutions is given by:

$$\rho(x,y,z) = \frac{3B}{r^2+x^2+y^2+z^2}$$
$$p(x,y,z) = \frac{-A^2B}{(r^2+x^2+y^2+z^2)^3}$$
$$v(x,y,z) = \frac{A}{(r^2+x^2+y^2+z^2)^2}\begin{pmatrix} 2(-ry+xz) \\ 2(rx+yz) \\ (r^2-x^2-y^2+z^2) \end{pmatrix}$$
$$g = 0$$
$$\mu = 0$$

   for arbitrary constants A and B. This is a solution in a non-viscous gas (compressible fluid) whose density, velocities and pressure goes to zero far from the origin.

3. If the initial velocity $V(x,t)$ is sufficiently small then the statement is true: there are smooth and globally defined solutions to the Navier-Stokes equations.

4. Given an initial velocity $V_0(x)$ there exists a finite time T, depending on $V_0(x)$ such that the Navier-Stokes equations on $\mathbb{R}^3 \times (0, T)$ have smooth solutions $v(x, t)$ and $p(x, t)$. It is not known if the solutions exist beyond that blowup time T.

5. The mathematician Jean Leray in 1934 proved the existence of weak solutions to the Navier-Stokes equations, satisfying the equations in mean value, not point wise.

6. Terence Tao in February 2014 announced a finite time blowup result for an averaged version of the 3-dimensional Navier-Stokes equation. He found the result formalizes a supercriticality barrier for the global regularity problem for the true Navier-Stokes equations, and claims that the method of proof in fact hints at a possible route to establishing blowup for the true equations.

7. Terence Tao from Adelaide in 18 March, 2007 announced three possible strategies if one wants to solve the full Millennium Prize problem for the 3-dimensional Navier-Stokes equation (NSE) Why global regularity for Navier-Stokes is hard. Strategy 1 Solve the Navier-Stokes equation exactly and explicitly (or at least transform this equation exactly and explicitly to a simpler equation) is used in works: Navier-Stokes First Exact Transformation, Navier-Stokes Second Exact Transformation. The author of these works AlexandrKozachok has offered (in February 2008-Internet, in November 2013 and February 2014-International Journal) two exact NSE transformations to the simpler equations.

3. Conclusion

As Navier-Stokes equations are studied by European mathematician with various solutions methods like finite element method, Numerical approximations method, finite differences to get Weak solution, Exact solutions, Mild solution. But still the problem of finding smooth and regular solution to Navier- Stokes equations is open and unsolved.

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