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Packing Chromatic Number of Benes Network

Research Article

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Abstract: The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k for which there exists a mapping $f : V(G) \longrightarrow \{1, 2, ..., k\}$ such that any two vertices of color i are at distance at least $i+1$. In this paper, the packing chromatic number of benes network is obtained.

MSC: 05C15.

Keywords: Packing coloring, benes network, interconnection network.

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1. Introduction

Let G be a connected graph and k be an integer, $k \geq 1$. A packing k-coloring of a graph G is a mapping $f : V(G) \longrightarrow \{1, 2, ..., k\}$ such that any two vertices of color i are at distance at least $i + 1$. The packing chromatic number $\chi_\rho(G)$ of G is the smallest integer k for which G has packing k-coloring. The concept of packing coloring comes from the area of frequency assignment in wireless networks and was introduced by Goddard et al. [1] under the name broadcast coloring.

A connection pattern of the components in a system is called an interconnection network. The interconnection network is responsible for fast and reliable communication among the processing nodes in any parallel computer. Processing and distribution of data using interconnection networks have become indissoluble elements of the development of our society [4]. In this paper, we compute the packing chromatic number of an interconnection network, called benes network.

2. Benes Network

Efficient representations for butterfly and benes networks have been obtained by Manuel et al. [2]. Benes network is constructed from butterfly network. Thus, we define the following definition for butterfly network.

Definition 2.1 ([4]). *The n-dimensional butterfly network, denoted by $BF(n)$; has vertex set*

$$V = \{(x; i) : x \in V(Q_n), 0 \leq i \leq n\}$$

Two vertices $(x; i)$ and $(y; j)$ are linked by an edge in $BF(n)$ if and only if $j = i + 1$ and either (i) $x = y$, or (ii) x differs from y in precisely the jth bit.

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Definition 2.2 ([2]). *The n-dimensional Benes network consists of back-to-back butterfly, denoted by $BB(n)$. The $BB(n)$ has $2n+1$ levels, each with 2^n vertices. The first and last $n+1$ levels in the $BB(n)$ form two $BF(n)$'s respectively, while the middle level in $BB(n)$ is shared by these butterfly networks. A 2-dimensional benes network is shown in Figure 2.*

Proposition 2.3 ([1]). *Let H be a subgraph of G. Then $\chi_\rho(H) \leq \chi_\rho(G)$.*

Proposition 2.4 ([1]). *Let C_n be a cycle on n vertices. Then $\chi_\rho(C_n) = 3$, when n is a multiple of 4.*

Theorem 2.5 ([3]). *Let $BF(3)$ be an 3-dimensional butterfly network. Then $\chi_\rho(BF(3)) = 9$.*

Since a cycle on 4 vertices is a subgraph of $BB(1)$, by Propositions 2.3 and 2.4, we have the following Remark.

Remark 2.6. $\chi_\rho(BB(1)) = 3$.

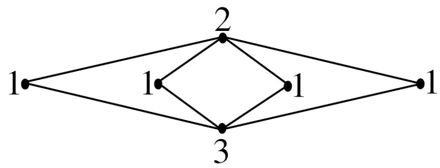

Figure 1. **Benes Network** $BB(1)$

Theorem 2.7. $\chi_\rho(BB(2)) = 7$.

Proof. Suppose $\chi_\rho(BB(2)) = 6$. Since $diam(BB(2)) = 4$, at most three vertices are colored 4, 5 and 6. Label $BB(2)$ as shown in Figure 2 (a).

Case 1: $c(a_1) = 3$

Since $d(a_1, a_i) = 4$ for $2 \leq i \leq 4$, we have $c(a_i) = 3$ for $2 \leq i \leq 4$. Since $d(a_i, \{c_j, d_j\}) = 1$ for $1 \leq i \leq 4$, $i = j$ and $d(a_i, e_j)$ $= 2$ for $1 \leq i, j \leq 4$, no other vertices of $BB(2)$ receives color 3.

Case 1.1: $c(b_1) = 2$

Since $d(b_1, b_i) = 4$ for $2 \leq i \leq 4$, we have $c(b_i) = 2$ for $2 \leq i \leq 4$. Since $d(b_i, \{c_j, d_j\}) = 1$ for $1 \leq i \leq 4$, $i = j$, and $d(b_i, e_j)$ $= 2$ for $1 \leq i, j \leq 4$, no other vertices of $BB(2)$ receive color 2.

Case 1.2: $c(c_1) = 1$

Since $d(c_1, c_i) = 2$ for $2 \leq i \leq 4$, we have $c(c_i) = 1$ for $2 \leq i \leq 4$. Since $d(c_i, e_j) \geq 2$ for $1 \leq i, j \leq 4$, $c(d_i) = 1$ for $\leq i \leq 4$. Since $d(c_i, \{e_1, e_2\}) = 1$ and $d(d_i, \{e_3, e_4\}) = 1$ for $1 \leq i \leq 4$, no other vertices of $BB(2)$ receive color 1.

From Cases 1, 1.1 and 1.2, at most four vertices with 3 and four vertices with 2 and eight vertices with 1 can be colored. Therefore, remaining one vertex should receive distinct color greater than 6.

Case 2: $c(d_1) = 3$

Since $d(d_1, c_2) = 4$, we have $c(c_2) = 3$. Since $d(d_1, \{d_3, d_4\}) = 2$, $d(c_2, \{c_3, c_4\}) = 2$, $d(c_2, \{a_i, b_i\}) \leq 3$ and $d(d_1, \{a_i, b_i\}) \leq 3$ for $1 \leq i \leq 4$, no other vertices of $BB(2)$ receive color 3.

Case 2.1: $c(d_2) = 2$

Since $d(d_2, c_1) = 4$, we have $c(c_1) = 2$. Since $d(d_2, \{a_i, b_i\}) = 3$ and $d(c_1, \{a_i, b_i\}) = 3$ for $3 \leq i \leq 4$, $c(a_3) = c(a_4) = 2$. Since $d(d_2, \{d_3, d_4\}) = 1$, $d(c_1, \{c_3, c_4\}) = 1$, $d(c_1, \{a_1, b_1\}) = 2$, $d(d_2, \{a_2, b_2\}) = 2$, $d(d_2, \{e_3, e_4\}) = 1$ and $d(c_1, \{e_1, e_2\}) = 1$, no other vertices of $BB(2)$ receive color 2.

Case 2.2: $c(a_1) = 1$

Since $d(a_1, \{a_2, b_1, b_2\}) = 2$, we have $c(a_2) = c(b_1) = c(b_2) = 1$. Since $d(e_i, \{a_j, b_j\}) = 2$ for $1 \leq i \leq 4$ and $1 \leq j \leq 2$, $c(e_i)$

$= 1$ for $1 \leq i \leq 4$. Then no other vertices of $BB(2)$ receive color 1.

From Cases 2, 2.1 and 2.2, at most two vertices with 3 and four vertices with 2 and eight vertices with 1 can be colored. Therefore, remaining three vertices should receive distinct color greater than 6.

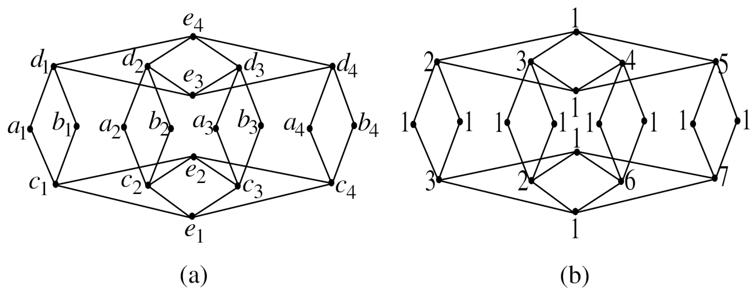

Figure 2. **Benes Network** $BB(2)$

Case 3: $c(e_1) = 3$

Since $d(e_1, e_3) = 4$, we have $c(e_3) = 3$. Since $d(e_1, \{a_i, b_i\}) = 2$, $d(e_1, c_i) = 1$ and $d(e_3, d_i) = 1$ for $1 \leq i \leq 4$, no a_i, b_i, c_i and d_i receive color 3.

Case 3.1: $c(e_2) = 2$

Since $d(e_2, e_4) = 4$, we have $c(e_4) = 2$. Since $d(e_2, \{a_i, b_i\}) = 2$, $d(e_2, c_i) = 1$ and $d(e_4, d_i) = 1$ for $1 \leq i \leq 4$, no a_i, b_i, c_i and d_i receive color 2 and no other vertices receive color 2.

Case 3.2: $c(b_1) = 1$

Since $d(b_1, \{a_i, b_j\}) \geq 2$ for $1 \leq i \leq 4$ and $2 \leq j \leq 4$, we have $c(a_i) = 1$ and $c(b_j) = 1$. Since $d(a_i, \{c_j, d_j\}) = 1$ and $d(b_i, \{c_j, d_j\}) = 1$ for $1 \leq i, j \leq 4$ and $i = j$, no other vertices of $BB(2)$ receive color 1.

From Cases 3, 3.1 and 3.2, at most two vertices with 3 and two vertices with 2 and eight vertices with 1 can be colored. Therefore, remaining five vertices should receive distinct color greater than 6. Thus $\chi_\rho(BB(2)) \geq 7$. The coloring in Figure 2 (b) proves that $\chi_\rho(BB(2)) = 7$. □

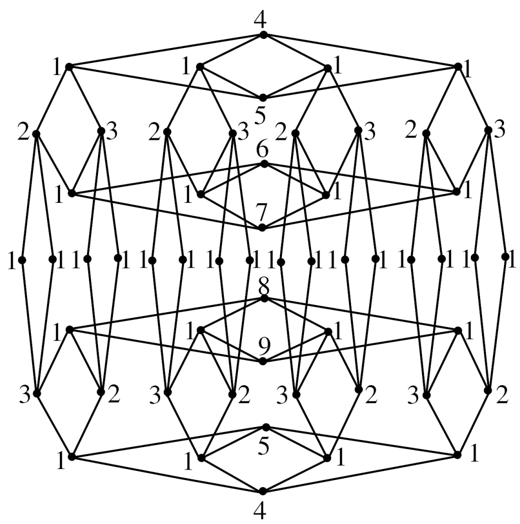

Figure 3. **Benes Network** $BB(3)$

Since $BF(3)$ is a subgraph of $BB(3)$, by Theorem 2.5, $\chi_\rho(BB(3)) \geq 9$. The packing coloring of $BB(3)$ is given in Figure 3.

Thus we have the following Theorem.

Theorem 2.8. $\chi_\rho(BB(3)) = 9$.

Open Problem

The packing chromatic number of $BB(r)$ is obtained for $r = 1$, 2 and 3. Finding $\chi_\rho(BB(r))$ for $r > 3$ is challenging and the problem remains open for $r > 3$. The packing chromatic number of butterfly network is obtained and accepted for the publication [3].

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