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Some Properties of Cyclotic Graph

Research Article

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Abstract: In the present work we have introduced a new graph called Cyclotic graph $G(n,k)$. We have given many examples of Cyclotic graph $G(n,k)$. We have proved some results about connectedness, planarity, regularity and number of edges in Cyclotic graph. We have also proved that $G(n,k)$ is not Eulerian graph and found some results which shows when $G(n,k)$ is bipartite and Hamiltonian.

Keywords: Cyclotic graph, connectedness in Cyclotic graph, planarity, regularity and number of edges in Cyclotic graph.

1. Introduction

We consider only simple, undirected and non-trivial graph $G = (V,E)$ with the vertex set V and edge set E. For various graph theoretic notations and terminology we follow Clark and Holton [2] whereas for number theory we follow D.M. Burton [1]. We will give brief summary of definitions and other information which are useful for the present investigations.

1.1. Definitions and Examples

Definition 1.1. *The cyclotic graph $G(n,k)$ with a positive integer $n > 1$ and integer k such that $0 \leq k \leq n-1$ are defined to be a graph with $V(G(n,k)) = \{v_i : 1 \leq i \leq n\}$ and $E(G(n,k)) = E_O \cup E_I$ where $E_O = \left\{v_{2i}v_{2i+1} : 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right\}$ and $E_I = \left\{v_1v_{1+(k+1)}, v_{1+(k+1)}v_{1+2(k+1)}, \ldots, v_{n-2k-1}v_{n-k}, v_{n-k}v_1\right\}$, here subscripts are taken modulo n. The elements of E_O are called outer edges and the elements of E_I are called inner edges. Cycle formed by all the inner edges is called inner cycle. i_e = The number of inner edges in graph $G(n,k)$.*

Example 1.2. *Following are some examples of $G(n,k)$ for some n and k.*

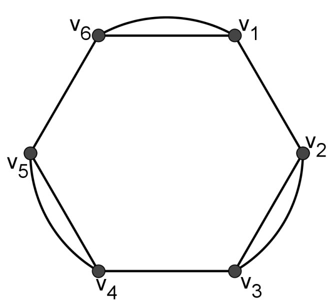

Figure 1. G(6,0)

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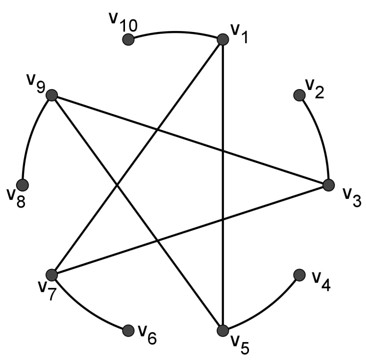


Figure 2. **G(10,3)**

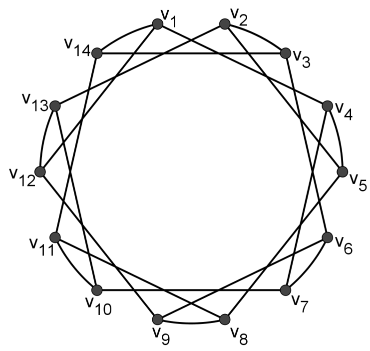


Figure 3. **G(14,2)**

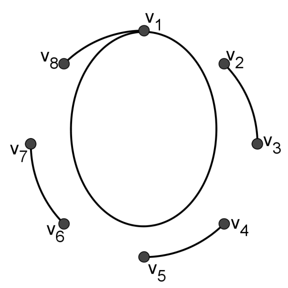


Figure 4. **G(8,7)**

Remark 1.3.

(1). No two outer edges have a vertex in common.

(2). $G(n,k)$ *has* $\frac{n}{2}$ *outer edges if* n *is even and* $\frac{n-1}{2}$ *outer edges if* n *is odd.*

(3). $G(n,k_1) = G(n,k_2)$ *iff* $k_1 = k_2$ *or* $k_1 + k_2 = n-2$*. Hence we study graph* $G(n,k)$ *for* $0 \leq k < \left\lfloor \frac{n}{2} \right\rfloor$ *and* $k = n-1$*.*

(4). Every $G(n,k)$ *has exactly one inner cycle. We call this cycle is starting and ending at* v_1*.*

(5). $G(n,k)$ *has inner cycle of length* i_e*.*

(6). Number of inner edges are always less then number of vertices.i.e., $i_e < n$*.*

(7). $G(n,n-1)$ *has exactly one loop which is formed by inner edge, it is the only inner edge of* $G(n,n-1)$*. The remaining edges of* $G(n,n-1)$ *are outer edges. So,* $k = n-1$ *is not much interesting case.*

(8). $G(n,0)$ has a cycle C_n. The remaining edges of $G(n,0)$ are outer edges. Also each of the outer edges is parallel to one edge of the inner cycle. And no two outer edges are parallel. So, $G(n,0)$ is always planar.

2. Main Section

Theorem 2.1. *For $G(n,k)$, if n is odd then $deg(v_1) = 2$.*

Proof. v_1 is a vertex of the inner cycle, so $deg(v_1) \geq 2$. If $deg(v_1) > 2$ then v_1 is incident at one of the outer edges of the form $v_{2i}v_{2i+1}$ for some $i \in \left\{1, 2, 3, ..., \left\lfloor \frac{n}{2} \right\rfloor\right\}$. $\therefore v_{2i+1} = v_1$. Thus, $2i + 1 \equiv 1 \ (mod\ n)$. So $2i \equiv 0 \ (mod\ n)$. As n is odd, we have $i \equiv 0 \ (mod\ n)$. So, $n|i$, which is not possible as $1 \leq i \leq \frac{n}{2}$. $\therefore deg(v_1) = 2$. □

Example 2.2. *In the following graph $deg(v_1) = 2$.*

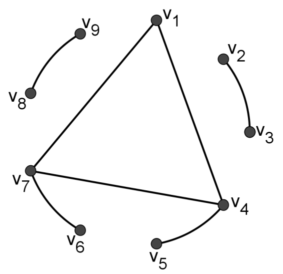


Figure 5. **G(9,2)**

Theorem 2.3. *For $G(n,k)$, if n is even then $deg(v_1) = 3$.*

Proof. v_1 is a vertex of the inner cycle so $deg(v_1) \geq 2$. v_1 is incident to an outer edge of the form $v_{2i}v_{2i+1}$ for some $i \in \left\{1, 2, 3, ..., \left\lfloor \frac{n}{2} \right\rfloor\right\}$. $\therefore 2i + 1 \equiv 1 \ (mod\ n)$. Thus, $2i \equiv 0 \ (mod\ n)$. So, $n|2i$. Hence,

$$n \leq 2i. \tag{1}$$

As $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$ we have

$$2 \leq 2i \leq 2\left\lfloor \frac{n}{2} \right\rfloor \leq n. \tag{2}$$

From (1) and (2), $2i = n$. Thus, $i = \frac{n}{2}$. Hence, v_1 is incident to the outer edge $v_n v_1$. $\therefore deg(v_1) = 3$. □

Example 2.4. *In the following graph $deg(v_1) = 2$.*

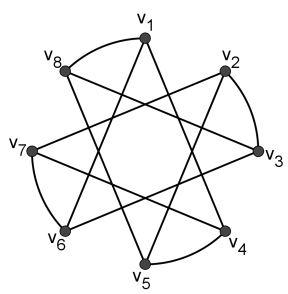


Figure 6. **G(8,2)**

Remark 2.5. *From Theorem 2.2 and Theorem 2.3 and Definition 1.1 we can say that all the vertices of $G(n,k)$ are adjacent to outer edges except v_1, if n is odd. Hence degree of all the vertices of $G(n,k)$ is atleast 1 for all n and k.*

Theorem 2.6. *$G(n,k)$ is 3-regular iff $n = i_e$, where n is even.*

Proof. Suppose $G(n,k)$ is 3-regular. Suppose if possible $n \neq i_e$ that is $n > i_e$. The inner cycle does not pass through atleast one vertex of $G(n,k)$. Let one of the such vertices be v_j. Then clearly $deg(v_j) = 1$, which contradicts our hypotesis that $G(n,k)$ is 3-regular. Hence, $n = i_e$.

Conversely, suppose that $n = i_e$ then every vertex of $G(n,k)$ is a vertex of the inner cycle. Every outer edge is of the form $v_{2i}v_{2i+1}$. So, degree of each vertex is 3. $\therefore G(n,k)$ is 3-regular. □

Example 2.7. *In the following figure $G(16,6)$ is 3-regular.*

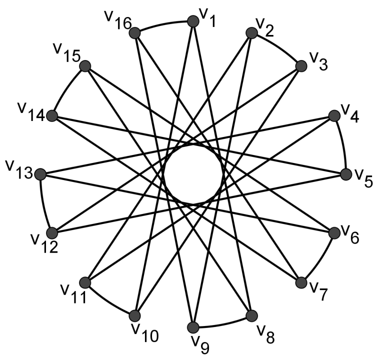


Figure 7. **G(16,6)**

Theorem 2.8. *If n is odd then $G(n,k)$ is not regular.*

Proof. From Theorem 2.2 $deg(v_1) = 2$. Let v_i be one of the vertices of the inner cycle other than v_1. Clearly v_i is incident to one of the outer edges. So, $deg(v_i) = 3$. $\therefore G(n,k)$ is not regular. □

Remark 2.9. *$deg(v_i) \leq 3$ for each vertex v_i of $G(n,k)$.*

Theorem 2.10. *$G(n,k)$ is not Eulerian graph.*

Proof.

case 1: n is even.

Then from Theorem 2.3, $deg(v_1) = 3$. Hence $G(n,k)$ is not Eulerian if n is even.

case 2: n is odd.

Then from Theorem 2.2, $deg(v_1) = 2$. Let v_i be one of the vertices of the inner cycle other than v_1. Clearly, v_i is incident to one of the outer edges. So, $deg(v_i) = 3$. Hence $G(n,k)$ is not Eulerian if n is odd. □

Theorem 2.11. *$G(n,k)$ has inner cycle of length i_e iff i_e is the smallest positive integer such that $n | i_e(k+1)$.*

Proof. Suppose $G(n,k)$ has inner cycle of length i_e. All the end vertices of the inner edges are of the form $v_{1+i(k+1)}$ where $i \in \left\{0, 1, 2, 3, ..., \left\lfloor \frac{n}{2} \right\rfloor\right\}$. $1 + i_e(k+1)$ is the smallest positive integer greater than 1 such that $v_{1+i_e(k+1)} = v_1$. *i.e.*, $1 + i_e(k+1) \equiv 1 \ (mod\ n) \Rightarrow i_e(k+1) \equiv 0 \ (mod\ n) \Rightarrow n | i_e(k+1)$.

Conversely, suppose that i_e is the smallest positive integer such that $n|i_e(k+1)$. i.e.,$i_e(k+1) \equiv 0\ (mod\ n) \Rightarrow 1+i_e(k+1) \equiv 1\ (mod\ n)$. $1+i_e(k+1)$ is the smallest positive integer greater than 1 such that $v_{1+i_e(k+1)} = v_1$. Thus the inner cycle which is $i.e$; $v_1v_{1+(k+1)}, v_{1+(k+1)}v_{1+2(k+1)}, \ldots, v_{1+(I_E-1)(k+1)}v_1$, is of length i_e. □

Theorem 2.12. *If $(k+1)|n$ then $G(n,k)$ has inner cycle of length $\frac{n}{k+1}$.*

Proof.

$$(k+1)|n \Rightarrow n = m(k+1). \tag{3}$$

Suppose $G(n,k)$ has cycle $v_1v_{1+(k+1)}, v_{1+(k+1)}v_{1+2(k+1)}, \ldots, v_{1+(x-1)(k+1)}v_{1+x(k+1)}$. Clearly, x is the smallest positive integer such that $1+x(k+1) \equiv 1\ (mod\ n) \Leftrightarrow x(k+1) \equiv 0\ (mod\ n) \Leftrightarrow n|x(k+1) \Leftrightarrow m(k+1)|x(k+1)$, (from (3)) $\Leftrightarrow m|x$. Since, x is the smallest positive integer satisfying above condition, we have $x = m$. Therefore, length of given cycle is m that is $\frac{n}{k+1}$. □

Example 2.13. *$G(8,3)$ has inner cycle of length 2 as shown in following figure.*

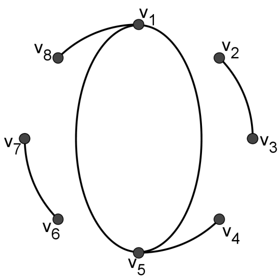


Figure 8. **G(8,3)**

Theorem 2.14. *If $(k+1)|n$ then $G(n,k)$ has exactly $\frac{n}{2}\left(\frac{k-1}{k+1}\right)+1$ components where n is even.*

Proof. $G(n,k)$ has $\frac{n}{2}$ outer edges. $G(n,k)$ has inner cycle of length $i_e = \frac{n}{k+1}$ say C. Each of the vertices of C is incident to exactly one outer edge. Also no two outer edges have a vertex in common. All such outer edges together with C forms a connected component of $G(n,k)$. From the remaining outer edges no two have a vertex in common. Hence, $G(n,k)$ has $\frac{n}{2} - \frac{n}{k+1} + 1 = \frac{n}{2}\left(\frac{k-1}{k+1}\right) + 1$ components. □

Example 2.15. *$G(12,2)$ has exactly 3 components as shown in the following figure.*

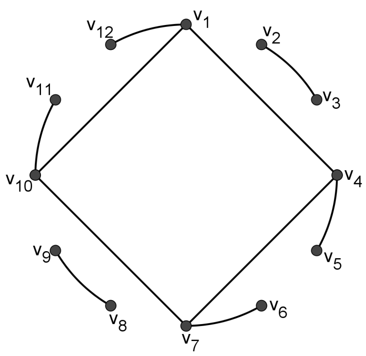


Figure 9. **G(12,2)**

Theorem 2.16. *$G(n,1)$ is always connected and unicyclic if n is even.*

Proof. By definition of $G(n,k)$, $V(G(n,1)) = \{v_1, v_2, \ldots, v_n\}$ and $E(G(n,1)) = E_O \cup E_I$ where $E_O = \{v_2v_3, v_4v_5, v_6v_7, \ldots, v_{n-2}v_{n-1}, v_nv_1\}$ and $E_I = \{v_1v_3, v_3v_5, v_5v_7, \ldots, v_{n-3}v_{n-1}, v_{n-1}v_1\}$. $G(n,1)$ has exactly one cycle which is the inner cycle i.e; $v_1v_3, v_3v_5, v_5v_7, \ldots, v_{n-3}v_{n-1}, v_{n-1}v_1$, of length $\frac{n}{2}$. Also each v_{2i} is adjacent to v_{2i+1}, where $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$. Hence $G(n,1)$ is connected and it has exactly one cycle. □

Example 2.17. *Following figure shows that $G(8,1)$ is connected and unicyclic.*

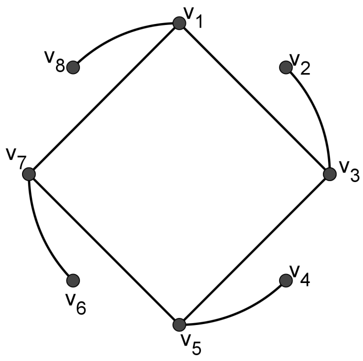


Figure 10. **G(8,1)**

Remark 2.18. *Theorem 2.16 is special case of Theorem 2.14.*

Theorem 2.19. *If n is odd and $(k+1)|n$ then $G(n,k)$ has exactly $\frac{n}{2}\left(\frac{k-1}{k+1}\right) + \frac{3}{2}$ components.*

Proof. $G(n,k)$ has $\frac{n-1}{2}$ outer edges. $G(n,k)$ has inner cycle of length $i_e = \frac{n}{k+1}$ say C. Each of the vertices of this cycle other than v_1 is incident to exactly one outer edge and no two outer edges have a vertex in common. All such outer edges together with C forms a connected component of $G(n,k)$. From the remaining outer edges no two have a vertex in common. Hence, $G(n,k)$ has $\frac{n-1}{2} - \left(\frac{n}{k+1} - 1\right) + 1 = \frac{n}{2}\left(\frac{k-1}{k+1}\right) + \frac{3}{2}$ components. □

Example 2.20. *Following figure shows that $G(15,4)$ has 6 components.*

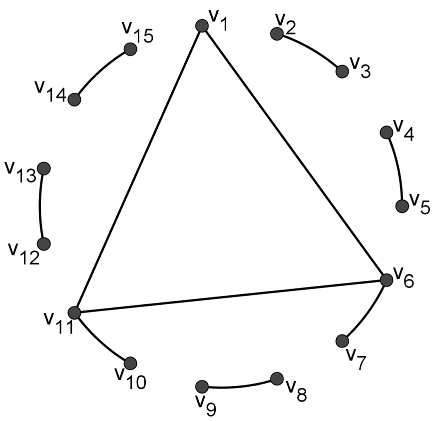


Figure 11. **G(15,4)**

Theorem 2.21. *If n is even and $(k+1)|n$ then $G(n,k)$ has exactly $\frac{n}{2}\left(\frac{k+3}{k+1}\right)$ edges.*

Proof. $G(n,k)$ has $\frac{n}{2}$ outer edges and inner cycle of length $\frac{n}{k+1}$. Hence, $G(n,k)$ has $\frac{n}{2} + \frac{n}{k+1} = \frac{n}{2}\left(\frac{k+3}{k+1}\right)$ edges. □

Example 2.22. $G(20,3)$ *has 15 edges as shown in the following figure.*

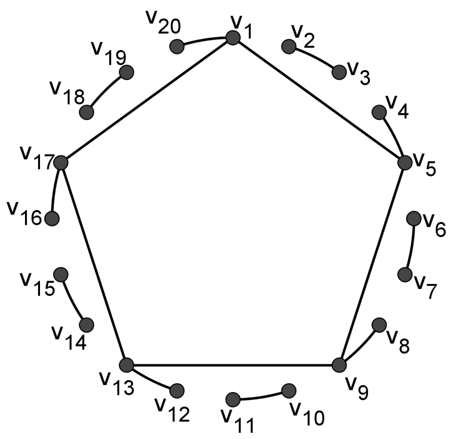


Figure 12. **G(20,3)**

Theorem 2.23. *If n is odd and $(k+1)|n$ then $G(n,k)$ has exactly* $\frac{n}{2}\left(\frac{k+3}{k+1}\right)-\frac{1}{2}$ *edges.*

Proof. $G(n,k)$ has $\frac{n-1}{2}$ outer edges and inner cycle of length $\frac{n}{k+1}$. Hence, $G(n,k)$ has $\frac{n-1}{2}+\frac{n}{k+1}=\frac{n}{2}\left(\frac{k+3}{k+1}\right)-\frac{1}{2}$ edges. □

Theorem 2.24. *If n is odd then $G(n,1)$ has* $\frac{3n-1}{2}$ *edges.*

Proof.

$$G(n,k)\ \text{ has }\ \frac{n-1}{2}\ \text{ outer edges.} \tag{4}$$

Suppose $G(n,1)$ has inner cycle $v_1v_{1+(1+1)}, v_{1+(1+1)}v_{1+2(1+1)}, \ldots, v_{1+(x-1)(1+1)}v_{1+x(1+1)}$. Clearly, x is the smallest positive integer such that $1+x(1+1)\equiv 1\ (mod\ n) \Leftrightarrow x(1+1)\equiv 0\ (mod\ n) \Leftrightarrow n|x(1+1) \Leftrightarrow n|2x$. As $gcd(n,2)=1$, we have $n|x$. Now, $x\leq n \Rightarrow x=n$.

$$\therefore\ G(n,1)\ \text{ has the inner cycle of length n.} \tag{5}$$

$\therefore$ From (4) and (5), $G(n,1)$ has $\frac{n-1}{2}+n=\frac{3n-1}{2}$ edges. □

Theorem 2.25. *$G(4n,2n-2)$ has $6n$ edges.*

Proof. $G(4n,2n-2)$ has $\frac{4n}{2}=2n$ outer edges. Suppose $G(4n,2n-2)$ has the inner cycle $v_1v_{1+(2n-1)}, v_{1+(2n-1)}v_{1+2(2n-1)}, \ldots, v_{1+(x-1)(2n-1)}v_{1+x(2n-1)}$. Clearly, x is the smallest positive integer such that $1+x(2n-1)\equiv 1\ (mod\ 4n) \Leftrightarrow x(2n-1)\equiv 0\ (mod\ n) \Leftrightarrow 4n|x(2n-1)$. $gcd(4n,2n-1)=gcd(n,2n-1)=gcd(n,-1)=gcd(n,1)=1$. Therefore $4n|x$. Since x is the smallest positive integer satisfying $4n|x$, we have $x=4n$. Thus, the length of the inner cycle is $4n$. $\therefore$ $G(4n,2n-2)$ has $2n+4n=6n$ edges. □

Theorem 2.26. *If n is odd prime then $G(n,1)$ has* $\frac{3n-1}{2}$ *edges.*

Proof. $G(n,k)$ has $\frac{n-1}{2}$ outer edges. Suppose $G(n,k)$ has the inner cycle $v_1v_{1+(k+1)}, v_{1+(k+1)}v_{1+2(k+1)}, \ldots, v_{1+(x-1)(k+1)}v_{1+x(k+1)}$. Clearly, x is the smallest positive integer such that $1+x(k+1)\equiv 1\ (mod\ n) \Leftrightarrow x(k+1)\equiv 0\ (mod\ n) \Leftrightarrow n|x(k+1)$. Since n is prime and $(k+1)<n$, we have $gcd(k+1,n)=1$. Thus, $n|x$. Since x is the smallest positive integer satisfying $n|x$, we have $x=n$. Hence, the length of the inner cycle is n. $\therefore$ $G(n,k)$ has $\frac{n-1}{2}+n=\frac{3n-1}{2}$ edges. □

Example 2.27. *$G(11,1)$ has 16 edges as shown in the following figure.*

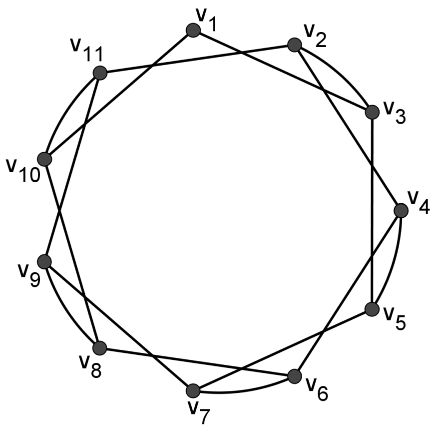


Figure 13. G(11,1)

Theorem 2.28. *$G(n,k)$ is Hamiltonian iff $i_e = n$.*

Proof. Suppose $i_e = n$. Hence the inner cycle passes through every vertex of $G(n,k)$. Thus the inner cycle is Hamiltonian cycle. i.e; $G(n,k)$ is Hamiltonian.

Conversely, suppose that $G(n,k)$ is Hamiltonian. Thus, there exist a cycle C of length n passes through every vertex of $G(n,k)$. If all the edges of cycle C are inner edges then cycle C is the inner cycle. Hence, $n = i_e$ and the result is proved.

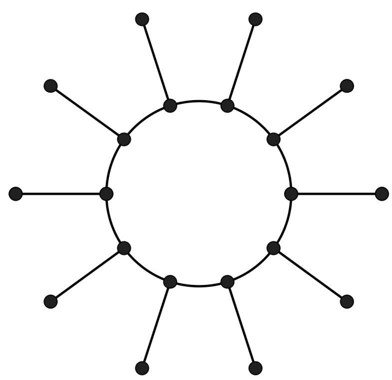
Suppose there exist an edge $e = v_iv_j$ of C which is not an inner edge. Thus either v_i or v_j is not incident to any of the inner edges. Hence either $deg(v_i) = 1$ or $deg(v_j) = 1$. It forces us that $G(n,k)$ is not Hamiltonian, which contradicts our hypothesis. Hence all the edges of cycle C are inner edges, i.e., C is inner cycle. So, $i_e = n$. □

Theorem 2.29. *If n is even and $i_e = \frac{n}{2}$, then $G(n,k)$ is planar.*

Proof. $G(n,k)$ has the inner cycle of length $\frac{n}{2}$ say C. Thus, for each vertex v_i of C, we have $deg(v_i) \geq 2$ and v_i is incident to exactly one outer edge.

$$deg(v_i) = \begin{cases} 3 & \text{,if } v_i \text{ is vertex of } C; \\ 1 & \text{,if } v_i \text{ is not a vertex of } C. \end{cases}$$

Hence we can draw $G(n,k)$ as



i.e; $G(n,k)$ is a crown graph. $\therefore$ $G(n,k)$ is planar. □

Example 2.30. $G(18,7)$ *and it's Plane drawing are shown in the following figures.*

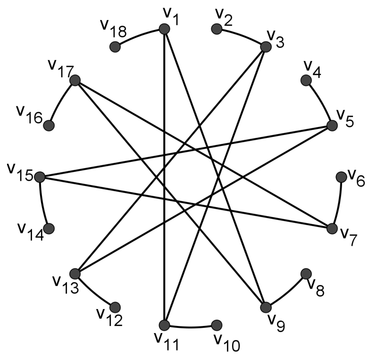


Figure 14. **G(18,7)**

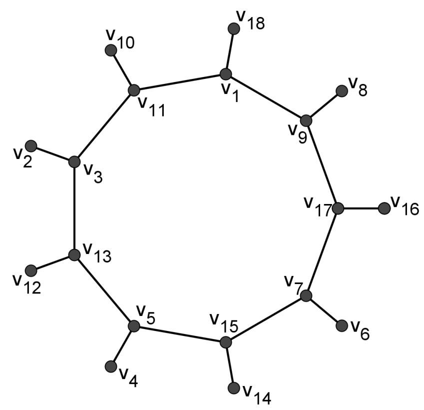


Figure 15. **Plane drawing of G(18,7)**

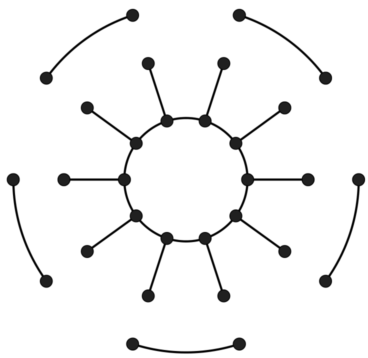
Theorem 2.31. *If* $i_e < \frac{n}{2}$, *then* $G(n,k)$ *is planar.*

Proof. **Case 1:** n is even.

$G(n,k)$ has inner cycle of length i_e say C. Thus, for each vertex v_i of C, we have $deg(v_i) \geq 2$ and v_i is incident to exactly one outer edge.

$$deg(v_i) = \begin{cases} 3 & \text{,if } v_i \text{ is vertex of } C; \\ 1 & \text{,if } v_i \text{ is not a vertex of } C. \end{cases}$$

That means each vertex of C is adjacent to exactly one vertex which is not a vertex of C. This forms a crown graph with $2i_e$ vertices. $i_e < \frac{n}{2} \Rightarrow 2i_e < n$. Thus, every vertex of $G(n,k)$ which is not a vertex of the crown graph has degree 1. Thus, $G(n,k)$ is a vertex disjoint union of a crown graph and several K_2. Hence we can draw $G(n,k)$ as



$\therefore G(n,k)$ is planar.

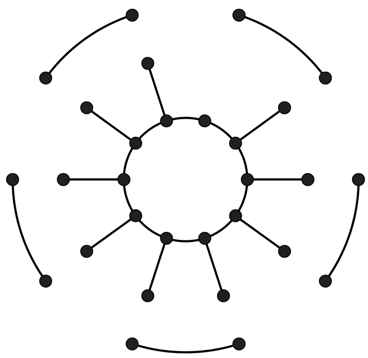
Case 2: n is odd.

$G(n,k)$ has inner cycle of length i_e say C'. Thus, for each vertex v_i of C', we have $deg(v_i) \geq 2$ and v_i is incident to exactly one outer edge.

$$deg(v_i) = \begin{cases} 3 & \text{,if } v_i \text{ is vertex of } C' \text{ and } i \neq 1; \\ 1 & \text{,if } v_i \text{ is not a vertex of } C'. \end{cases}$$

That means each vertex of C' except v_1 is adjacent to exactly one vertex which is not a vertex of C'. This forms a graph G_1 which is a subgraph of a crown graph with $2i_e - 1$ vertices. $i_e < \frac{n}{2} \Rightarrow 2i_e < n \Rightarrow 2i_e - 1 < n$. Thus every vertex of $G(n,k)$

which is not a vertex of the graph G_1 has degree 1. Thus $G(n,k)$ is a vertex disjoint union of a graph G_1 and several K_2. Hence we can draw $G(n,k)$ as



$\therefore G(n,k)$ is planar. □

Theorem 2.32. *If $(k+1)|n$ then $G(n,k)$ is bipartite iff $\dfrac{n}{k+1}$ is even.*

Proof. Let $G(n,k)$ be bipartite graph. Hence it does not have any odd cycle. Also $(k+1)|n$ we can say that $G(n,k)$ has the inner cycle of length $\dfrac{n}{k+1}$. Therefore, $\dfrac{n}{k+1}$ is even.
Conversely, suppose $\dfrac{n}{k+1}$ is even. Now suppose if possible $G(n,k)$ is not bipartite. Thus, $G(n,k)$ has a cycle of odd length say C. Since $(k+1)|n$, $G(n,k)$ has the inner cycle C' of length $\dfrac{n}{k+1}$. Therefore, for each vertex v_i of C', we have $deg(v_i) \geq 2$ and v_i is incident to exactly one outer edge.

$$deg(v_i) = \begin{cases} 3 & \text{,if } v_i \text{ is vertex of } C'; \\ 1 & \text{,if } v_i \text{ is not a vertex of } C'. \end{cases} \tag{6}$$

$C \neq C'$ implies that there exist atleast one outer edge $e' = v_{2j}v_{2j+1}$ of C which is not in C'. Thus, $deg(v_{2j}) \geq 2$ and $deg(v_{2j+1}) \geq 2$. So from (6), e' must be an edge of C', which is not possible because C' contains only inner edges. Hence $G(n,k)$ is bipartite. □

3. Conclusion and Open Problems

We have introduced a new graph namely Cyclotic graph. We have derived several results using the elementary concepts of graph theory for the Cyclotic graph. However, large number of problems for the cyclotic graph remains open. Some of them are following:

(1). Number of inner edges always divides number of vertices in $G(n,k)$.

(2). Either number of inner edges are less than or equal to number of vertices of $G(n,k)$ or exactly equal to number of vertices of $G(n,k)$.

(3). If n is even then $G(n,k)$ is connected iff $i_e \leq \dfrac{n}{2}$.

(4). If n is odd then $G(n,k)$ is connected iff $i_e \leq \dfrac{n-1}{2}$.

(5). If n is odd and $i_e < \dfrac{n-1}{2}$ then $G(n,k)$ has $\dfrac{n-1}{2} - (i_e - 1)$ components.

(6). If n is even and $i_e < \dfrac{n}{2}$ then $G(n,k)$ has $\left(\dfrac{n}{2} - i_e\right)$ components.

(7). $G(n,k)$ is not planar if n is prime.

(8). If n is even and k is odd then $G(n,k)$ is planar.

(9). If n is even and k is even then $G(n,k)$ is not planar where $k \notin \{0, n-2\}$.

(10). If n is odd, composite and $k \equiv 2 \ (mod\ 3)$ then $G(n,k)$ is planar.

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