International Journal of Mathematics And its Applications
Volume 4, Issue 2–C (2016), 29–39.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Intuitionistic Fuzzy g'''-continuous Mappings

Research Article

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Abstract: In this paper, we introduce the concepts of intuitionistic fuzzy g'''-continuous mappings and intuitionistic fuzzy g'''-irresolute mappings. Further, we study some of their properties.

MSC: 54A40, 03F55.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy g'''-continuous mapping, Intuitionistic fuzzy continuous mapping, Intuitionistic fuzzy generalized continuous mapping.

1. Introduction

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker [3] introduced the basic definitions and properties of intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets.

In this paper we introduce Intuitionistic fuzzy g'''-continuous mappings and Intuitionistic fuzzy g'''-irresolute mappings. Also the interconnections between the intuitionistic fuzzy continuous mappings and the intuitionistic fuzzy irresolute mappings are investigated. Some examples are given to illustrate the results.

2. Preliminaries

Definition 2.1 ([1]). *Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and the function $\nu_A : X \to [0, 1]$ is called the non-membership function and $\nu_A(x)$ denotes the degree to which $x \notin A$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote IFS(X), the set of all intuitionistic fuzzy sets in X.

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Throughout the paper, X denotes a non empty set.

Definition 2.2 ([1]). *Let A and B be any two IFSs of the form* $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(1). $A \subseteq B$ *if and only if* $\mu_A(x) \le \mu_B(x)$ *and* $\nu_A(x) \ge \nu_B(x)$ *for all* $x \in X$,

(2). $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,

(3). $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(4). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(5). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ *are called the empty set and the whole set of X respectively.*

Definition 2.4 ([1]). *Let A and B be any two IFSs of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(1). $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(2). $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(3). $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,

(4). $(A \cup B)^c = A^c \cap B^c$ *and* $(A \cap B)^c = A^c \cup B^c$,

(5). $((A)^c)^c = A$,

(6). $(1_\sim)^c = 0_\sim$ *and* $(0_\sim)^c = 1_\sim$.

Definition 2.5 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms :*

(1). $0_\sim, 1_\sim \in \tau$,

(2). $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(3). $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS in short) in X.*
The complement A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.6 ([3]). *Let* (X, τ) *be an IFTS and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.7 ([3]). *For any IFSs A and B in* (X, τ), *we have*

(1). $int(A) \subseteq A$,

(2). $A \subseteq cl(A)$,

(3). A *is an IFCS in* $X \Leftrightarrow cl(A) = A$,

(4). A *is an IFOS in* $X \Leftrightarrow int(A) = A$,

(5). $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ *and* $cl(A) \subseteq cl(B)$,

(6). $int(int(A)) = int(A)$,

(7). $cl(cl(A)) = cl(A)$,

(8). $cl(A \cup B) = cl(A) \cup cl(B)$,

(9). $int(A \cap B) = int(A) \cap int(B)$.

Proposition 2.8 ([3]). *For any IFS A in* (X, τ), *we have*

(1). $int(0_\sim) = 0_\sim$ *and* $cl(0_\sim) = 0_\sim$,

(2). $int(1_\sim) = 1_\sim$ *and* $cl(1_\sim) = 1_\sim$,

(3). $(int(A))^c = cl(A^c)$,

(4). $(cl(A))^c = int(A^c)$.

Proposition 2.9 ([3]). *If A is an IFCS in* (X, τ) *then* $cl(A) = A$ *and if A is an IFOS in* (X, τ) *then* $int(A) = A$. *The arbitrary union of IFCSs is an IFCS in* (X, τ).

Definition 2.10 ([3]). *Let X and Y be two nonempty sets and* $f : X \to Y$ *be a mapping. If* $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle \mid y \in Y \}$ *is an IFS in Y, then the preimage of B under f denoted by* $f^{-1}(B)$, *is an IFS in X defined by* $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) \rangle \mid x \in X \}$ *where* $f^{-1}(\mu_B)(x) = \mu_B(f(x))$.

Definition 2.11 ([3]). *Let X and Y be two nonempty sets and* $f : X \to Y$ *be a mapping. If* $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ *is an IFS in X, then the image of A under f denoted by f(A), is an IFS in Y defined by* $f(A) = \{ \langle y, f(\mu_A)(y), f(\nu_A)(y) \rangle \mid y \in Y \}$, *where*

$$f(\mu_A)(y) = \begin{cases} sup\ \mu_A(x) & if\ f^{-1}(y) \neq \emptyset \quad and \quad x \in f^{-1}(y) \\ 0 & otherwise \end{cases} \quad and$$

$$f(\nu_A)(y) = \begin{cases} inf\ \nu_A(x) & if\ f^{-1}(y) \neq \emptyset \quad and \quad x \in f^{-1}(y) \\ 1 & otherwise \end{cases} \quad for\ each\ y \in Y.$$

Result 2.12 ([3]). *Let* A, A_1, A_2 *be IFSs in X and* B, B_1, B_2 *be IFSs in Y. Let* $f : X \to Y$ *be a mapping. Then*

(1). $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(2). $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(3). $A \subseteq f^{-1}(f(A))$,

(4). $f(f^{-1}(B)) \subseteq B$,

(5). If f is injective mapping, then $f^{-1}(f(A)) = A$,

(6). If f is surjective mapping, then $f(f^{-1}(B)) = B$,

(7). $f^{-1}(0_\sim) = 0_\sim$,

(8). $f^{-1}(1_\sim) = 1_\sim$,

(9). $f(0_\sim) = 0_\sim$,

(10). If f is surjective mapping, then $f(1_\sim) = 1_\sim$,

(11). $f^{-1}(B^c) = (f^{-1}(B))^c$,

(12). If f is bijective mapping, then $(f(A))^c = f(A^c)$.

Definition 2.13. *An IFS A in an IFTS (X, τ) is said to be an*

(1). intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$, [5]

(2). intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$, [4]

(3). intuitionistic fuzzy semi pre closed set (IFSPCS in short) if $int(cl(int(A))) \subseteq A$. [17]

The complement of an IFαCS (resp. an IFSCS, an IFSPCS) is called an IFαOS (resp. an IFSOS, an IFSPOS).

Definition 2.14 ([14]). *Let A be an IFS in an IFTS (X, τ). Then the α-interior of A ($\alpha int(A)$ in short) and the α-closure of A ($\alpha cl(A)$ in short) are defined as follows:*

$$\alpha int(A) = \cup \{G \mid G \text{ is an IF}\alpha\text{OS in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap \{K \mid K \text{ is an IF}\alpha\text{CS in } (X, \tau) \text{ and } A \subseteq K\}.$$

$sint(A)$, $scl(A)$, $spint(A)$ and $spcl(A)$ are similarly defined. For any IFS A in (X, τ), we have $\alpha cl(A^c) = (\alpha int(A))^c$ and $\alpha int(A^c) = (\alpha cl(A))^c$.

Definition 2.15. *An IFS A in (X, τ) is said to be an*

(1). intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [17].

(2). intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [13]. The complement of an IFGSCS is called an IFGSOS.

(3). intuitionistic fuzzy semi generalized closed set (IFSGCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [16].

(4). intuitionistic fuzzy α generalized closed set (IFαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [14].

(5). intuitionistic fuzzy α generalized semi closed set (IFαGSCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ), [6].

(6). intuitionistic fuzzy ω closed set (IFωCS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSOS in (X, τ), [16]. The complement of an IFωCS is called an IFωOS.

(7). intuitionistic fuzzy generalized semi pre closed set (IFGSPCS in short) if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, τ), [11].

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets. The family of all IFωOSs of (X, τ) is denoted by IFωO(X).

Definition 2.16 ([9]). *An IFS A in (X, τ) is said to be an*

(1). intuitionistic fuzzy g'''-closed set (IFG$'''$CS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFGSOS in (X, τ).

(2). intuitionistic fuzzy g_s'''-closed set (IFG$_S'''$CS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFGSOS in (X, τ).

(3). intuitionistic fuzzy g_α'''-closed set (IFG$_\alpha'''$CS in short) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFGSOS in (X, τ).

The complement of an IFG$'''$CS (resp. IFG$_S'''$CS, IFG$_\alpha'''$CS) is called an IFG$'''$OS (resp. IFG$_S'''$CS, IFG$_\alpha'''$OS).

Definition 2.17. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

(1). intuitionistic fuzzy continuous (IF continuous in short) if f^{-1}(B) $\in$ IFO(X) for every B $\in \sigma$, [4].

(2). intuitionistic fuzzy semi continuous (IFS continuous in short) if f^{-1}(B) $\in$ IFSO(X) for every B $\in \sigma$, [7].

(3). intuitionistic fuzzy ω continuous (IFω continuous in short) if f^{-1}(B) $\in$ IFωO(X) for every B $\in \sigma$, [16].

(4). intuitionistic fuzzy generalized continuous (IFG continuous in short) if f^{-1}(B) is an IFGCS in (X, τ) for every IFCS B of (Y, σ), [17].

(5). intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if f^{-1}(B) is an IFGSCS in (X, τ) for every IFCS B of (Y, σ), [13].

(6). intuitionistic fuzzy generalized semi pre continuous (IFGSP continuous in short) if f^{-1}(B) is an IFGSPCS in (X, τ) for every IFCS B of (Y, σ), [12].

(7). intuitionistic fuzzy α-generalized continuous (IFαG continuous in short) if f^{-1}(B) is an IFαGCS in (X, τ) for every IFCS B of (Y, σ), [10].

(8). intuitionistic fuzzy α-generalized semi continuous (IFαGS continuous in short) if f^{-1}(B) is an IFαGSCS in (X, τ) for every IFCS B of (Y, σ), [6].

(9). intuitionistic fuzzy α-continuous (IFα continuous in short) if f^{-1}(B) is an IFαCS in (X, τ) for every IFCS B of (Y, σ), [7].

3. Intuitionistic Fuzzy g'''-continuous Mappings

In this section we introduce intuitionistic fuzzy g'''-continuous functions and study some of its properties.

Definition 3.1. *A function f : (X, τ) $\to$ (Y, σ) is called an intuitionistic fuzzy g'''-continuous (IFG$'''$ continuous in short) if f^{-1}(B) is an IFG$'''$CS in (X, τ) for every IFCS B of (Y, σ).*

Example 3.2. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f^{-1}(B)$ is an IFG$'''$CS in (X, τ) for every IFCS B of (Y, σ). Therefore f is an IFG$'''$ continuous function.*

Example 3.3. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.6, 0.5), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.3, 0.4), (0.6, 0.5)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function.*

Theorem 3.4. *Every IF continuous function is an IFG$'''$ continuous function.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF continuous function. Let B be an IFCS in Y. Since f is an IF continuous function, $f^{-1}(B)$ is an IFCS in X. Since every IFCS is an IFG$'''$CS [9], $f^{-1}(B)$ is an IFG$'''$CS in X. Hence f is an IFG$'''$ continuous function. □

The converse of Theorem 3.4 need not be true as seen from the following Example.

Example 3.5. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$ and $T_2 = \langle y, (0.8, 0.7), (0.1, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.1, 0.2), (0.8, 0.7)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFCS in (X, τ). Therefore f is not an IF continuous function. But f is an IFG$'''$ continuous function.*

Theorem 3.6. *Every IFG$'''$ continuous function is an IFGSP continuous function.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFG$'''$ continuous function. Let B be an IFCS in Y. Since f is an IFG$'''$ continuous function, $f^{-1}(B)$ is an IFG$'''$CS in X. Since every IFG$'''$CS is an IFGSPCS [9], $f^{-1}(B)$ is an IFGSPCS in X. Hence f is an IFGSP continuous function. □

The converse of Theorem 3.6 need not be true as seen from the following Example.

Example 3.7. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.6, 0.5), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.3, 0.4), (0.6, 0.5)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFGSP continuous function.*

Theorem 3.8. *Every IFG$'''$ continuous function is an IFω continuous function.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IFG$'''$ continuous function. Let B^c be an IFOS in Y. Then B is an IFCS in Y. Since f is an IFG$'''$ continuous function, $f^{-1}(B)$ is an IFG$'''$CS in X. Since every IFG$'''$CS is an IFωCS [9], $f^{-1}(B)$ is an IFωCS in X and $X \setminus f^{-1}(B) = f^{-1}(B^c)$ is IFωOS in X. Hence f is an IFω continuous function. □

The converse of Theorem 3.8 need not be true as seen from the following Example.

Example 3.9. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFω continuous function.*

Theorem 3.10. *Every IFG''' continuous function is an IFG continuous function.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFG''' continuous function. Let B be an IFCS in Y. Since f is an IFG''' continuous function, f^{-1}(B) is an IFG'''CS in X. Since every IFG'''CS is an IFGCS [9], f^{-1}(B) is an IFGCS in X. Hence f is an IFG continuous function. □

The converse of Theorem 3.10 need not be true as seen from the following Example.

Example 3.11. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.6, 0.5), (0.3, 0.4)\rangle$ and $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG'''CS in (X, τ). Therefore f is not an IFG''' continuous function. But f is an IFG continuous function.*

Theorem 3.12. *Every IFG''' continuous function is an IFαG continuous function.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFG''' continuous function. Let B be an IFCS in Y. Since f is an IFG''' continuous function, f^{-1}(B) is an IFG'''CS in X. Since every IFG'''CS is an IFαGCS [9], f^{-1}(B) is an IFαGCS in X. Hence f is an IFαG continuous function. □

The converse of Theorem 3.12 need not be true as seen from the following Example.

Example 3.13. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ and $T_2 = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG'''CS in (X, τ). Therefore f is not an IFG''' continuous function. But f is an IFαG continuous function.*

Theorem 3.14. *Every IFG''' continuous function is an IFGS continuous function.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFG''' continuous function. Let B be an IFCS in Y. Since f is an IFG''' continuous function, f^{-1}(B) is an IFG'''CS in X. Since every IFG'''CS is an IFGSCS [9], f^{-1}(B) is an IFGSCS in X. Hence f is an IFGS continuous function. □

The converse of Theorem 3.14 need not be true as seen from the following Example.

Example 3.15. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG'''CS in (X, τ). Therefore f is not an IFG''' continuous function. But f is an IFGS continuous function.*

Definition 3.16. *A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuituionistic fuzzy g_s'''-continuous (IFG_S''' continuous in short) if $f^{-1}(B)$ is an IFG_S'''CS in (X, τ) for every IFCS B of (Y, σ).*

Theorem 3.17. *Every IFG''' continuous function is an IFG_S''' continuous function.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFG''' continuous function. Let B be an IFCS in Y. Since f is an IFG''' continuous function, f^{-1}(B) is an IFG'''CS in X. Since every IFG'''CS is an IFG_S'''CS [9], f^{-1}(B) is an IFG_S'''CS in X. Hence f is an IFG_S''' continuous function. □

The converse of Theorem 3.17 need not be true as seen from the following Example.

Example 3.18. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle y, (0.6, 0.5), (0.3, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.3, 0.4), (0.6, 0.5)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFG$'''_S$ continuous function.*

Theorem 3.19. *Every IFG$'''$ continuous function is an IFαGS continuous function.*

Proof. Let f : $(X, \tau) \to (Y, \sigma)$ be an IFG$'''$ continuous function. Let B be an IFCS in Y. Since f is an IFG$'''$ continuous function, $f^{-1}(B)$ is an IFG$'''$CS in X. Since every IFG$'''$CS is an IFαGSCS [9], $f^{-1}(B)$ is an IFαGSCS in X. Hence f is an IFαGS continuous function. □

The converse of Theorem 3.19 need not be true as seen from the following Example.

Example 3.20. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$ and $T_2 = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFαGS continuous function.*

Definition 3.21. *A function $f : (X, \tau) \to (Y, \sigma)$ is called an intuituionistic fuzzy g'''_α-continuous (IFG$'''_\alpha$ continuous in short) if $f^{-1}(B)$ is an IFG$'''_\alpha$CS in (X, τ) for every IFCS B of (Y, σ).*

Theorem 3.22. *Every IFG$'''$ continuous function is an IFG$'''_\alpha$ continuous function.*

Proof. Let f : $(X, \tau) \to (Y, \sigma)$ be an IFG$'''$ continuous function. Let B be an IFCS in Y. Since f is an IFG$'''$ continuous function, $f^{-1}(B)$ is an IFG$'''$CS in X. Since every IFG$'''$CS is an IFG$'''_\alpha$CS [9], $f^{-1}(B)$ is an IFG$'''_\alpha$CS in X. Hence f is an IFG$'''_\alpha$ continuous function. □

The converse of Theorem 3.22 need not be true as seen from the following Example.

Example 3.23. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.6), (0.6, 0.3)\rangle$ and $T_2 = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFG$'''_\alpha$ continuous function.*

Remark 3.24. *IFα continuous functions and IFG$'''$ continuous functions are independent.*

Example 3.25. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$ and $T_2 = \langle y, (0.9, 0.6), (0.1, 0.4)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.1, 0.4), (0.9, 0.6)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFα continuous function.*

Example 3.26. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$, $T_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_3 = \langle y, (0.25, 0.15), (0.65, 0.75)\rangle$. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ and $\sigma = \{0_\sim, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.65, 0.75), (0.25, 0.15)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFαCS in (X, τ). Therefore f is not an IFα continuous function. But f is an IFG$'''$ continuous function.*

Remark 3.27. *IFS continuous functions and IFG$'''$ continuous functions are independent.*

Example 3.28. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_2 = \langle y, (0.5, 0.4), (0.4, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.4, 0.5), (0.5, 0.4)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFG$'''$CS in (X, τ). Therefore f is not an IFG$'''$ continuous function. But f is an IFS continuous function.*

Example 3.29. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$, $T_2 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $T_3 = \langle y, (0.1, 0.2), (0.8, 0.7)\rangle$. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ and $\sigma = \{0_\sim, T_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then for the IFCS $B = \langle y, (0.8, 0.7), (0.1, 0.2)\rangle$ of (Y, σ), $f^{-1}(B)$ is not an IFSCS in (X, τ). Therefore f is not an IFS continuous function. But f is an IFG$'''$ continuous function.*

Theorem 3.30. *A function $f : X \to Y$ is an IFG$'''$ continuous if and only if the inverse image of every IFOS in Y is an IFG$'''$OS in X.*

Proof. Necessary Part: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Since f is an IFG''' continuous, $f^{-1}(A^c)$ is an IFG'''CS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFG'''OS in X. Hence the inverse image of an IFOS in Y is an IFG'''OS in X.

Sufficient Part: Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFG'''OS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFG'''CS in X. Hence f is an IFG''' continuous function. □

Let us introduce IF$_{g'''}$T$_{1/2}$ and IF$_{g'''a}$T$_{1/2}$ spaces.

Definition 3.31. *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{g'''}T_{1/2}$ (IF$_{g'''}T_{1/2}$ in short) if every IFG$'''$CS in (X, τ) is an IFCS in (X, τ).*

Definition 3.32. *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{g'''a}T_{1/2}$ (IF$_{g'''a}T_{1/2}$ in short) if every IFGCS in (X, τ) is an IFG$'''$CS in (X, τ).*

Theorem 3.33. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFG$'''$ continuous function. Then f is an IF continuous function if X is an IF$_{g'''}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. By hypothesis, $f^{-1}(A)$ is an IFG'''CS in X. Since X is an IF$_{g'''}$T$_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous function. □

Theorem 3.34. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IFG continuous function. Then f is an IFG$'''$ continuous function if X is an IF$_{g'''a}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. By hypothesis, $f^{-1}(A)$ is an IFGCS in X. Since X is an IF$_{g'''a}$T$_{1/2}$ space, $f^{-1}(A)$ is an IFG'''CS in X. Hence f is an IFG''' continuous function. □

Remark 3.35. *The composition of two IFG$'''$ continuous functions need not be an IFG$'''$ continuous function.*

Example 3.36. *Let $X = \{a, b\}$, $Y = \{c, d\}$ and $Z = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ and $T_2 = \langle z, (0.8, 0.7), (0.1, 0.2)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$, $\sigma = \{0_\sim, 1_\sim\}$ and $\delta = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X, Y and Z respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$ and $f(b) = d$, and define a function $g : (Y, \sigma) \to (Z, \delta)$ by $g(c) = u$ and $g(d) = v$. Then f and g are IFG$'''$ continuous functions. Consider an IFS $A = \langle z, (0.1, 0.2), (0.8, 0.7)\rangle$. Then A is an IFCS in Z. But $(g \circ f)^{-1}(A)$ is not an IFG$'''$CS in X. Hence the composition of two IFG$'''$ continuous functions need not be an IFG$'''$ continuous function.*

Theorem 3.37. *Let f : (X, τ) → (Y, σ) be an IFG‴ continuous function. Let g : (Y, σ) → (Z, δ) be an IF continuous. Then g ∘ f : (X, τ) → (Z, δ) is an IFG‴ continuous function.*

Proof. Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y. Since f is an IFG''' continuous function, $f^{-1}(g^{-1}(A))$ is an IFG'''CS in X. Hence g ∘ f is an IFG''' continuous function. □

4. Intuitionistic Fuzzy g'''-irresolute Functions

In this section, we introduce intuitionistic fuzzy g'''-irresolute functions and study some of its properties.

Definition 4.1. *A function f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy g'''-irresolute (IFG''' irresolute, in short) if $f^{-1}(A)$ is an IFG''' CS in (X, τ) for every IFG''' CS A in (Y, σ).*

Example 4.2. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ and $T_2 = \langle y, (0.7, 0.8), (0.2, 0.1)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is not an IFG''' irresolute function.*

Theorem 4.3. *Let f : (X, τ) → (Y, σ) be an IFG''' irresolute. Then f is an IFG''' continuous function.*

Proof. Let f be an IFG''' irresolute function. Let A be any IFCS in Y. Since every IFCS is an IFG'''CS [9], A is an IFG'''CS in Y. By hypothesis, $f^{-1}(A)$ is an IFG'''CS in X. Hence f is an IFG''' continuous function. □

The converse of Theorem 4.3 need not be true as seen from the following Example.

Example 4.4. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$ and $T_2 = \langle y, (0.8, 0.7), (0.2, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function f : (X, τ) → (Y, σ) by f(a) = 1−u and f(b) = 1−v. Then f is an IFG''' continuous function. But f is not an IFG''' irresolute function. We have the IFS $A = \langle y, (0.1, 0.2), (0.9, 0.8)\rangle$ is an IFG''' CS in Y, but $f^{-1}(A) = \langle x, (0.9, 0.8), (0.1, 0.2)\rangle$ is not an IFG''' CS in X. Hence f is not an IFG''' irresolute function.*

Theorem 4.5. *Let f : (X, τ) → (Y, σ) be an IFG''' irresolute. Then f is an IF continuous function if X is an $IF_{g'''}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then A is an IFG'''CS in Y. By hypothesis, $f^{-1}(A)$ is an IFG'''CS in X. Since X is an $IF_{g'''}T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous function. □

Theorem 4.6. *Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be IFG''' irresolute functions. Then g ∘ f : (X, τ) → (Z, δ) is an IFG''' irresolute function.*

Proof. Let A be an IFG'''CS in Z. Then, since g is an IFG''' irresolute function, $g^{-1}(A)$ is an IFG'''CS in Y. Since f is an IFG''' irresolute function, $f^{-1}(g^{-1}(A))$ is an IFG'''CS in X. Hence g ∘ f is an IFG''' irresolute function. □

Theorem 4.7. *Let f : (X, τ) → (Y, σ) be an IFG''' irresolute function and g : (Y, σ) → (Z, δ) be an IFG''' continuous function. Then g ∘ f : (X, τ) → (Z, δ) is an IFG''' continuous function.*

Proof. Let A be an IFCS in Z. Since g is an IFG''' continuous function, $g^{-1}(A)$ is an IFG'''CS in Y. Since f is an IFG''' irresolute function, $f^{-1}(g^{-1}(A))$ is an IFG'''CS in X. Hence g ∘ f is an IFG''' continuous function. □

Theorem 4.8. *Let f : (X, τ) → (Y, σ) be an IFG''' irresolute function. Then f is an IFG continuous function if X is an $IF_{g'''a}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then A is an IFG'''CS in Y. By hypothesis, f^{-1}(A) is an IFG'''CS in X and hence IFGCS in X. Since X is an IF$_{g'''a}$T$_{1/2}$ space, f^{-1}(A) is an IFG'''CS in X and hence IFGCS in X. Hence f is an IFG continuous function. □

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