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Total Global Domination in Permutation Graphs

Research Article

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Abstract: A total dominating set D of a graph $G_\pi = (V_\pi, E_\pi)$ is a total global dominating set if D is also a total dominating set of $\bar{G}_\pi$. The total global domination number $\gamma_{tg}(G_\pi)$ of G_π is the minimum cardinality of a total global dominating set. In this paper, we permutation characterize total global dominating sets and bounds are obtained for $\gamma_{tg}(G_\pi)$. we exhibit inequalities involving variations on domination numbers and vertex covering number. Finally we found the total global domination number of a permutation and also derived the total global domination number of permutation graph through the permutation.

Keywords: Permutation graph, Global domination, Total domination-Total global domination, Total global domination number.

1. Introduction

Sampathkumar [5] introduced the Global Domination Number of a Graph. V.R.Kulli and B.Janakiram [4] introduced the Total Global Domination Number of a Graph. J.Chithra, S.P.Subbiah and V.Swaminathan [2] introduced the concept of Domination in Permutation graphs. If i, j belongs to a permutation on n symbols $\{1, 2, ..., n\}$ and i is less than j then there is an edge between i and j in the permutation graph if i appears after j. (i. e) inverse of i is greater than the inverse of j. So the line of i crosses the line of j in the permutation. So there is a one to one correspondence between crossing of lines in the permutation and the edges of the corresponding permutation graph. S.Vijayakumar and C.V.R. Harinarayanan [3] introduced global domination in permutation graphs. In this paper we found the total global domination number of a permutation and also derived the total global domination number of permutation graph through the permutation.

2. Permutation Graphs

Definition 2.1. *Let π be a permutation on n symbols $\{a_1, a_2, ..., a_n\}$ where image of a_i is $a_i^{'}$. Then the permutation graph G_π is given by (V_π, E_π) where $V_\pi = \{a_1, a_2, ..., a_n\}$ and $a_i, a_j \in E_\pi$ if $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j)) < 0$.*

Definition 2.2. *Let π be a permutation on a finite set $A = \{a_1, a_2, a_3, ..., a_n\}$ given by*

$$\pi = \begin{pmatrix} a_1 & a_2 & a_3 & a_4... & a_n \\ a_1^{'} & a_2^{'} & a_3^{'} & a_4^{'}... & a_n^{'} \end{pmatrix},$$

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where $|a_{i+1} - a_i| = c, c > 0, 0 < i \leq n-1$. *The sequence of* π *is given by* $s(\pi) = \{a_1^{'}, a_2^{'}, a_3^{'}, ..., a_n^{'}\}$. *When elements of* A *are ordered in* L_1 *and the sequence of* π *are represented in* L_2, *then a line joining* a_i *in*L_1 *and* a_i *in* L_2 *is represented by* l_i. *This is known as line representation of* a_i *in* π.

Example 2.3. *Let* $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$, *Then the line* l_1 *crosses* l_3 *and* l_5 ; l_2*crosses* l_3, l_4 *and* l_5 ; l_3 *crosses* l_1 *and* l_2; l_4 *crosses* l_2 *and* l_5 ; l_5 *crosses* l_1, l_2 *and* l_4.

Definition 2.4. *Let* $a_i, a_j \in A$. *Then the residue of* a_i *and* a_j *in* π *is denoted by* $Res(a_i, a_j)$ *and is given by* $(a_i - a_j)(\pi^{-1}(a_i) - \pi^{-1}(a_j))$.

Definition 2.5. *Let* l_i *and* l_j *denote the lines corresponding to the elements* a_i *and* a_j *respectively. Then* l_i *crosses* l_j *if* $Res(a_i, a_j) < 0$. *If* l_i *crosses* l_j *then* $(a_i, a_j) \in E_\pi$.

Example 2.6. *Let* π *be a permutation on a finite set*$A = \{a_1, a_2, a_3, ..., a_p\}$ *given by* $\pi = \begin{pmatrix} a_1 & a_2 & a_3 & a_4... & a_p \\ a_1^{'} & a_2^{'} & a_3^{'} & a_4^{'}... & a_p^{'} \end{pmatrix}$, *where* $|a_{i+1} - a_i| = c, c > 0, 0 < i \leq p-1$. *Then the* π *permutation Graph* G_π *is given by* $G_\pi = (V_\pi, E_\pi)$ *where* $V_\pi = \{a_1, a_2, ..., a_p\}$ *and* $a_i a_j \in E_\pi$, *if* $Res(a_i, a_j) < 0$.

Example 2.7. *Let* $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$, *Then* $G_\pi = (V_\pi, E_\pi)$ *where* $V_\pi = \{1, 2, 3, 4, 5\}$ *and* $E_\pi = \{(1,3), (1,5), (2,3), (2,4), (2,5), (4,5)\}$.

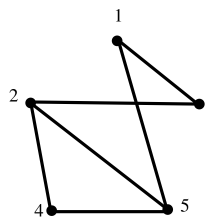

Figure 1. **Permutation graph of** G_π

3. Total Domination of a Permutation

Definition 3.1. *A graph* $G_\pi = (V_\pi, E_\pi)$, $D \subseteq V_\pi$ *is a total dominating set of* G_π *if every vertex in* V_π *is dominated by some vertex in* D. *Thus, A dominating set* D *is a total dominating set such that the subgraph* $< D >$,*induced by* D *has no isolated vertices.*

Note 3.2. *This parameter is defined only for graphs without isolated vertices.*

Definition 3.3. *The smallest number of a vertices in any total dominating set of* G_π *is called the total domination number and is denoted by* $\gamma_t(G_\pi)$ *or* γ_t.

Note 3.4. *Since any total dominating set is a dominating set,* $\gamma \leq \gamma_t$.

Definition 3.5. *The element* a_i *is said to dominate* a_j *if their lines cross each other in* π. *The set of collection of elements of* π *whose lines cross all the lines of the elements* $a_1, a_2, ..., a_n$ *in* π *is said to be a dominating set of* π. $V_\pi = \{a_1, a_2, ..., a_n\}$ *is always a dominating set.*

Definition 3.6. *According to this domination, we have the figure 1 is. Total dominating number of* G_π *is 2, since*$\{1, 5\}$ *is a minimum total dominating set.*

4. Total Global Domination of a Permutation

Definition 4.1. *A dominating set D of G_π is a global dominating set of a graph G_π if D is also a dominating set of the complement of G_π. The global domination number $\gamma_g(G_\pi)$ is the minimum cordinality of a global dominating set.*

Definition 4.2. *The dominating number of a permutation π is the minimum cardinality of a set in MDS(π) and is denoted by $\gamma(\pi)$. The global dominating number of a permutation π is the minimum cardinality of a set in MDS(π) and is denoted by $\gamma_g(\pi)$.*

Theorem 4.3. *The global domination number of a permutation π is $\gamma_g(\pi) = \gamma_g(G_\pi)$, the minimum cardinality of the minimal (global) dominating sets ($MGDS$) of G_π [3].*

Definition 4.4. *A total dominating set D of G_π is a total global dominating set if D is also a total dominating set of $\bar{G}_\pi$. The total global domination number $\gamma_{tg}(G_\pi)$ of G_π is the minimum cordinality of a total global dominating set.*

Note 4.5. *A γ_g- is a minimum global dominating set and γ_t- is a minimum total dominating set. Also similarly γ_{tg} is a minimum total global dominating set.*

Theorem 4.6. *A dominating set D of G_π is a global dominating set iff for each $a_j \in V_\pi - D$, there exists a $a_i \in D$ such that a_i is not adjacent to a_j. Let $\bar{\gamma}(\pi) = \gamma(\bar{G}_\pi)$ and $\bar{\gamma}_g(\pi) = \gamma_g(\bar{G}_\pi)$. Then the permutation graph $\gamma_g(\pi) = \bar{\gamma}_g(\pi)$ [3].*

Theorem 4.7. *A total dominating set D of G_π is a total global dominating set if and only if for each vertex $a_i \in V_\pi$ there exists a vertex $a_j \in D$ such that a_i is not adjacent to a_j.*

Theorem 4.8. *Let G_π be a graph such that neither G_π nor $\bar{G}_\pi$ have an isolated vertex. Then*

(1). $\gamma_{tg}(G_\pi) = \gamma_{tg}(\bar{G}_\pi)$;

(2). $\gamma_t \leq \gamma_{tg}(G_\pi)$;

(3). $\gamma_g \leq \gamma_{tg}(G_\pi)$;

(4). $\frac{\gamma_t(G_\pi)+\gamma_t(\bar{G}_\pi)}{2} \leq \gamma_{tg}(G_\pi) \leq \gamma_t(G_\pi) + \gamma_t(\bar{G}_\pi)$.

Example 4.9. *let* $G_\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 2 & 7 & 1 & 8 & 3 & 6 & 4 \end{pmatrix}$, *Here $D = \{4, 5\}$ is minimal total global dominating set.* $\gamma_{tg}(\pi) = \gamma_{tg}(G_\pi) = 2$.

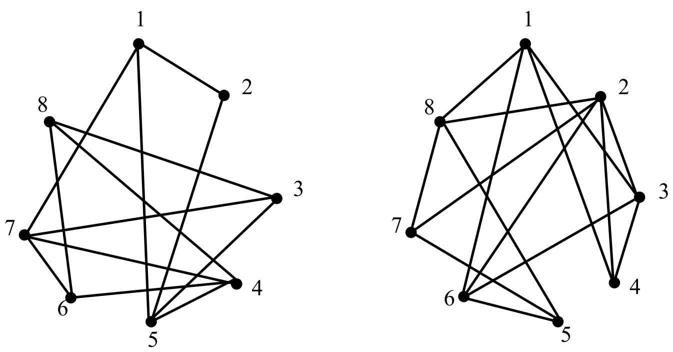

Figure 2. **Total Global domination in permutation graph G_π and $\bar{G}_\pi$**

5. Some Theorems of Total Global Domination

Theorem 5.1.

(1). For a graph G_π with p vertices, $\gamma_{tg}(G_\pi) = p$ iff $G_\pi = K_p$ or $\bar{K}_p$.

(2). $\gamma_{tg}(K_{m,n}) = 2$ for all $m, n \geq 1$.

(3). $\gamma_{tg}(C_4) = 2, \gamma_{tg}(C_5) = 3$ and $\gamma_{tg}(C_n) = \lceil \frac{n}{3} \rceil$for all $m, n \geq 6$.

(4). $\gamma_{tg}(P_n) = 2$ for $n = 2, 3$ and $\gamma_{tg}(P_n) = \lceil \frac{n}{3} \rceil$ for $n \geq 6$.

Proof. we prove only (1) and (2)-(4) are obvious. Clearly, $\gamma_{tg}(K_p) = \gamma_{tg}(\bar{K}_p) = p$. Suppose $\gamma_{tg}(G_\pi) = p$ and $G_\pi \neq K_p$ or $\bar{K}_p$ Then G_π has at least one edge uv and a vertex w not adjacent to, say v. Then $V_\pi - \{v\}$ is a total global domination set and $\gamma_{tg}(G_\pi) = p - 1$. For some graphs including trees, γ_{tg} is almost equal to γ. □

Theorem 5.2. *Let D be a minimum dominating set of G_π. If there exists a vertex v in $V - D$ adjacent to only vertices in D, then $\gamma_{tg} \leq \gamma + 1$.*

Proof. This follows since $D \cup \{v\}$ is a total global dominating set. □

Corollary 5.3. *Let $G_\pi = (V_1 \cup V_2, E_\pi)$ be a bipartite graph without isolates, where$\mid V_1 \mid= m, \mid V_2 \mid= n$ and $m \leq n$. Then $\gamma_{tg} \leq m + 1$.*

Proof. This follows from $\gamma_{tg} \leq \gamma + 1$ since $m \leq n$. □

Corollary 5.4. *For any graph with a pendant vertex, $\gamma_{tg} \leq \gamma + 1$ holds. In particular, $\gamma_{tg} \leq \gamma + 1$ holds for a tree.*

Corollary 5.5. *If $V - D$ is independent, then $\gamma_{tg} \leq \gamma + 1$ holds. Let α_0 and β_0 respectively denote the covering and independence number of a graph.*

Theorem 5.6. *For a (p, q) graph G_π without isolates $\frac{2q-p(p-3)}{2} \leq \gamma_{tg} \leq p - \beta_0 + 1$.*

Proof. Let D be a minimum total global dominating set. Then every vertex in $V_\pi - D$ is not adjacent to atleast one vertex in D. This implies $q \leq pC_2 - (p - \gamma_{tg})$ and the lower bound follows. To establish the upper bound, let B be an independent set with β_0 vertices. Since G_π has no isolates. $V - B$ is a dominating set of G_π. Clearly, for any $V \in B$, $(V - B) \cup \{V\}$ is a total global dominating set of G_π, and the upper bound follows. Since $\alpha_0 + \beta_0 = p$ for eny graph of order p without isolates. □

Corollary 5.7. *$\gamma_{tg} \leq \alpha_0 + 1$. The independent domination number $i(G)$ of G_π is the minimum cardinality of a dominating set which is also independent. It is well-known that $\gamma \leq i \leq \beta_0$.*

Corollary 5.8. *For any graph G_π of order p without isolates.*

(1). $\gamma + \gamma_{tg} \leq p + 1$,

(2). $i + \gamma_{tg} \leq p + 1$.

Theorem 5.9. *For any graph $G_\pi = (V_\pi, E_\pi)$, $\gamma_{tg} \leq max\{\chi(G_\pi).\chi(\bar{G}_\pi)\}$, where $\chi(G_\pi)$ is the chromatic number of G_π.*

Proof. we know this Theorem proved [3]. So Corollary use for total global domination. □

Corollary 5.10. *For any graph G_π of order p, $\gamma_{tg} \leq max\{\Delta+1, \bar{\Delta}+1\} = max\{p-\bar{\delta}, p-\delta\}$ and If G_π is neither complete nor an odd cycle $\gamma_{tg} \leq max\{\Delta, \bar{\Delta}\} = max\{p-1-\bar{\delta}, p-1-\delta\}$, since $\gamma \leq \gamma_{tg}$ and $\bar{\gamma} \leq \gamma_{tg}$*

Corollary 5.11. *Let $t = \gamma$ or $\bar{\gamma}$. For any graph G_π, $t \leq max\{\Delta+1, \bar{\Delta}+1\}$ if g_π is neither complete nor an odd cycle $t \leq max\{\Delta, \bar{\Delta}\}$. Let k and $\bar{k}$ respectively denote the connectivity of G_π and $\bar{G}_\pi$. it is well know that $k \leq \delta$.*

Corollary 5.12. *For any graph G_π of order p, $\gamma_{tg} \leq max\{p-k-1, p-\bar{k}-1\}$. For $v \in V_\pi$, let $N(v) = \{u \in V_\pi : uv \in E_\pi\}$ and $N[v] = (v) \cup \{v\}$. A set $D \subset V_\pi$ is full if $N(v) \cap V_\pi - D \neq \emptyset$ for all $v \in D$. Also D is tg-full if $N(v) \cap V_\pi - D \neq \emptyset$ both in G_π and $\bar{G}_\pi$. The full number$f = f(G_\pi)$ of G_π is the maximum cardinality of a full set of G_π and the tg- full number$f_{tg} = f_{tg}(G_\pi)$ of G_π is the maximum cardinality of a tg-full set of G_π. Clearly $f_{tg}(G_\pi) = f_{tg}(\bar{G}_\pi)$.*

Proposition 5.13. *If G_π is of order $\gamma + f = p$.*

Theorem 5.14. *If G_π is of order $\gamma_{tg} + f_{tg} = p$.*

Proof. Let D be a minimum global dominating set and $v \in V_\pi - D$. Then $N(v) \cap D \neq \emptyset$ both in G_π and $\bar{G}_\pi$. Hence $V_\pi - D$ is g- full and $p - \gamma_{tg} = |V_\pi - D| \leq f_{tg}$. On the otherhand. Suppose D V_π is g-full with $|D| = f_{tg}$. Then, for all $v \in D$, $N(v) \cap V_\pi - D \neq \emptyset$ both in G_π and $\bar{G}_\pi$. This implies that $V_\pi - D$ is a global dominating set. Hence $\gamma_{tg} \leq |V_\pi - D| = p - f_{tg}$. □

6. Total Global Domination Number

A partition $\{a_1, a_2, ..., a_n\}$ of V is a domination (total global domination) partition of G_π if each V_i is a dominating set(total global dominating set). The domination number $d = d(G_\pi)$ (total global domination number $d = d(G_\pi)$) of G_π is the maximum order of a domination (total global domination) partition of G_π. Clearly, for any graph G_π, $d_{tg}(G_\pi) = d_{tg}(\bar{G}_\pi)$

Proposition 6.1.

(1). $d_{tg}(K_n) = d_{tg}(\bar{K}_n)$=1.

(2). For any $n \geq 1$, $d_{tg}(C_{3n}) = 3$, and $d_{tg}(C_{3n+1}) = d_{tg}(C_{3n+2})$=2.

(3). For any $2 \leq m \leq n$, $d_{tg}(K_{m,n}) = n$. when $\bar{d} = d(\bar{G}_\pi)$ and $\bar{d}_{tg} = d_{tg}(\bar{G}_\pi)$.

Proposition 6.2. *If G_π is of order p, then $\gamma + d \leq p+1$ and $\gamma_{tg} + d_{tg} \leq p+1$ if and only if $G_\pi = K_p$ or $\bar{K}_p$.*

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