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g*s* - Closed Sets in Topological Spaces

Research Article

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Abstract: In this paper, a new class of sets, namely g^*s^*-closed sets was introduced and some of their properties were studied. Further the notion of g^*s^*-continuous maps, g^*s^*-irresolute maps, Tb^*-spaces, gTb^*-spaces, $^*gTb^*$-spaces,g^*s^*-compactness, g^*s^*-connectedness were introduced and its properties are investigated.

Keywords: g^*s^*-closed set, g^*s^*-continuous maps, g^*s^*-irresolute maps, Tb^*-spaces, gTb^*-spaces, $^*gTb^*$-spaces, g^*s^*-compactness, g^*s^*-connectedness.

1. Introduction

The concept of generalized closed sets and semi-open sets were introduced and studied by Norman Levine [7] respectively. Arya and Nour [4] defined generalized semi-closed sets for obtaining some characterizations of s-normal spaces. Bhattacharya and Lahiri [5] introduced and investigated semi-generalized closed sets. The concept of generalized semi-pre closed sets was introduced by Dontchev [6]. Palaniappan and Rao [14] introduced rg-closed sets. Pauline Mary Helen, Ponnuthai and Veronica [15] introduced and studied g^{**}-closed sets. Anitha [3] introduced g^*s-closed sets. $g\alpha-$closed sets and αg-closed sets were introduced by Maki et. al. [10] and some of their properties were investigated. In this paper we introduce a new class of called g^*s^*-closed sets and study the relationship of g^*s^*-closed sets with the above mentioned sets. We also obtain basic properties of g^*s^*-closed sets and introduced g^*s^*-continuous maps and g^*s^*-irresolute maps.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset of a space (X,τ), $cl(A)$, $int(A)$ and $scl(A)$ denote the *closure* of A, the *interior* of A and $semi-closure$ of A respectively. The class of all subsets of a space (X,τ) is denoted by C(X,τ).

Definition 2.1. *A subset A of a topological space (X,τ) is called*

(i). a $semi-openset$ [8] if $A\subseteq cl(int(A))$ and semi-closed set if $int(cl(A))\subseteq A$.

(ii). a $semi-preopenset$ [2] (=$\beta-open$ [1]) if $A\subseteq cl(int(cl(A)))$ and a $semi-preclosed$ [2] set (=$\beta-closed$ [1]) if $int(cl(int(A)))\subseteq A$.

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Definition 2.2. *A subset A of a topological space (X, τ) is called*

(i). g-closed set [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(ii). gs-closed set [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(iii). w-closed set [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).

(iv). g^*-closed set [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

(v). g^{**}-closed set [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g*$-open in (X, τ).

(vi). gsp-closed set [6] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(vii). g^*s-closed set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in (X, τ).

(viii). rg-closed set [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open in (X, τ).

Definition 2.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is called*

(i). g^*-continuous [19] if the inverse image $f^{-1}(V)$ of every closed set in (Y, σ) is g^*-closed in (X, τ).

(ii). gs-continuous [4] if the inverse image $f^{-1}(V)$ of every closed set in (Y, σ) is gs-closed in (X, τ).

(iii). gsp-continuous [6] if the inverse image $f^{-1}(V)$ of every closed set in (Y, σ) is gsp-closed in (X, τ).

(iv). g^*s-continuous [3] if the inverse image $f^{-1}(V)$ of every closed set in (Y, σ) is g^*s-closed in (X, τ).

(v). g^{**}-continuous [19] if the inverse image $f^{-1}(V)$ of every closed set in (Y, σ) is g^{**}-closed in (X, τ).

Definition 2.4. *A topological space (X, τ) is said to be*

(i). a $T^*_{\frac{1}{2}}$-space [19] if every g^*-closed set in (X, τ) is closed in (X, τ).

(ii). a $T^{**}_{\frac{1}{2}}$-space [15] if every g^{**}-closed set in (X, τ) is closed in (X, τ).

3. Properties of g^*s^*-closed sets

We now introduce the following definition.

Definition 3.1. *A subset I of (X, τ) is said to be a g^*s^*-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in (X, τ). The class of all g^*s^*-closed subset of (X, τ) is denoted by $G^*S^* - C(X, \tau)$.*

Proposition 3.2. *Every closed set is g^*s^*-closed.*

The converse of the above preposition need not be true and in general it can be seen from the following example.

Example 3.3. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$. Let $A = \{a, b\}$, then A is g^*s^*-closed but not closed. So, the class of g^*s^*-closed sets is properly contained in the class of closed sets.*

Proposition 3.4. *Every g^*-closed set is g^*s^*-closed set.*

The converse of the above preposition need not be true and in general it can be seen from the following example.

Example 3.5. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}\}$. Let $A = \{b, c\}$, then A is g^*s^*-closed but not g^*-closed.*

Proposition 3.6. *Every $g^{**}-$closed set is g^*s^*-closed set.*

The converse of the above preposition need not be true and in general it can be seen from the following example.

Example 3.7. *Let X= $\{a,b,c\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$. Let $A = \{b\}$, then A is g^*s^*-closed but not $g^{**}-$closed.*

Proposition 3.8. *Every g^*s-closed set is g^*s^*-closed set.*

The converse of the above preposition need not be true and in general it can be seen from the following example.

Example 3.9. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{b\}, \{b,c\}\}$. Let $A = \{a,b\}$, then A is g^*s^*-closed but not g^*s-closed set.*

Proposition 3.10. *Every g^*s^*-closed set is gs-closed set.*

The converse of the above preposition need not be true and in general it can be seen from the following example.

Example 3.11. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}, \{b,c\}\}$. Let $A = \{b,c\}$,then A is gs-closed but not g^*s^*-closed.*

Proposition 3.12. *Every g^*s^*-closed set is gsp-closed set.*

The converse of the above preposition need not be true and in general it can be seen from the following example.

Example 3.13. *In Example 3.11, $A = \{c\}$. Then A is gsp-closed but not g^*s^*-closed.*

Remark 3.14. *g^*s^*-closedness is independent of g-closedness.*

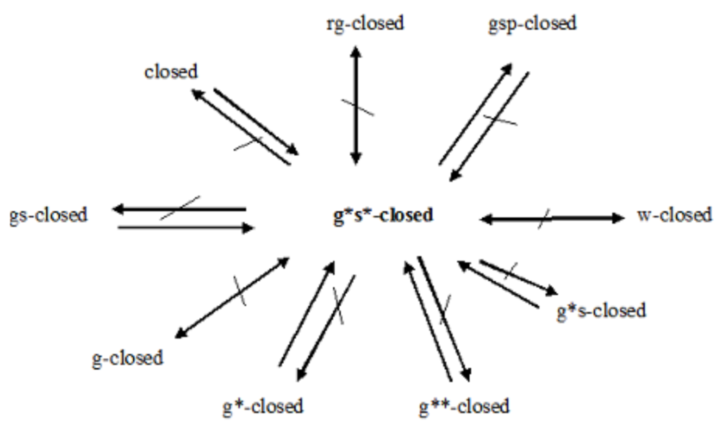
Example 3.15. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{a\}, \{a,c\}\}$. Let $A = \{c\}$, then A is g^*s^*-closed but not g-closed set.In example* [3.11],*$A = \{b\}$. Then A is g-closed but not g^*s^*-closed.*

Remark 3.16. *g^*s^*-closedness is independent of w-closedness (or) s*g-closedness.*

Example 3.17. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, X, \{c\}\}$. Let $A = \{b,c\}$. In Example 3.11, A is g^*s^*-closed but not w-closed set (or) s*g-closed.*

Remark 3.18. *g^*s^*-closedness is independent of rg-closedness.*

In Example 3.7, A $= \{a,b\}$. Then A is rg-closed but not g^*s^*-closed. In Example 3.7, A $= \{b\}$. Then A is g^*s^*-closed but not rg-closed. Thus we have the following diagram.



where $A \to B$ implies B and $A \not\to B$ represents A does not imply B (resp. A and B are independent).

4. g^*s^*-Continuous Maps and g^*s^*-Irresolute Maps.

Definition 4.1. *A map $f : (X, \tau) \to (Y, \sigma)$ from a topological space (X, τ) to a topological space (Y, σ) is called g^*s^*-continuous if the inverse image of every closed set in (Y, σ) is g^*s^*-closed in (X, τ).*

Theorem 4.2. *Every continuous map is g^*s^*-continuous.*

Proof. Let f: $(X, \tau) \to (Y, \sigma)$ be continuous. Let F be a closed set in (Y, σ) then $f^{-1}(F)$ is closed in (X, τ). Since every closed set is g^*s^*-closed, $f^{-1}(F)$ is g^*s^*-closed in (X, τ). f is g^*s^*-continuous in (X, τ). □

The converse of the above theorem need not be true in general and it can be seen from the following example.

Example 4.3. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$, $\sigma = \{\phi, Y, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b\}\}$. Let f: $(X, \tau) \to (Y, \sigma)$ be the identity map. The inverse image of every closed set in (Y, σ) is g^*s^*-closed, but, $f^{-1}(\{c\}) = \{c\}$ is not closed set in (X, τ).*

Theorem 4.4. *Every g^*s-continuous map is g^*s^*-continuous.*

Proof. Let f: $(X, \tau) \to (Y, \sigma)$ be g^*s-continuous. Let F be a closed set in (Y, σ) then is $f^{-1}(F)$ g^*s-closed in (X, τ). Since every g^*s-closed set is g^*s^*-closed, $f^{-1}(F)$ is g^*s^*-closed in (X, τ). f is g^*s^*-continuous in (X, τ). □

The converse of the above theorem need not be true in general and it can be seen from the following example.

Example 4.5. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}, \{b, c\}\}$, $\sigma = \{\phi, X, \{c\}, \{a, b\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. The inverse image of every closed set in (Y, σ) is g^*s^*-closed, but $f^{-1}(\{b, c\}) = \{b, c\}$ is not g^*s-closed in (X, τ).*

Theorem 4.6. *Every g^*-continuous map is g^*s^*-continuous.*

Proof. Let f: $(X, \tau) \to (Y, \sigma)$ be g*-continuous. Let F be a closed set in (Y, σ) then $f^{-1}(F)$ is g*-closed in (X, τ). Since every g*-closed set is g^*s^*-closed, $f^{-1}(F)$ is g^*s^*-closed in (X, τ). f is g^*s^*-continuous in (X, τ). □

The converse of the above theorem need not be true in general and it can be seen from the following example.

Example 4.7. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}\}$, $\sigma = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$. Let f: $(X, \tau) \to (Y, \sigma)$ be the identity map. The inverse image of every closed set in (Y, σ) is g^*s^*-closed, but $f^{-1}(\{b, c\}) = \{b, c\}$ which is not g^*-closed in (X, τ).*

Theorem 4.8. *Every g^{**}-continuous map is g^*s^*-continuous.*

Proof. Let f: $(X, \tau) \to (Y, \sigma)$ be g**-continuous. Let F be a closed set in (Y, σ) then $f^{-1}(F)$ is g**-closed in (X, τ). Since every g**-closed set is g^*s^*-closed, $f^{-1}(F)$ is g^*s^*-closed in (X, τ). f is g^*s^*-continuous in (X, τ). □

The converse of the above theorem need not be true in general and it can be seen from the following example.

Example 4.9. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}, \sigma = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. The inverse image of every closed set in (Y, σ) is g^*s^*-closed, but $f^{-1}(\{a, b\}) = \{a, c\}$ which is not g^{**}-closed in (X, τ).*

Theorem 4.10. *Every g^*s^*-continuous map is gs-continuous.*

Proof. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be g^*s^* -continuous. Let F be a closed set in (Y, σ) then $f^{-1}(F)$ is g^*s^* -closed in (X, τ). Since every g^*s^*-closed set is gs-closed, $f^{-1}(F)$ is gs-closed in (X, τ). f is gs-continuous in (X, τ). □

The converse of the above theorem need not be true in general and it can be seen from the following example.

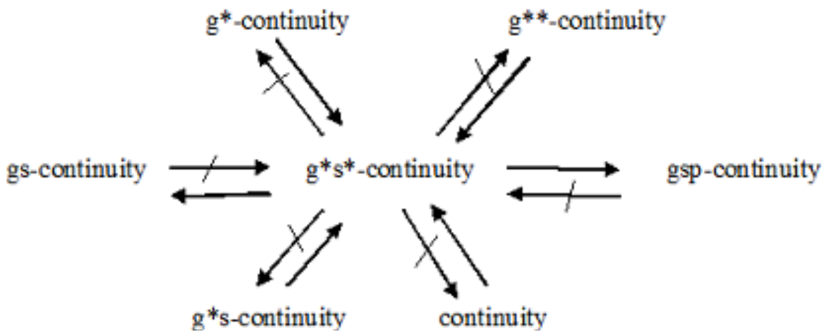
Example 4.11. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b, c\}\}, \sigma = \{\phi, X, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping defined by $f(a) = a, f(b) = c, f(c) = b$. The inverse image of every closed set in (Y, σ) is gs-closed, but $f^{-1}(\{b\}) = \{c\}$ which is not g^*s^*-closed in (X, τ).*

Theorem 4.12. *Every g^*s^*-continuous map is gsp-continuous.*

Proof. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be g^*s^* -continuous. Let F be a closed set in (Y, σ) then $f^{-1}(F)$ is g^*s^*-closed in (X, τ). Since every g^*s^*-closed set is gsp-closed, $f^{-1}(F)$ is gsp-closed in (X, τ). f is gsp-continuous in (X, τ). □

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example 4.13. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b, c\}\}, \sigma = \{\phi, X, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping defined by $f(a) = a, f(b) = c, f(c) = b$. The inverse image of every closed set in (Y, σ) is gsp-closed, but $f^{-1}(\{b\}) = \{c\}$ which is not g^*s^*-closed in (X, τ). Thus, we have the following diagram.*



Definition 4.14. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ from a topological space (X, τ) to a topological space (Y, σ) is called g^*s^*-irresolute if the inverse image of every g^*s^*-closed set in (Y, σ) is g^*s^*-closed in (X, τ).*

Theorem 4.15. *Every g^*s^*-irresolute map is g^*s^*-continuous.*

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example 4.16. *Let $X = Y = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}, \sigma = \{\phi, X, \{b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping defined by $f(a) = b, f(b) = c, f(c) = a$. Let $\{a, c\}$ be a closed set in (Y, σ). But $f^{-1}(\{a, c\}) = \{a, b\}$ which is not g^*s^*-closed set in (X, τ). f is g^*s^*-continuous. $\{c\}$ is a g^*s^*-closed set in (Y, σ). But $f^{-1}(\{c\}) = \{a\}$ which is not g^*s^*-closed in (X, τ). f is not g^*s^*-irresolute.*

Theorem 4.17. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$, then*

(i). $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is g^*s^*-continuous if f is g^*s^*-irresolute and g is g^*s^*-continuous.

(ii). $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is g^*s^*-irresolute if f and g are g^*s^*-irresolute.

(iii). $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is g^*s^*-continuous if f is g^*s^*-continuous and g is g^*s^*-irresolute.

5. Applications Of g^*s^*-Closed Set.

As application of g^*s^*-closed sets, new spaces namely, Tb*-space, gTb*-space and * gTb*-space are introduced. We introduce the following definitions.

Definition 5.1. *A space (X,τ) is said to be a Tb*-space if every g^*s^*-closed set in (X,τ) is closed in (X,τ).*

Theorem 5.2. *Every Tb*-space is $T_{1/2}*$-space.*

Proof. Let (X,τ) be a Tb*-space. Let A be a g*-closed set in (X,τ). But by preposition (3.4), every g*-closed set is g^*s^*-closed. Since (X,τ) is a Tb*-space, A is closed in (X,τ). (X,τ) is a $T_{1/2}*$-space. □

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example 5.3. *In example* [3.5]*, Let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\}\}$. Here, (X,τ) is a $T_{1/2}*$-space and the set $\{b,c\}$ is g^*s^*-closed but not closed. (X,τ) is not a Tb*-space.*

Theorem 5.4. *Let f : $(X,\tau)\to(Y,\sigma)$ be a g^*s^*-continuous mapping. If (X,τ) is Tb*-space, then f is continuous.*

Proof. Let f:$(X,\tau)\to(Y,\sigma)$ be g^*s^*-continuous. Let F be a closed set in (Y,σ). Then $f^{-1}(F)$ is g^*s^*-closed in (X,τ). Since (X,τ) is Tb*-space, $f^{-1}(F)$ is closed in (X,τ). f is continuous. □

Theorem 5.5. *Every Tb*-space is $T_{1/2}**$-space.*

Proof. Let (X,τ) be a Tb*-space. Let A be g**-closed set in (X,τ). But by preposition (3.6), every g**-closed set is g^*s^*-closed. Since (X,τ) is Tb*-space, A is closed in (X,τ), which implies, g^*s^*-closed set is closed. (X,τ) is a $T_{1/2}**$-space. □

The converse of the above theorem need not be true and in general it can be seen from the following example.

Example 5.6. *In example* [3.7]*, Let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$. Here, (X,τ) is a $T_{1/2}**$-space and the sets $\{a\},\{b\}$ are g^*s^*-closed but not closed. (X,τ) is not Tb*-space.*

Definition 5.7. *A space (X,τ) is said to be gTb*-space if every g^*s^*-closed set in (X,τ) is g*-closed in (X,τ).*

Theorem 5.8. *Every Tb*-space is gTb*-space.*

Proof. Let (X,τ) be a Tb*-space. Let A be g^*s^*-closed set in (X,τ). Since (X,τ) is a Tb*-space, A is closed in (X,τ). But we know that, every closed set is g*-closed. Hence, A is g*-closed set in (X,τ). (X,τ) is a gTb*-space. □

Theorem 5.9. *Let f : $(X,\tau)\to(Y,\sigma)$ be g^*s^*-continuous mapping. If (X,τ) is gTb*-space, then f is g*-continuous.*

Proof. Let f: $(X,\tau)\to(Y,\sigma)$ be g^*s^*-continuous. Let F be a closed set in (Y,σ). Then $f^{-1}(F)$ is g^*s^*-closed in (X,τ). Since (X,τ) is a gTb*-space, $f^{-1}(F)$ is g*-closed in (X,τ). f is g*-continuous. □

Definition 5.10. *A space (X,τ) is said to be *gTb*-space if every g^*s^*-closed set in (X,τ) is g*s-closed in (X,τ).*

Theorem 5.11. *Every Tb*-space is *gTb*-space.*

Proof. Let (X,τ) be a Tb*-space. Let A be g^*s^*-closed set in (X,τ). Since (X,τ) is Tb*-space, A is closed in (X,τ). But we know that, every closed set is g*s-closed. Hence, A is g*s-closed set in (X,τ). (X,τ) is a *gTb*-space. □

Theorem 5.12. *Let f : $(X,\tau)\to(Y,\sigma)$ be g^*s^*-continuous mapping. If (X,τ) is *gTb*-space, then f is g*s-continuous.*

Proof. Let f: $(X,\tau)\to(Y,\sigma)$ be g^*s^*-continuous. Let F be a closed set in (Y,σ). Then $f^{-1}(F)$ is g^*s^*-closed in (X,τ). Since (X,τ) is a *gTb*-space, $f^{-1}(F)$ is g*s-closed in (X,τ). f is g*s-continuous. □

6. G*S*-Compactness

Definition 6.1. *A collection $\{A_i/i \in A\}$ of g^*s^*-open sets in a topological space X is called a g^*s^*-open cover of a subset B of X if $B \subset \cup_{i\in A} A_i$.*

Definition 6.2. *A topological space X is G^*S^*-compact if every g^*s^*-open cover of X has a finite sub cover.*

Definition 6.3. *A subset B of a topological space X is said to be G^*S^*-compact relative to X if for every collection of g^*s^*-open subsets of X such that $B \subset \cup_{i\in A} A_i$, there exists a finite subset A_0 of A such that, $B \subset \cup_{i\in A_0} A_i$.*

Definition 6.4. *A subset B of X is G^*S^*-compact if B is G^*S^*-compact as a subspace of X.*

Proposition 6.5. *A g^*s^*-closed subset of G^*S^*-compact space is G^*S^*-compact relative to X.*

Proof. Let A be a g^*s^*-closed subset of G^*S^*-compact space X. Then A^c is g^*s^*-open in X. Let M be a cover of A by g^*s^*-open sets in X. Then M, A^c is a g^*s^*-open cover of X. Since X is G^*S^*-compact,it has a finite sub-cover, namely $G1, G2, \ldots, G_n$. Therefore, we have obtained a finite g^*s^*-open sub-cover of A.Thus, A is G^*S^*-compact relative to X. □

Proposition 6.6.

(i). A g^*s^*-continuous image of a G^*S^*-compact space is compact.

(ii). If a map $f : X \to Y$ is g^*s^*-irresolute and a subset B of X is G^*S^*-compact relative to X, then the image f(B) is G^*S^*-compact relative to X.

Proof.

(i). Let $f : X \to Y$ be a g^*s^*-continuous map from a G^*S^*-compact space onto a topological space Y. Let $A_i : i \in A$ be an open cover of Y. Then $\left\{f^{-1}(A_i) : i \in A\right\}$ is a g^*s^*-open cover of X. Since X is G^*S^*-compact, it has a finite subcover, namely $\left\{f^{-1}(A_1), f^{-1}(A_2), ...f^{-1}(A_n)\right\}$. Since f is onto, $A_1, A_2, ...A_n$ is an open cover of Y and so Y is compact.

(ii). Let $A_i : i \in A$ be any collection of g^*s^*-open subsets of Y such that $f(B) \subset \cup A_i : i \in A$. Then $B \subset \cup f^{-1}(A_i) : i \in A$ holds. By using assumption, there exists a finite subset A_0 of A such that $B \subset \cup f^{-1}(A_i) : i \in A_0$. Therefore, we have $f(B) \subset \cup A_i : i \in A_0$, which shows that $f(B)$ is G*S*-compact relative to Y.

□

Theorem 6.7. *If the product space of two non-empty spaces is G^*S^*-compact, then each of he factor space is G^*S^*-compact.*

Proof. Let $X \times Y$ be the product space of non-empty spaces X and Y. Obviously, the projection p: $X \times Y \to Y$ from $X \times Y$ onto X is g^*s^*-irresolute map. In fact, let F be any g^*s^*-closed set of X. Then it follows that, $F \times Y (= p^{-1}(F))$ is g^*s^*-closed in $X \times Y$ and hence p is g^*s^*-irresolute Now, suppose that $X \times Y$ is G^*S^*-compact. By using Preposition , we obtain that the g^*s^*-irresolute image $p(X \times Y)(= X)$ is G^*S^*-compact. For Y, the proof is similar to the case of X. □

7. G^*S^*-Connectedness

Definition 7.1. *A topological space X is G^*S^*-connected if X cannot be written as a disjoint union of two non-empty g^*s^*-open sets. A subset V of X is G^*S^*-connected if V is G^*S^*-connected as a subspace.*

Proposition 7.2. *For a topological space X, the following conditions are equivalent.*

(i). X is G^*S^*-connected.

(ii). The only subsets of X which are both g^*s^*-open and g^*s^*-closed are empty set and X.

(iii). Each g^*s^*-continuous map of X into a discrete space Y with at least two points is a constant map.

Proof.
(i) ⇒ (ii): Let U be a g^*s^*-open and g^*s^*-closed subset of X. Then X-U is both g^*s^*-closed and g^*s^*-open. Since X is the disjoint union of the g^*s^*-open sets U and X-U, one of these must be empty, that is U = ϕ o U =X.
(ii) ⇒ (i): Suppose X = $A \cup B$ where A and B are disjoint non-empty $g * s*$-open subsets of X. Since A is a $g * s*$-open subset of X, by condition (ii), it may be $g * s*$-closed and $A = \phi$ or A = X. If A = ϕ, X=B. If A=X, B=ϕ. Thus, X is g^*s^*-connected.
(ii) ⇒ (iii): Let f:X → Y be a g^*s^*-continuous map. Then X is covered by g^*s^*-open and g^*s^*-closed covering $\{f^{-1}(y)/y \in Y\}$. By assumption, $f^{-1}(y) = \phi$ or X for each $x \in X$. If $f^{-1}(y) = \phi$ for all $y \in Y$ then f fails to be a map. Then, there exists only one point $y \in Y$ such that $f^{-1}(y) \neq \phi$ and hence $f^{-1}(y) = X$ which shows that f is a constant map.
(iii) ⇒ (ii): Let U be both g^*s^*-open and g^*s^*-closed in X. Suppose U ≠ ϕ. Let f: X → Y be a g^*s^*-continuous map defined by $f(U) = \{y\}$ and $f(X - U) = \{w\}$ for some distinct points y and w in Y. By assumption, f is a constant. Therefore, U =X. □

It is obvious that every G^*S^*-connected space is connected . The following example shows that the converse is not true.

Example 7.3. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$. Then the topological space is (X, τ) is connected. However, since $\{b\}$ is both g^*s^*-closed and g^*s^*-open in X. By Preposition 7.2, X is not G^*S^*-connected.*

Proposition 7.4. *If X is T_b^*- space and connected, then X is g^*s^*-connected.*

Proof. Let X be T_b^*-space and connected. Assume that X can be written in the form $X = A \cup B$ where A and B are nonempty disjoint and g^*s^*-open sets in X. Since X is T_b^*-space, every g^*s^*-open set is open and so X= $A \cup B$ where A and B are disjoint nonempty and open sets in X. This contradicts the fact that X is connected. Therefore X is g^*s^*-connected. □

Proposition 7.5. *If f:X→Y is g^*s^*-continuous surjection and X is g^*s^*-connected then Y is connected.*

Proof. Suppose that Y is not connected. Let Y= $A \cup B$ where A and B are disjoint nonempty open sets in Y. Since f is g^*s^*-continuous and onto, X= $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty g^*s^*-open in X. This contradicts the fact that X is g^*s^*-connected. Hence Y is connected. □

Proposition 7.6. *If f:X→Y is g^*s^*-continuous map from a connected space X into a topological space Y, then Y is g^*s^*-connected.*

Proof. Let Y be not g^*s^*-connected. Then Y can be written as Y= $A \cup B$ where A and B are disjoint nonempty g^*s^*-open sets in Y. Since f is g^*s^*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are open sets in X. Also X=$f^{-1}(A) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$. This contradicts the fact that X is connected. Therefore Y is g^*s^*-connected. □

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