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# Intuitionistic Fuzzy L-Filters

Research Article

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**Abstract:** This paper contains some definitions and results in intuitionistic fuzzy L-filters. some of its basic properties are investigated. We also study the intuitionistic fuzzy L-filters and homomorphism and some related properties are discussed.

**MSC:** 03E72.

**Keywords:** Fuzzy sublattice, Fuzzy L-ideals, Fuzzy L-filter, Intuitionistic fuzzy sublattice, Intuitionistic fuzzy L-ideals, Intuitionistic fuzzy L-filter, Intuitionistic fuzzy L-ideals and homomorphism.


## 1. Introduction

The theory of fuzzy set was introduced by L.A Zadeh [1] in 1965. In that line the fuzzy group was introduced by Rosenfield [2]. The concept of intuitionistic fuzzy set was introduced by K.T Atanassov [3] as a generalization of the notion of fuzzyset. M.Mullai [4] has studied the concepts of fuzzy L-ideals and fuzzy L-filters. The idea of intuitionistic L-fuzzy semi filter was introduced by M.Maheswari and M.Palanivelrajan [5]. The intuitionistic L-fuzzy ideal and the homomorphism of intuitionistic L-fuzzy ideal established by N.Palaniappan [6]. In particular K.V.Thomas [7] developed the theory of fuzzy sublattice. The main purpose of this work is to study the generalization of these concepts for intuitionistic fuzzy lattices. The main motivation in this work is to introduce the concept of intuitionistic fuzzy L-filter and establish some results on it.

## 2. Preliminaries

**Definition 2.1.** *Let $L$ be a lattice. Let $\mu$ be a fuzzy set in $L$. Then $\mu$ is called a fuzzy sublattice of $L$. if $\forall\ x,\ y \in L$.*

*(1).* $\mu(x \vee y) \geq \min\{\mu(x),\ \mu(y)\}$.

*(2).* $\mu(x \wedge y) \geq \min\{\mu(x),\ \mu(y)\}$.

**Definition 2.2.** *A fuzzy subset $\mu\ : L \to [0,\ 1]$ of $L$ is called a fuzzy L-ideal of $L$. $\forall\ x,\ y \in L$.*

*(1).* $\mu(x \vee y) \geq \min\{\mu(x),\ \mu(y)\}$.

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*(2).* $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$.

**Definition 2.3.** *A fuzzy subset* $\mu : L \to [0,1]$ *of* $L$ *is called a fuzzy L-filter of* $L$ *if* $\forall\ x, y \in L$.

*(1).* $\mu(x \vee y) = \max\{\mu(x), \mu(y)\}$.

*(2).* $\mu(x \wedge y) = \min\{\mu(x), \mu(y)\}$.

**Definition 2.4.** *Let* $L$ *be a lattice and* $A = \{< x, \mu_A(x), v_A(x) > / x \in L\}$ *be an Intuitionistic fuzzy set of* $L$*. Then* $A$ *is called an intuitionistic fuzzy sublattice (Intuitionistic fuzzy lattice) of* $L$ *if the following conditions are satisfied.*

*(1).* $\mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$.

*(2).* $\mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$.

*(3).* $V_A(x \vee y) \leq \max\{V_A(x), V_A(y)\}$.

*(4).* $V_A(x \wedge y) \leq \max\{V_A(x), V_A(y)\}, \ \forall\ x, y \in L$.

**Definition 2.5.** *An Intuitionistic fuzzy set* $A$ *of* $L$ *is called an intuitionistic fuzzy L-ideal of* $L$*, if the following conditions are satisfied.*

*(1).* $\mu_A(x \vee y) = \min\{\mu_A(x), \mu_A(y)\}$.

*(2).* $\mu_A(x \wedge y) = \max\{\mu_A(x), \mu_A(y)\}$.

*(3).* $V_A(x \vee y) = \max\{V_A(x), V_A(y)\}$.

*(4).* $V_A(x \wedge y) = \min\{V_A(x), V_A(y)\}, \ \forall\ x, y \in L$.

# 3. Intuitionistic Fuzzy L-filter

**Definition 3.1.** *An Intuitionistic fuzzy set* $A$ *of* $L$ *is called an Intuitionistic fuzzy L-filter of* $L$ *if the following axioms are satisfied.*

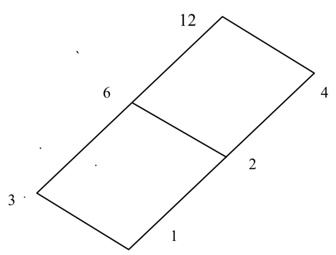
*(1).* $\mu_A(x \wedge y) = \max\{\mu_A(x), \mu_A(y)\}$

*(2).* $\mu_A(x \vee y) = \min\{\mu_A(x), \mu_A(y)\}$

*(3).* $V_A(x \wedge y) = \min\{V_A(x), V_A(y)\}$

*(4).* $V_A(x \vee y) = \max\{V_A(x), V_A(y)\} \ \forall\ x, y \in L$.

**Example 3.2.** *Consider the lattice* $L = \{1, 2, 3, 4, 6, 12\}$ *of divisors of 12.*




*We define* $A = \{< x, \mu_A(x), v_A(x) > / x \in L\}$ *by* $\{< 1, .2, .5 >, < 2, .5, .5 >, < 3, .3, .6 >, < 4, .5, .4 >, < 6, .5, .5 >, < 12, .5, .4 >\}$*. Then* $A$ *is an Intuitionistic fuzzy L-filter.*

**Theorem 3.3.** *If A and B are two Intuitionistic fuzzy filters of a lattice L then $A \cap B$ is an Intuitionistic fuzzy L-filter.*

*Proof.* Let $A = \{< x,\ \mu_A(x),\ v_A(x) > /x \in L\}$ and then $A \cap B = \{< x,\ \mu_{A\cap B}(x),\ v_{A\cap B}(x) > /x \in L\}$, where

$$\mu_{A\cap B}(x) = \min\{\mu_A(x),\ \mu_B(x)\}$$
$$v_{A\cap B}(x) = \max\{v_A(x),\ v_B(x)\}$$

so

$$\begin{aligned}\mu_{A\cap B}(x \vee y) &= \min\{\mu_A(x \vee y),\ \mu_B(x \vee y)\}\\ &= \min\{\min\{\mu_A(x),\ \mu_A(y)\},\ \min\{\mu_B(x),\ \mu_B(y)\} \text{ as A and B are Intuitionistic fuzzy L-filters.}\\ &= \min\{\min\{\mu_A(x),\ \mu_B(x)\},\ \min\{\mu_A(y),\ \mu_B(y)\}\}\\ &= \min\{\mu_{A\cap B}(x),\ \mu_{A\cap B}(y)\},\ \forall\ x,\ y \in L.\end{aligned}$$

Similarly, we get $\mu_{A\cap B}(x \wedge y) \leq \max\{\mu_{A\cap B}(x),\ \mu_{A\cap B}(y)\},\ \forall\ x,\ y \in L.$ Also

$$\begin{aligned}v_{A\cap B}(x \vee y) &= \max\{v_A(x \vee y),\ v_B(x \vee y)\}\\ &\geq \max\{\max\{v_A(x),\ v_A(y)\},\ \max\{v_B(x),\ v_B(y)\}\} \text{ as A and B are Intuitionistic fuzzy L-filters.}\\ &= \max\{\max\{v_A(x),\ v_B(x)\},\ \max\{v_A(y),\ v_B(y)\}\}\\ &= \max\{v_{A\cap B}(x),\ v_{A\cap B}(y)\},\quad \forall\ x,\ y \in L.\end{aligned}$$

Similarly, we get $v_{A\cap B}(x \wedge y) \geq \min\{v_{A\cap B}(x),\ v_{A\cap B}(y)\},\ \forall\ x,\ y \in L.$ □

**Proposition 3.4.** *A is an Intuitionistic fuzzy L-filter if and only if [A] and $< A >$ are Intuitionistic fuzzy L-filters.*

*Proof.* Firstly assume that A is an intuitionistic fuzzy L-filter. We have $[A] = \{< x,\ \mu_A(x),\ \mu_A^C(x) > /x \in L\}$, where $\mu_A^C(x) = 1 - \mu_A(x),\ \forall\ x,\ y \in L.$

$$\begin{aligned}\mu_A(x \wedge y) &\leq \max\{\mu_A(x),\ \mu_A(y)\}\\ \mu_A(x \vee y) &\leq \min\{\mu_A(x),\ \mu_A(y)\}\\ \mu_A^C(x \vee y) &= 1 - \mu_A(x \vee y)\\ &= 1 - \min\{\mu_A(x),\ \mu_A(x)\} \quad \text{as A is an Intuitionistic fuzzy L-filter.}\\ &= \max\{1 - \mu_A(x),\ 1 - \mu_A(x)\}\\ &= \max\{\mu_A^C(x),\ \mu_A^C(y)\},\ \forall\ x,\ y \in L.\end{aligned}$$

Similarly

$$\mu_A^C(x \wedge y) = \min\{\mu_A^C(x),\ \mu_A^C(y)\},\ \forall\ x,\ y \in L.$$

Hence [A] is an Intuitionistic fuzzy L-filter. We have $< A >= \{< x,\ v_A^C(x),\ v_A(x) >\}$, where $v_A^C(x) = 1 - v_A(x)$, $\forall\ x,\ y \in L$.

$$\begin{aligned}v_A(x \vee y) &= \max\{v_A(x),\ v_A(y)\}\\ v_A(x \wedge y) &= \min\{v_A(x),\ v_A(y)\},\ \text{since A is an Intuitionistic fuzzy L-filter.}\end{aligned}$$

Now

$$\begin{aligned} v_A^C(x \vee y) &= 1 - v_A(x \vee y) \\ &= 1 - \max\{v_A(x),\ v_A(y)\} \text{ as A is an Intuitionistic fuzzy L-filter.} \\ &= \min\{1 - v_A(x),\ 1 - v_A(y)\} \\ &= \min\{v_A^C(x),\ v_A^C(y)\},\ \forall\ x,\ y \in L. \end{aligned}$$

Similarly

$$v_A^C(x \wedge y) = \max\{v_A^C(x),\ v_A^C(y)\},\ \forall\ x,\ y \in L.$$

Hence $< A >$ is an Intuitionistic fuzzy L-filter.

Conversely assume that [A] and $< A >$ are Intuitionistic fuzzy L-filters.

To prove that, A is an intuitionistic fuzzy L-filter.

$$\begin{aligned} \mu_A^C(x \vee y) &= \max\{\mu_A^C(x),\ \mu_A^C(y)\},\ \forall\ x,\ y \in L. \\ &= \max\{1 - \mu_A(x),\ 1 - \mu_A(x)\} \\ &= 1 - \min\{\mu_A(x),\ \mu_A(x)\} \text{ as A is an Intuitionistic fuzzy L-filter.} \\ &= 1 - \mu_A(x \vee y) \end{aligned}$$

Similarly

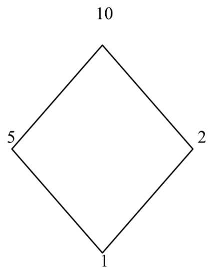
$$\begin{aligned} \mu_A^C(x \wedge y) &= \min\{\mu_A^C(x),\ \mu_A^C(y)\},\ \forall\ x,\ y \in L. \\ v_A^C(x \vee y) &= \min\{v_A^C(x),\ v_A^C(y)\},\ \forall\ x,\ y \in L. \\ &= \min\{1 - v_A(x),\ 1 - v_A(y)\} \\ &= 1 - \max\{v_A(x),\ v_A(y)\} \text{ as A is an Intuitionistic fuzzy L-filter.} \\ &= 1 - v_A(x \vee y). \end{aligned}$$

Similarly

$$v_A^C(x \wedge y) = \max\{v_A^C(x),\ v_A^C(y)\},\ \forall\ x,\ y \in L.$$

Hence A is an Intuitionistic fuzzy L-filter. □

**Example 3.5.** *Consider the lattice of divisors of 10. That is* $L = \{1,\ 2,\ 5,\ 10\}$.




*Let* $A = \{< x,\ \mu_A(x),\ v_A(x) > / x \in L\}$ *be given by* $\{< 1,\ .5,\ .1 >,\ < 2,\ .4,\ .5 >,\ < 5,\ .4,\ .3 >,\ < 10,\ .7,\ .3 >\}$. *Then A is an Intuitionistic fuzzy lattice of L.*

**Remark 3.6.** *The union of two Intuitionistic fuzzy lattice's need not be an Intuitionistic fuzzy lattices. Consider the lattice given in Example 3.5.*

$$A = \{< x,\ \mu_A(x),\ v_A(x) > /x \in L\}\ by\ \{< 1,\ .7,\ .2 >,\ < 2,\ .4,\ .5 >,\ < 5,\ .1,\ .5 >,\ < 10,\ .2,\ .4 >\}$$

$$A = \{< x,\ \mu_B(x),\ v_B(x) > /x \in L\}\ by\ \{< 1,\ .6,\ .1 >,\ < 2,\ .1,\ .5 >,\ < 5,\ .3,\ .3 >,\ < 10,\ .2,\ .3 >\}$$

*Here A and B are Intuitionistic fuzzy lattices. Also* $A \cup B = \{< x,\ \mu_{A\cup B}(x),\ v_{A\cup B}(x) > /x \in L\}$ *is* $\{< 1,\ .7,\ .1 >,\ < 2,\ .4,\ .5 >,\ < 5,\ .3,\ .3 >,\ < 10,\ .2,\ .3 >\}$. *But* $\mu_{A\cup B}(x \vee y) = \min\{\mu_{A\cup B}(x), \mu_{A\cup B}(y)\}$,

$$\mu_{A\cup B}(5 \vee 2) = \mu_{A\cup B}(10) = .2$$

$$\mu_{A\cup B}(5) = .3,\ \mu_{A\cup B}(2) = .4$$

*Therefore* $.2 \ngeq .3$. *Hence* $A \cup B$ *is not an Intuitionistic fuzzy lattices.*

**Remark 3.7.** *Every Intuitionistic fuzzy L-filter is an Intuitionistic fuzzy lattice. But the converse is not true. Consider the lattice L given Example 3.5. Define* $A = \{< x,\ \mu_A(x),\ v_A(x) > /x \in L\}$ *by* $\{< 1,\ .5,\ .1 >,\ < 2,\ .4,\ .3 >,\ < 5,\ .4,\ .5 >,\ < 10,\ .7,\ .3 >\}$. *A is an Intuitionistic fuzzy lattice but not an Intuitionistic fuzzy L-filter because*

$$\mu_A(x \vee y) = \min\{\mu_A(x),\ \mu_B(x)\}$$

$$\mu_A(2 \vee 10) = \min \mu_A(10) = .7$$

$$\mu_A(2) = .4;\ \ \mu_A(10) = .7$$

$$\therefore\ .2 = \min\{.4,\ .7\}$$

$$.7 = .4$$

*Therefore it's not Intuitionistic fuzzy L-filter. Hence every Intuitionistic fuzzy L-ideal is an Intuitionistic fuzzy lattice. But the converse is not true.*

**Remark 3.8.** *The union of two Intuitionistic fuzzy L-filter's need not be an Intuitionistic fuzzy L-filter, which follow from Remarks 3.6 and 3.7.*

**Remark 3.9.** *If A is an Intuitionistic fuzzy L-filter and B an Intuitionistic fuzzy lattice. Then* $A \cap B$ *is an Intuitionistic fuzzy lattice but not an Intuitionistic fuzzy L-filter. Consider the lattice given in Example 3.2. Define* $A = \{< x,\ \mu_A(x),\ v_A(x) > /x \in L\}$ *by* $\{< 1,\ .2,\ .7 >,\ < 2,\ .5,\ .5 >,\ < 3,\ .3,\ .6 >,\ < 4,\ .5,\ .4 >,\ < 6,\ .5,\ .5 >,\ < 12,\ .5,\ .4 >\}$ *and* $B = \{< x,\ \mu_B(x),\ v_B(x) > /x \in L\}$ *by* $\{< 1,\ .2,\ .7 >,\ < 2,\ .4,\ .4 >,\ < 3,\ .2,\ .5 >,\ < 4,\ .3,\ .6 >,\ < 6,\ .5,\ .5 >,\ < 12,\ .6,\ .3 >\}$. *Here A is an Intuitionistic fuzzy L-filter, and B is an IFL of L. Then*

$$A \cap B = \min\{< x,\ \mu_{A\cap B}(x),\ v_{A\cap B}(x) /x \in L >\}\ \ is\ l$$

$\{< 1,\ .2,\ .7 >,\ < 2,\ .4,\ .5 >,\ < 3,\ .2,\ .5 >,\ < 4,\ .3,\ .6 >,\ < 6,\ .5,\ .5 >,\ < 12,\ .4,\ .5 >\}$. *Clearly* $A \cap B$ *is an Intuitionistic fuzzy lattice.*

$$\mu_{A\cap B}(x \vee y) = \min\{\mu_{A\cap B}(x),\ \mu_{A\cap B}(y)\}$$

$$\mu_{A\cap B}(x \vee y) = \mu_{A\cap B}(2 \vee 3) = \mu_{A\cap B}(6) = .5$$

$$\mu_{A\cap B}(x) = \mu_{A\cap B}(2) = .4,\ \mu_{A\cap B}(3) = .2$$

*Therefore* $.5 = \min\{.4,\ .2\}$; $.5 \nleq .2$. *But not an Intuitionistic fuzzy L-filter.*

# 4. Homomorphism

**Definition 4.1.** *If $f : L \to L'$ be a mapping from a lattice L to another lattice L' and $A = \{< x,\ \mu_A(x),\ v_A(x) > /x \in L\}$ be an Intuitionistic fuzzy set of L then the image f(A) is defined by $f(A) = \{< y,\ f(\mu_A)(y),\ f(v_A)(y) > /y \in L\}$, where*

$$f(\mu_A)(y) = \sup\{\mu_A(x)/x \in f^{-1}(y)\},\ f^{-1}(y) \neq \emptyset$$
$$= 0,\ if,\ f^{-1}(y) \neq \emptyset\ textand$$
$$f(v_A)(y) = \inf\{v_A(x)/x \in f^{-1}(y)\},\ f^{-1}(y) \neq \emptyset$$
$$= 1,\ if,\ f^{-1}(y) \neq \emptyset$$

*Similarly if $A' = \{< y,\ \mu'_A(y),\ v'_A(y) > /y \in L'\}$ is an Intuitionistic fuzzy set of L' then $f^{-1}(A')$ is defined by $f^{-1}(A') = \{x,\ f^{-1}(\mu'_A)(x),\ f^{-1}(v'_A)(x) > /x \in L\}$, where $f^{-1}(\mu'_A)(x) = \mu'_A(f(x))$ and $f^{-1}(v'_A)(x) = v'_A(f(x))$.*

**Theorem 4.2.** *If $f : L \to L'$ is a lattice epimorphism and A is an Intuitionistic fuzzy L-filter then f(A) is an Intuitionistic fuzzy L-filter of $L'$.*

*Proof.* Let $A = \{< x,\ \mu_A(x),\ v_A(x) > /x \in L\}$ be an Intuitionistic fuzzy L-filter. Then $f(A) = \{< y,\ f(\mu_A)(y),\ f(v_A)(y) > /y \in L'\}$. Let $y,\ z \in L'$. Then

$$f(\mu_A)(y \vee z) = \sup\{\mu_A(x)/\ x \in f^{-1}(y \vee z)\}$$
$$= \sup\{\mu_A(u \vee v)/\ u \in f^{-1}(y),\ \ v \in f^{-1}(z)$$
$$= \sup\{\min\{\mu_A(u),\ \mu_A(v)\}\ /\ u \in f^{-1}(y),\ v \in f^{-1}(z)\}$$
$$= \min\{\sup \mu_A(u)/u \in f^{-1}(y),\ \sup \mu_A(v)\ \ /v \in f^{-1}(z)\}$$
$$= \min\{f(\mu_A)\ (y),\ f(\mu_A)(z)\}$$

Also

$$f(\mu_A)(y \wedge z) = \sup\{\mu_A(x)/\ x \in f^{-1}(y \wedge z)\}$$
$$= \sup\{\mu_A(u \wedge v)/\ u \in f^{-1}(y),\ v \in f^{-1}(z)\}$$
$$= \sup\{\max\{\mu_A(u),\ \mu_A(v)\}\ /\ u \in f^{-1}(y),\ v \in f^{-1}(z)\} \text{ as A Intuitionistic fuzzy L-filter.}$$
$$= \max\{\sup \mu_A(u)/u \in f^{-1}(y),\ \sup \mu_A(v)\ \ /v \in f^{-1}(z)\}$$
$$= \max\{f(\mu_A)\ (y),\ f(\mu_A)(z)\}$$

Again

$$f(v_A)(y \vee z) = \inf\{v_A(x)/\ x \in f^{-1}(y \wedge z)\}$$
$$= \inf\{v_A(u \vee v)/\ u \in f^{-1}(y),\ v \in f^{-1}(z)\}$$
$$= \inf\{\max\{v_A(u),\ v_A(v)\}\ /\ u \in f^{-1}(y),\ v \in f^{-1}(z)\} \text{ since A Intuitionistic fuzzy L-filter.}$$
$$= \max\{\inf f(v_A)(u)/u \in f^{-1}(y),\ \inf f(v_A)(v)\ \ /v \in f^{-1}(z)\}$$
$$= \max\{f(v_A)(y),\ f(v_A)(z)\}$$

And

$$
\begin{aligned}
f\left(v_A\right)(y \wedge z) &= \inf\{v_A(x)/\ x \in f^{-1}(y \wedge z)\} \\
&= \inf\{v_A(u \wedge v)/\ u \in f^{-1}(y),\ v \in f^{-1}(z)\} \\
&= \inf\{\min\{v_A(u),\ v_A(v)\}\ /\ u \in f^{-1}(y),\ v \in f^{-1}(z)\} \text{ since A Intuitionistic fuzzy L-filter.} \\
&= \min\{\inf f(v_A)(u)\ /u \in f^{-1}(y),\ \inf f(v_A)(v)\ \ /v \in f^{-1}(z)\} \\
&= \max\{f(v_A)\ (y),\ f(v_A)(z)\}
\end{aligned}
$$

Hence f(A) is an Intuitionistic fuzzy L-filter of $L'$. □

**Theorem 4.3.** *If $f:\ L \to L'$ is a lattice homomorphism and $A'$ is a n Intuitionistic fuzzy L-filter of $L'$,then $f^{-1}(A')$ is an Intuitionistic fuzzy L- filter of L.*

*Proof.* Let $A' = \{< y,\ \mu'_A(y),\ v'_A(y) > /y \in L'\}$ be an L-filter of $L'$. We have $f^{-1}(A') = \{x,\ f^{-1}(\mu'_A)(x),\ f^{-1}(v'_A)(x) > /x \in L\}$. Then for any $x,\ y \in\ L$, we have

$$
\begin{aligned}
f^{-1}(\mu_{A'})(x \vee y) &= (\mu_{A'})[f(x \vee y)] \\
&= (\mu_{A'})[f(x) \vee f(y)] \\
&= \min\{(\mu_{A'})f(x)),\ (\mu_{A'})f(y)\} \text{ Since } A' \text{ is an Intuitionistic fuzzy L-filter of } L'. \\
&= \min\{f^{-1}(\mu_{A'})(x),\ f^{-1}(\mu_{A'})(y)\}.
\end{aligned}
$$

and

$$
\begin{aligned}
f^{-1}(\mu_{A'})(x \wedge y) &= (\mu_{A'})[f(x \vee y)] \\
&= (\mu_{A'})[f(x) \wedge f(y)] \\
&\leq \max\{(\mu_{A'})f(x),\ (\mu_{A'})f(y)\} \text{ Since } A' \text{ is an Intuitionistic fuzzy L-filter of } L'. \\
&= \max\{f^{-1}(\mu_{A'})(x),\ f^{-1}(\mu'_A)(y)\}.
\end{aligned}
$$

Also

$$
\begin{aligned}
f^{-1}(v_{A'})(x \vee y) &= (v_{A'})[f(x \vee y)] \\
&= (v_{A'})[f(x) \vee f(y)] \\
&= \max\{(v_{A'})f(x)),\ (v_{A'})f(y)\} \text{ Since } A' \text{ is Intuitionistic fuzzy L-filter of } L'. \\
&= \max\{f^{-1}(v_{A'})(x),\ f^{-1}(v_{A'})(y)\}.
\end{aligned}
$$

And

$$
\begin{aligned}
f^{-1}(v_{A'})(x \wedge y) &= (v_{A'})[f(x \vee y)] \\
&= (v_{A'})[f(x) \vee f(y)] \\
&= \min\{(v_{A'})f(x)),\ (v_{A'})f(y)\},\ A' \text{ is an Intuitionistic fuzzy L-filter of } L'. \\
&= \min\{f^{-1}(v_{A'})(x),\ f^{-1}(v_{A'})(y)\}.
\end{aligned}
$$

Hence $f^{-1}(A')$ is an Intuitionistic fuzzy L-filter of L. □

**Theorem 4.4.** *If $f : L \to L'$ is an onto mapping and A and $A'$ are Intuitionistic fuzzy sets of the lattices L and $L'$, respectively. Then*

*(i).* $f[f^{-1}(A)] = A'$,

*(ii).* $A \subseteq f^{-1}[f(A)]$.

*Proof.*

(i). Let $y \in L'$. Then we have

$$\begin{aligned} f(f^{-1}(\mu_{A'})(y)) &= \sup\{f^{-1}(\mu_{A'})(x)/x \in f^{-1}(y)\} \\ &= \sup\{\mu_{A'}\ f(x)/x \in\ L,\ f(x) = y\} \\ &= \mu_{A'}(y). \end{aligned}$$

Since f is an onto mapping for every $y \in L'$ there exist $x \in L$ such that $f(x) = y$. Similarly $f(f^{-1}(v_{A'})(y)) = v_{A'}(y)$. Hence $f[f^{-1}(A)] = A'$.

(ii). Let $x \in L$. Then we have

$$\begin{aligned} f^{-1}(f(\mu_A))(x) &= f(\mu_A)(f(x)) \\ &= \sup\{\mu_A(x)/x \in f^{-1}(x)\} \\ &= \mu_A(x) \text{ and} \\ f^{-1}(f(v_A)(x) &= f(v_A)(f(x)) \\ &= \inf\{v_A(x)/x \in f^{-1}(f(x)\} \\ &= v_A(x). \end{aligned}$$

Hence $A \subseteq f^{-1}[f(A)]$.

□

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