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An Optimization Approach for Identification of Petri Nets

Research Article

V.Padma [1]* and K.Bhuvaneswari[2]

1 Research scholar, Department of Mathematics, Mother Teresa Women's University, Kodaikanal, India.

2 Head of Department, Department of Mathematics, Mother Teresa Women's University, Kodaikanal, India.

Abstract: In this paper we deal with the problem of identifying a Petri net system, given a finite language that it generates .The set of transitions and the number of places is assumed to be known. Identification problem consists in determining a Petri net system starting from example/counter example of its language. Our optimization approach requires the constraints whose set of unknowns contains the elements of the pre and post incidence matrices and the initial marking of the net. We continue to discuss the linear programming techniques for the identification of Petri nets and study their properties.

Keywords: Petri Net, Optimization, Identification, and Incidence matrix.

1. Introduction

Identification is a classical problem in system theory: given an observed behavior it consists in determining a system whose behavior approximate the observed one [1]. In the context of Petri nets having a set of P-places and T-Transitions, it is common to consider as observed behavior the language of the net i.e. the set of transition sequences that can be fired starting from the initial marking. Assume that a language $\mathcal{L} \subset T^*$ is given, where T is a given set of n transitions. Let this language be finite, prefix-closed and k an integer greater than (or) equal to the length of the longest string it contains.
The identification problem is that of determining the structure of a net $\mathbb{N}^{m\times n}$, i.e., the matrices Pre,$Post \in \mathbb{N}^{m\times n}$, and its initial marking $M_0 \in \mathbb{N}^{m\times n}$ such that the set of all firable transitions sequences of length less than (or) equal to k is denoted by $L_k(N, M_0) = \mathcal{L}$ [2]. Note that the $\mathcal{L}$ explicitly lists *positive examples*, i.e., strings that are known to belong to the language, but also, implicitly defines several counter examples, namely all those string of length less than (or) equal to k that do not belong to that language. Thus from the observed language one can construct a set of enabling constraints $\mathcal{E}$. i.e., set of pairs (y, t), such that transition t should be enabled after sequence σ has fired, where y is the firing vector of σ and a set of disabling constraints $\mathcal{D}$, i.e., set of pairs (y, t), such that transition t should not be enabled after sequence σ has fired, where y is the firing vector of σ. The number of variables grows exponentially with the length of the longest string in $\mathcal{L}$ and problems of this kind may easily become intractable.

2. Preliminaries

In this section we first recall the Petri net formalism used in the paper, referring to [3] for a comprehensive introduction to Petri nets. Then we define a special class of linear constraint sets and prove an important property of such a class that will

* *E-mail: padmagurumoorthy@gmail.com*

be useful in the solution of our identification problem.

2.1. Background on Petri nets [4]

A place/transition net (P/T net) is a structure $\mathrm{N} = (P, T, Pre, Post)$, where P is a set of m places; T is a set of n transitions; $Pre : P \times T \to \mathbb{N}$ and $Post : P \times T \to \mathbb{N}$ are the $Pre-$ and $Post-$ incidence functions that specify the arcs; $C = Post - Pre$ is the incidence matrix. A *marking* is a vector $M : P \times T \to \mathbb{N}$ that assigns to each place of a P/T net a nonnegative integer number of tokens, represented by black dots. We denote $M(p)$ the marking of place p. A P/T *system or net system* $\langle N, M_0\rangle$ is a net N with an initial marking M_0.

A transition t is enabled at M iff $M \geq Pre(\cdot, t)$ put dot before t and may fire yielding the marking $M' = M + C(\cdot, t) = M + C \cdot \vec{t}$ where $\vec{t} \in \mathbb{N}^n$ is a vector whose components are all equal to 0 except the component associated to transition t that is equal to 1. We write $M[\sigma\rangle$ to denote that the sequence of transitions σ is enabled at M, and we write $M[\sigma\rangle M'$ to denote that the firing of σ yields M. Note that in this paper we always assume that two or more transitions cannot simultaneously fire (non-concurrency hypothesis). A marking M is *reachable* in $\langle N, M_0\rangle$ iff there exists a firing sequence σ such that $M_0[\sigma\rangle M$. In such a case the state equation $M = M_0 + C \cdot \vec{\sigma}$ where $\vec{\sigma} \in \mathbb{N}^n$ is the *firing vector* of σ, i.e., the vector whose ith entry represents the number of times the transition t , is contained in σ. The set of all markings reachable from M_0 defines the *reachability set* of $\langle N, M_0\rangle$ and is denoted $R\langle N, M_0\rangle$. Given a Petri net system $\langle N, M_0\rangle$ we define its free-language as the set of its firing sequences.

$$L(N, M_0) = \{\sigma \in T^* | M_0[\sigma\rangle\}$$

We also define the set of firing sequences of length less than or equal to $k \in \mathbb{N}$ as:

$$L_k(N, M_0) = \{\sigma \in L(N, M_0) || \sigma| \leq k\}$$

Finally given a language $\mathcal{L} \subset T^*$ and a vector write this $y \in \mathbb{N}^n$ we denote

$$\mathcal{L}(y) = \{\sigma \in L | \vec{\sigma} = y\}$$

the set of all sequences in $\mathcal{L}$ whose firing vector is y.

2.2. Special constraint sets

We define a special class of linear constraint sets (CS).

Definition 2.1. *Given $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^{m \times n}$, consider the linear constraint set:*

$$C(A, b) = \{x \in \mathbb{R}^m | Ax \geq b\}$$

The set $C(A, b)$ is called:

- ***ideal**: if $x \in C(A, b)$ implies $\alpha x \in C(A, b)$ for all $\alpha \geq 1$*
- ***rational**: if $A \in \mathbb{Q}^{m \times n}$ and $b \in \mathbb{Q}^m$, i.e., if the entries of matrix A and of vector b are rational.*

The following result provides a simple characterization of ideal CSs.

Proposition 2.2. *A linear constraint set $C(A, b)$ is ideal if $b \geq 0$.*

Proof. Since $Ax \geq b \geq 0$ then for all $\alpha \geq 1$ it holds $A(\alpha x) \geq Ax \geq b$, hence it is ideal. □

Proposition 2.3. *If a CS is ideal and rational, then it has a feasible solution if and only if it has a feasible integer solution.*

Proof. The if part is trivial. To prove the only if part, we reason as follows. If there exists a solution there exists a basis solution x_B , i.e., such that

$$x_B = A_B^{-1} b$$

where A_B is obtained by A selecting a set of basis columns. If the CS is rational the entries of A_B and b are rational, hence the entries of A_B^{-1} and of x_B are rational as well. If the CS is ideal, we just need to multiply the rational vector x_B by a suitable positive integer to obtain an integer solution. □

3. P/T Net Identification

The problem we consider in this paper can be formally stated as follows.

Example 3.1. *Let $\mathcal{L} \subset T^*$ be a finite prefix-closed language, and $k \geq max|\sigma|$; $\sigma \in \mathcal{L}$; be an integer greater than or equal to the length of the longest string in $\mathcal{L}$. We want to identify the structure of a net $N = (P, T, Pre, Post)$ and an initial marking M_0 such that $L_k(N, M_0) = \mathcal{L}$. The unknowns we want to determine are the elements of the two matrices Pre, $Post \in \mathbb{N}^{m\times n}$ and the elements of the vector $M_0 \in \mathbb{N}^m$ Associated to an identification problem are the two sets defined in the following.*

Definition 3.2. *Let $\mathcal{L} \subset T^*$ be a finite prefix-closed language and let $k \in \mathbb{N}$ be defined as in Example 3.1. We define the set of enabling conditions*

$$\mathcal{E} = \{(y, t)|(\exists\sigma \in \mathcal{L}) : |\sigma| < k, \sigma \in \mathcal{L}(y), \sigma t \in \mathcal{L}\} \leq \mathbb{N}^n \times T \tag{1}$$

and the set of disabling conditions

$$\mathcal{D} = \{(y, t)|(\exists\sigma \in \mathcal{L}) : |\sigma| < k, \sigma \notin \mathcal{L}(y), \sigma t \in \mathcal{L}\} \leq \mathbb{N}^n \times T \tag{2}$$

Clearly, a solution to Example 3.1 is a net $\langle N,\ M_0\rangle$ such that:

- *for all $(y, t) \in \mathcal{E}$ transition t is enabled after the firing of all $\sigma \in \mathcal{L}(y)$, i.e., $M_0[\sigma\rangle M_y[t\rangle$, where $M_y = M_0 + C \cdot y$ represents the marking reached after the firing of sequence σ.*
- *for all $(y, t) \in \mathcal{D}$ transition t is disabled after the firing of $\sigma \in \mathcal{L}(y)$, i.e., $M_0[\sigma\rangle M_y\neg[t\rangle$. We can characterize the number of places required to solve our identification problem.*

Definition 3.3. *Let as $\mathcal{L}$ be a finite prefix-closed language on alphabet T, whose words have length less than or equal to k. Given the set of disabling conditions (2) let $m_\mathcal{D} = |\mathcal{D}|$. We say that a Petri net system $\langle N, M_0\rangle$ with set of places P and $L_k(N, M_0) = \mathcal{L}$, is $\mathcal{D}-canonical$ if*

1). $|P| = m_\mathcal{D}$;

2). There exists a bijective mapping $h : \mathcal{D} \to P$ such that, for all $(y, t) \in \mathcal{D}$, place $p = h(y, t)$ satisfies

$$M_y(p) \triangleq M_0(p) + C(p, \cdot) \cdot y < Pre(p, t)$$

i.e., place p disables t after any $\sigma \in \mathcal{L}(y)$.

In simple words, a net system $\langle N, M_0\rangle$ is $\mathcal{D}-$canonical if a different place is associated to each element in the set of disabling constraints $\mathcal{D}$.

Theorem 3.4. *Let us consider a finite prefix-closed language on alphabet T, whose words have length less than or equal to k and let $\mathcal{E}$ and $\mathcal{D}$ be the corresponding sets of enabling and disabling conditions [2]. Let*

$$\mathcal{N}(\mathcal{E},\mathcal{D}) \triangleq \begin{cases} M_0 + Post \cdot y & \\ -Pre \cdot (y + \vec{t}) \geq 0 & \forall (y,t) \in \mathcal{E} \\ M_0(p_{(y,t)}) + Post(p_{(y,t)}, \cdot) \cdot y & \\ -Pre(p_{(y,t)}, \cdot) \cdot (y + \vec{t}) \leq -1 & \forall (y,t) \in \mathcal{D} \\ M_0 \in \mathbb{R}^{m_{\mathcal{D}}}_{\leq 0} & \\ Pre, Post \in \mathbb{R}^{m_{\mathcal{D}}}_{\leq 0} & \end{cases}$$

Consider a net system $\langle N, M_0\rangle$ with $N = (P, T, Pre, Post)$. the system $\langle N, M_0\rangle$ is $\mathcal{D}-$canonical solution of the identification Example 3.1 iff $Pre, Post, M_0$ are integer solutions of CS (3).

Proof. We first show that any integer solution $\langle N, M_0\rangle$ of CS (3) is a solution of Example 3.1

- Any constraint $M_0 + Post \cdot y - Pre \cdot (y + \vec{t}) \geq 0$ can be rewritten as $M_y = M_0 + (Post - Pre) \cdot y \geq Pre(\cdot, t)$ or equivalently $M_y \geq Pre(\cdot, t)$ where $M_0[\sigma\rangle M_y$ for all $\sigma \in \mathcal{L}(y)$ for all. This shows that transition t is enabled on $\langle N, M_0\rangle$ from marking M_y and by induction on the length of σ (since language $\mathcal{L}$ is prefix-closed) we conclude that $\sigma t \in \mathcal{L}$.

- Assume that sequence $\sigma \in \mathcal{L}$ is firable on the net and $M_0[\sigma\rangle M_y$. If for at least a place p in the net it holds $M_0(p) + Post(p, \cdot) \cdot y - Pre(p, \cdot) \cdot (y + \vec{t}) \leq -1$, then $M_y = M_0 + (Post - Pre) \cdot y \ngeq Pre(\cdot, t)$ or equivalently $M_y \ngeq Pre(\cdot, t)$. This shows that transition t is not enabled on (N, M_0) from marking M_y and we conclude that $\sigma t \notin \mathcal{L}$.

Since net $\langle N, M_0\rangle$ satisfies all enabling and disabling constraints,$L_k(N, M_0) = \mathcal{L}$. We now show that any solution of CS (3) is $\mathcal{D}-$ canonical. In fact, the mapping $h(y,t) = p_{(y,t)}$ for each couple $(y,t) \in \mathcal{D}$ is bijective.
We now show that any $\mathcal{D}-$canonical net system $\langle N, M_0\rangle$ with $L_k(N, M_0) = \mathcal{L}$ is a solution of CS (3). In fact, let $h : \mathcal{D} \to P$ be the bijective function of the net system. If we define $p_{(y,t)} = h(y,t)$ for all $(y,t) \in \mathcal{D}$, then all equations in CS (3) are satisfied. □

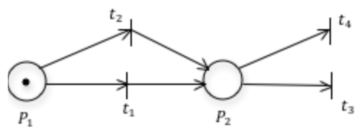
Proposition 3.5. *The linear CS (3) is ideal and rational follows from Proposition 2.2. The following theorem provides a practical and efficient procedure to solve our identification problem.*

Theorem 3.6. *The identification Example 3.1 admits a solution if the (linear) CS(3) is feasible.*

Proof. If By Proposition 2.2 CS (3) is ideal and rational thus by Proposition 2.3 if it has solutions, then it also has feasible integer solutions. However, by Theorem 3.4 this also implies that such integer solutions are also solutions of the identification in Example 3.1 once a solution of CS (3) is found, if this solution is rational we can always find an integer solution by simply multiplying $Pre, Post$ and M_0 by a suitable $\alpha \geq 1$. We illustrate the above concept with the following example. □

Example 3.7. *Let us consider the following example for the identification problem. Let $f(M_0, Pre, Post)$ can be a given performance index. The solution to the identification problem that minimizes $f(M_0, Pre, Post)$ can be computed by solving the IPP problem*

$$\begin{cases} f(M_0, Pre, Post) \\ \quad s.t.\ \mathcal{N}(\mathcal{E}, \mathcal{D}) \end{cases}$$



For the above Petri net we have

$$P = \{p_1, p_2\} \qquad T = \{t_1, t_2, t_3, t_4\}$$

$$Pre = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad Post = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

$$M_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 2 & 1 & -1 & -1 \end{bmatrix}$$

Let us consider a language $\mathcal{L} = \{\mathcal{E}, t_1, t_2, t_1t_3, t_1t_4, t_2t_3, t_2t_4\}$ and let $k = 2$. We have additional information: the transitions t_1 and t_2 have only one input place and transitions t_3 and t_4 are in a free choice relation. The set of enabling constraints and disabling constraints are respectively.

$$\mathcal{E} = \{(\mathcal{E}, t_1), (\mathcal{E}, t_2), (t_1, t_3), (t_1, t_4), (t_2, t_3), (t_2, t_4)\} \text{ and}$$
$$\mathcal{D} = \{(\mathcal{E}, t_3), (\mathcal{E}, t_4), (t_1, t_1), (t_2, t_1), (t_3, t_1), (t_4, t_1), (t_2, t_2), (t_3, t_2), (t_4, t_2)\}$$

$m = 2$, $k = 2$, $n = 4$ and $L_2(N, M_0) = \mathcal{L}$. A solution has been determined associating a linear objective function $f(M_0, Pre, Post)$ to $N(\mathcal{E}, D)$ and solving the resulting linear programming problem. In particular, by assuming $f(M_0, Pre, Post) = 1^T \cdot M_0 + 1^T \cdot Pre \cdot 1 + 1^T \cdot Post \cdot 1$ gives the solution as integer.

4. Conclusion

We have presented a procedure to identify a Petri net from a finite prefix of a language. The procedure is based on constraints, which is used in optimizing the linear programming problem. The number of places of resulting net may not be minimal, but we will discuss the techniques that maybe used to reduce it. As a possible line for future research, we extend this discussion to linear programming techniques for the identification of Petri nets [5].

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