International Journal of Mathematics And its Applications
Volume 4, Issue 2–D (2016), 29–33.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Packing Parameters of Bloom Graph

Research Article

D.Antony Xavier[1]*, S.Kulandai Therese[1,2], Andrew Arokiaraj[1] and Elizabeth Thomas[1]

1 Department of Mathematics, Loyola College, Chennai, Tamilnadu, India.

2 Department of Mathematics, St.Mary's College, Thoothukudi, Tamilnadu, India.

Abstract: Packing in graphs is an effective tool as it has lots of applications in applied sciences. In this paper, we determine the maximum H-packing for bloom graph in algorithmic manner with distinct vertices when H is isomorphic to P_3. Moreover we discuss the maximum F-packing for bloom graph with distinct vertices and disjoint edges when F is isomorphic to a family of cycles.

Keywords: Interconnection networks, Bloom graph, H-packing, F-packing.

1. Introduction

Packing is one of the most extensively studied problem in computer science, mainly due to its combinatorial aspects and algorithmic implementations. The theory of packing has wide applications in the field of scheduling, wireless sensor tracking, wiring-board design, code optimization and many others [1, 13]. The packing problem is also used in dynamic channel assignment for cellular radio communication systems [7]. In addition to this, academic researchers work to create tiny circuits using nodes that automatically arrange themselves into useful patterns [5].

An Hpacking of a graph $G(V,E)$ is a set of vertex(edge) disjoint subgraphs of G, each of which is isomorphic to a fixed graph H [9].The maximum cardinality of an H-packing with vertices(edges) is denoted by $\lambda_v(G,H)$ $(\lambda_e(G,H))$. An H-packing is perfect if it covers all the vertices(edges) of G. If H is the complete graph K_2, the vertex-packing problem becomes the maximum matching problem. The study of packing problems has a rich history. Kirkpatrick and Hell proved that the maximum H-packing problem is NP-complete if H is a connected graph with at least three vertices [6]. Felzenbaum et al. studied packing lines in a hypercube. Algorithms were developed for packing almost stars into the complete graph and dense packing of trees [2, 4, 12].

An F-packing is a natural generalization of H-packing concept. For a given family F of graphs, the problem is to identify a set of vertex(edge)-disjoint sub graphs of G, each isomorphic to a member of F. The F-packing problem is to find an F-packing in a graph G that covers the maximum number of vertices(edges) of G. For sufficiently large n, it is possible to pack hypercube with its edge-disjoint copies of G [8]. E.Dobson proved that sequence of trees may be packed into K_{2n+1} [3].

* *E-mail: antonyxavierlc@gmail.com*

A new interconnection network topology called the bloom graph has been recently introduced by Xavier et al. [10, 11]. The bloom graph $B_{m,m}$ is identical with $P_m \times C_m$ as cylinder in structure. $B_{m,m}$ yields better average distance, message traffic density, network cost, graph density, density, total connectivity and network throughput. In this paper we determine the maximum P_3-packing number of bloom graph and F-packing number of bloom graph with cycles.

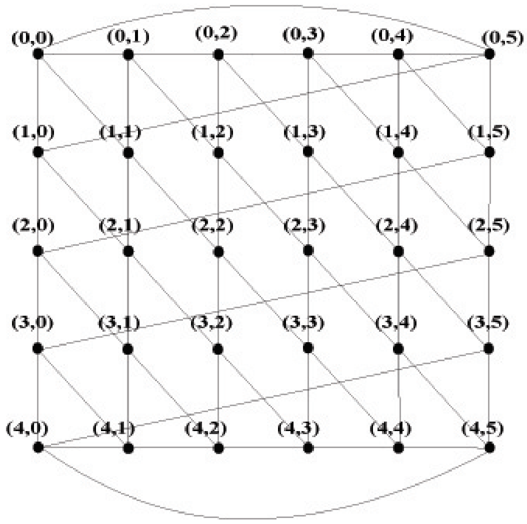

Figure 1. **Bloom graph** $B_{5,6}$

2. Bloom Graph

Definition 2.1 ([11]). *The bloom graph denoted by* $B_{m,n}$; m ,$n > 2$ *consists of vertex set* $V(B_{m,n}) = \{(x,y) : 0 \le x \le m-1, 0 \le y \le n-1\}$, *two distinct vertices* (x_1, y_1) *and* (x_2, y_2) *being adjacent if and only if*

(i). $x_2 = x_1 + 1$ *and* $y_1 = y_2$

(ii). $x_1 = x_2 = 0$ *and* $y_1 + 1 \equiv y_2 \,(mod\, n)$

(iii). $x_1 = x_2 = m-1$ *and* $y_1 + 1 \equiv y_2 \,(mod\, n)$

(iv). $x_2 = x_1 + 1$ *and* $y_1 + 1 \equiv y_2 \,(mod\, n)$

The first condition describes the vertical edges, the second and third condition describe the horizontal edges in the top most and the lower most rows respectively. Condition four describes the slant edges (See Figure 1). Bloom graph has mn vertices and $2mn$ *edges. The vertex connectivity and the edge connectivity of Bloom graph is 4. Bloom graph is planar, 4-regular and tripartite.The diameter of* $B_{m,n} = \begin{cases} m-1, & \text{if } m > n; \\ \lceil (m+n)/2 \rceil, & \text{if } m \le n; \end{cases}$

2.1. Construction of Layers in Bloom Graph

Bloom graph $B_{m,n}$ has m rows and n columns namely R_i and C_j , $\forall i, j \; 0 \le i \le m-1, 0 \le j \le \; n-1$. The edges of bloom graph $B_{m,n}$ are partitioned into m+1 layers denoted by L_i, where $0 \le i \le m$, defined as follows:

(i). The layer L_0 is the cycle $\{((0,j)\,,(0, j \oplus_n 1))\,,\ \forall j \; 0 \le j \le n-1\}$.

(ii). The intermediate layers L_{i+1}, $\forall i \quad 0 \leq i \leq m-2$ consists of the edge set $\{((i,j)\ ,\ (i+1,j)), ((i,j)\ ,\ (i+1, j \oplus_n 1), \forall j\ 0 \leq j \leq n-1)\}$. (i.e) The cycle of length 2n formed by the vertices of two consecutive rows $R_{i-1} \cup R_i$ of $B_{m,n}$ is known as intermediate layer L_i, $\forall i\ 1 \leq i \leq m-1$.

(iii). The layer L_m is the cycle $\{((m-1,j)\ ,(m-1,j \oplus_n 1))\ ,\ \forall j\ 0 \leq j \leq n-1\}$.

Definition 2.2. *Petal layer PL_k is the subgraph induced by $L_k \cup L_{k+1}, \forall k\ 0 \leq k \leq m-1$.*

Lemma 2.3 ([14]). *If G(V,E) is a graph,then $\lambda_v(G,H) \leq \frac{|V(G)|}{|V(H)|}$, and that $\lambda_e(G,H) \leq \frac{|E(G)|}{|E(H)|}$. When equality arises in $\lambda_v(G,H)$ $(\lambda_e(G,H))$, G has an H-factor (H-decomposition) .*

3. H-packing of Bloom Graph

In this section we study the H-packing of $B_{m,n}$ when H is isomorphic to P_3.

Theorem 3.1. *Let $B_{m,n}$ be a bloom graph, where m,n $\geq$ 3 and $H \simeq P_3$. If $m \equiv 0\,(mod\ 3)$ or $n \equiv 0\,(mod\ 3)$, then $B_{m,n}$ has a perfect H-packing. Otherwise, it has a near perfect H-packing.*

Proof.

Case 1: When $m \equiv 0(mod\ 3)$

The packing $\{(3i,j) \longrightarrow (3i+1,j) \longrightarrow (3i+2,j),\ \forall i\ ,\ j\ \ 0 \leq i \leq \lfloor m/3 \rfloor - 1,\ 0 \leq j \leq n-1\}$ splits the vertices of $B_{m,n}$ with P_3. Since m is a multiple of 3, $\frac{mn}{3}$ number of P_3-paths define perfect H-packing.

Case 2: When $m \equiv 1(mod\ 3)$

a) When $n \equiv 0(mod\ 3)$

The vertices of rows $R_i, \forall i\ \ 0 \leq i \leq m-2$ are perfectly H-packed using case 1. The packing $\{(m-1,3j) \longrightarrow (m-1,3j+1) \longrightarrow (m-1,3j+2),\ \forall j\ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ splits row R_{m-1} into $\frac{n}{3}$ number of P_3-paths. Hence perfect H-packing is obtained.

b) When $n \equiv 1(mod\ 3)$

The vertices of rows $R_i, \forall i\ \ 0 \leq i \leq m-2$ are perfectly H-packed using case 1. The packing $\{(m-1,3j) \longrightarrow (m-1,3j+1) \longrightarrow (m-1,3j+2),\ \forall j\ \ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ splits row R_{m-1} into $\frac{n-1}{3}$ number of P_3 paths. There is an isolated vertex $(m-1,n-1)$. Hence near perfect P_3-packing occurs.

c) When $n \equiv 2(mod\ 3)$

The vertices of rows $R_i, \forall i\ \ 0 \leq i \leq m-2$ are perfectly H-packed using case 1. The packing $\{(m-1,3j) \longrightarrow (m-1,3j+1) \longrightarrow (m-1,3j+2),\ \forall j\ \ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ splits row R_{m-1} into $\frac{n-1}{3}$ number of P_3 paths. There are two isolated vertices $(m-1,n-2)$ and $(m-1,n-1)$. Hence near perfect P_3-packing occurs.

Case 3: When $m \equiv 2(mod\ 3)$

a) When $n \equiv 0(mod\ 3)$

The vertices of rows $R_i, \forall i\, 0 \leq i \leq m-3$ are perfectly H-packed using case 1. The packings $\{(m-1,3j) \longrightarrow (m-2,3j) \longrightarrow (m-1,3j+1),\ \forall j\ \ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ and $\{(m-2,3j+1) \longrightarrow (m-1,3j+2) \longrightarrow (m-2,3j+2),\ \forall j\ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ split rows R_i, where $i = m-2, m-1$ into $\frac{2n}{3}$ number of P_3-paths. Hence perfect H-packing is obtained.

b) When $n \equiv 1(mod\ 3)$

The vertices of rows $R_i, \forall i\ \ 0 \leq i \leq m-3$ are perfectly H-packed using case 1. The packings $\{(m-1,3j) \longrightarrow (m-2,3j) \longrightarrow (m-1,3j+1),\ \forall j\ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ and $\{(m-2,3j+1) \longrightarrow (m-1,3j+2) \longrightarrow (m-2,3j+2),\ \forall j\ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor - 1\}$ split rows R_i , where $i = m-2, m-1$ into $\frac{2(n-1)}{3}$ number of P_3-paths.Two vertices $(m-2,n-1)$ and $(m-1,n-1)$ are left out as isolated vertices.

c) When $n \equiv 2 (mod\ 3)$

The vertices of rows $R_i, \forall i \ 0 \leq i \leq m-3$ are perfectly H-packed using case 1. The packings $\{(m-1,3j) \longrightarrow (m-2,3j) \longrightarrow (m-1,3j+1),\ \forall j \ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor -1\}$ and $\{(m-2,3j+1) \longrightarrow (m-1,3j+2) \longrightarrow (m-2,3j+2),\ \forall j \ 0 \leq j \leq \lfloor \frac{n}{3} \rfloor -1\}$ split rows R_i, where $i = m-2, m-1$ into $\frac{2(n-2)}{3}$ number of P_3-paths. A vertex $(m-2,n-1)$ is left out as an isolated vertex.

Thus in all the above cases, there exist a perfect or near perfect H-packing of $B_{m,n}$ with $\lfloor |V(B_{m,n})| / |P_3| \rfloor$. Hence, $\lambda_v (B_{m,n}, H) = \lfloor mn/3 \rfloor$ □

4. F-Packing of Bloom Graph

In the following theorems, we consider a family F of subgraphs which packs the bloom graph $B_{m,n}$ with disjoint edges.

Theorem 4.1. *Let G be a Bloom Graph $B_{m,n}$, where m is odd and $m,n \geq 3$. If $H_1 \backsimeq C_4$, $H_2 \backsimeq C_3$ and $F = \{H_1, H_2\}$, then there exists an F-factor of $B_{m,n}$.*

Proof. The petal layer PL_0 saturates the edges of $\{(0,j) \longrightarrow (0, j \oplus_n 1) \longrightarrow (1, j \oplus_n 1) \longrightarrow (0,j), \forall j \ 0 \leq j \leq n-1\}$. The petal layer PL_{m-1} saturates the edges of $\{(m-2,j) \longrightarrow (m-1,j) \longrightarrow (m-1, j \oplus_n 1) \longrightarrow (m-2,j), \forall j \ 0 \leq j \leq n-1\}$. Hence PL_0 and PL_{m-1} pack $2n$ cycles of length three, which pack $6n$ edges.

Moreover, the intermediate petal layers $PL_{2i}, \forall i \ 1 \leq i \leq m-3/2$ saturate the edges of $\{(2i-1,j) \longrightarrow (2i,j) \longrightarrow (2i+1, j \oplus_n 1) \longrightarrow (2i, j \oplus_n 1) \longrightarrow (2i-1,j), \forall j \ 0 \leq j \leq n-1\}$ which pack $\left(\frac{m-3}{2}\right) n$ cycles of length four. Hence $2(m-3)n$ edges are packed by intermediate petal layers. □

Theorem 4.2. *Let G be a Bloom Graph $B_{m,n}$, where m is even and $m,n \geq 3$. If $H_1 \backsimeq C_4$, $H_2 \backsimeq C_3$, $H_3 \backsimeq C_n$ and $F = \{H_1, H_2, H_3\}$, then there exists an F-factor of $B_{m,n}$.*

Proof. The intermediate petal layers $PL_{2i}, \forall i \ 1 \leq i \leq \left(\frac{m}{2}-1\right)$ saturate the edges of $\{(2i-1,j) \longrightarrow (2i,j) \longrightarrow (2i+1, j \oplus_n 1) \longrightarrow (2i, j \oplus_n 1) \longrightarrow (2i-1,j), \forall j \ 0 \leq j \leq n-1\}$ which pack $n\left(\frac{m}{2}-1\right)$ copies of cycles of length four. (i.e) $PL_{2i}, \forall i \ 1 \leq i \leq \left(\frac{m}{2}-1\right)$ pack $2(m-2)n$ edges. The petal layer PL_0 saturates the edges of $\{(0,j) \longrightarrow (0, j \oplus_n 1) \longrightarrow (1, j \oplus_n 1) \longrightarrow (0,j), \forall j \ 0 \leq j \leq n-1\}$, packing n copies of cycles of length three. (i.e) PL_0 packs 3n edges. The bottom layer L_m saturates the edges of $\{(m-1,0) \longrightarrow (m-1,1) \ldots (m-1,n-1) \longrightarrow (m-1,0), \forall j \ 0 \leq j \leq n-1\}$, which is a cycle of length n. (i.e) L_m packs n edges. Since F packs $2mn$ edges, $B_{m,n}$ has a perfect F-packing. □

In the following theorem, we consider a family F of subgraphs which packs the bloom graph $B_{m,n}$ with distinct vertices.

Theorem 4.3. *Let G be a Bloom Graph $B_{m,n}$, where $m,n \geq 3$. If $H_1 \backsimeq C_{2n}$, $H_2 \backsimeq C_n$ and $F = \{H_1, H_2\}$, then there exists a F-factor of $B_{m,n}$.*

Proof.

Case 1: When m is odd.

The packing $\left\{\{(2i-1,j), (2i,j)\} \cup \{(2i-1,j), (2i, j \oplus_n 1)\}, \forall i,\ j \ 1 \leq i \leq \frac{m-1}{2}, 0 \leq j \leq n-1\right\}$ saturates vertices of $\frac{m-1}{2}$ copies of cycles of length $2n$. (i.e) It packs $n(m-1)$ vertices of intermediate layers $L_{2i}, \forall i \ 1 \leq i \leq \left(\frac{m-1}{2}\right)$. Moreover the top layer L_0 saturates the vertices of $\{(0,j), \forall j \ 0 \leq j \leq n-1\}$ which packs a cycle of length n. It packs n vertices. Altogether $\frac{m-1}{2}$ copies of cycles of length $2n$ and a cycle of length n perfectly pack $B_{m,n}$.

Case 2: When m is even.

The packing $\left\{\{(2i,j), (2i+1,j)\} \cup \{(2i,j), (2i+1, j \oplus_n 1)\}, \forall i,\ j \ 0 \leq i \leq \frac{m}{2}-1,\ 0 \leq j \leq n-1\right\}$ saturates vertices of $\frac{m}{2}$

copies of cycles of length $2n$. (i.e) The vertices of intermediate layers $L_{2i+1}, \forall i \; 0 \leq i \leq \left(\frac{m}{2} - 1\right)$ perfectly pack $B_{m,n}$.

□

5. Conclusion

In this paper we have determined the vertex disjoint H-packing of $B_{m,n}$, when $H \simeq P_3$. F-packing using cycles are determined separately to pack edges as well as to pack vertices. Packing of certain chemical graphs and interconnection networks are under study.

References

[1] R.Bejar, B.Krishnamachari, C.Gomes and B.Selman, *Distributed constraint satisfaction in a wireless sensor tracking system*, Workshop on Distributed Constraint Reasoning, International Joint Conference on Artificial Intelligence, (2001).

[2] E.Dobson, *Packing almost stars into the complete graph*, J. Graph Theory, 10(1997), 169-172.

[3] E.Dobson, *Packing Trees into the Complete Graph*, Combinatorics, Probability and Computing, 11(2002), 263-272.

[4] A.Felzenbaum, R.Holzman and D.J.Kleitman, *Packing lines in a hypercube*, Discrete Mathematics, 117(1993), 107-112.

[5] L.Hardesty, *Self-assembling computer chips*, MIT News Office, Cambridge, MA, 2010, (Posted on 16 March 2010).

[6] P.Hell and D.Kirkpatrick, *On the complexity of a generalized matching problem*, Proceedings of Tenth ACM Symposium On Theory of Computing, (1978), 309-318.

[7] J.Kind, T.Niessen and R.Mathar, *Theory of maximum packing and related channel assignment strategies for cellular radio networks*, Mathematical Methods of Operations Research, 48(1)(1997), 1-16.

[8] D.Offner, *Packing the hypercube*, Discussiones Mathematicae Graph Theory, 34(1)(2014), 85-93.

[9] I.Rajasingh, A.Muthumalai, R.Bharati and A.S.Shanthi, *Packing in honeycomb networks*, Journal of Mathematical Chemistry, 50(5)(2012), 1200-1209.

[10] D.A.Xavier and C.J.Deeni, *Bloom Graph*, International Journal of Computing Algorithm, 48(1)(1997), 1-16.

[11] D.A.Xavier, M.Rosary, E.Thomas and A.Arokiaraj, *Broadcasting in Bloom Graph*, International Journal of Mathematics and soft computing, 6(2)(2016), 57-64 .

[12] H.P.Yap, *Packing of graphsa survey*, Discrete Mathematics, 72(1988), 395-404.

[13] R.B.Yehuda, M. Halldorsson, J.Naor, H.Shachnai and I.Shapira, *Scheduling split intervals*, Proceedings of Thirteenth Annual ACM-SIAM Symposium On Discrete Algorithms, (2002), 732-741.

[14] R.Yuster, *Combinatorial and computational aspects of graph packing and graph decomposition*, Computerscience Review, 1(2007), 12-26.