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# Nano Regular Generalized Star $b$-continuous Functions in Nano Topological Spaces

**Research Article**

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**Abstract:** In this paper, we introduce and investigate the notions of nano regular generalized star $b$ continuous functions in terms of nano regular generalized star $b$ closed sets in nano topological spaces.

**Keywords:** Nano topology, $Nr$-continuous, $Ng^*$-continuous, $Nrg^*$-continuous, $Nrg^*b$-continuous, $Nrg^*b$-open and closed function.


## 1. Introduction

The concept of nano topology was introduced by Lellis Thivagar [7] in the year 2013, which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined a nano continuous functions, nano open mappings, nano closed mappings and nano homeomorphisms and their representations in terms of nano closure and nano interior. In this paper we introduced a new class of nano generalized closed sets and nano regular generalized star $b$ continuous, nano regular generalized star $b$ open and closed function in nano topological spaces.

## 2. Preliminaries

**Definition 2.1** ([7]). *Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation. Then $U$ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.*

*(1). The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and it is denoted by $L_R(X)$. i.e., $L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.*

*(2). The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $U_R(X)$. i.e.,$U_R(X) = \cup_{x\in U}\{R(x) : R(x) \bigcap X \neq \phi\}$.*

*(3). The boundary region of $X$ with respect to $R$ is the set of all objects, which can be classified neither as $X$ nor as not-$X$ with respect to $R$ and it is denoted by $B_R(X)$. i.e.,$B_R(X) = U_R(X) - L_R(X)$.*

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**Property 2.2** ([7]). *IF $(U, R)$ is an approximation space and $X, Y \subseteq U$, then*

*(1).* $L_R(X) \subseteq X \subseteq U_R(X)$

*(2).* $L_R(\phi) = U_R(\phi) = (\phi)$ *and* $L_R(U) = U_R(U) = U$

*(3).* $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

*(4).* $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$

*(5).* $L_R(X \cup Y) = L_R(X) \cup L_R(Y)$

*(6).* $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

*(7).* $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$

*(8).* $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$

*(9).* $U_R U_R(X) = L_R U_R(X) = U_R(X)$

*(10).* $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

**Definition 2.3** ([7]). *Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by Property 2.1, $\tau_R(X)$ satisfies the following axioms:*

*(1). $U$ and $\phi \in \tau_R(X)$.*

*(2). The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

*(3). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$. i.e.,$\tau_R(X)$ is a topological space with nano topology on $U$ with respect to $X$. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets.*

**Remark 2.4** ([8]). *If $\tau_R(X)$ is the nano topology on $U$ with respect to $X$, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

**Definition 2.5** ([8]). *IF $(U, \tau_R(X))$ is a nano topological space with respect to $X$ where $X \subseteq U$ and if $A \subseteq U$, Then the nano interior of $A$ is defined as the union of all nano open subsets of $A$ and it is denoted by $Nint(A)$. i.e.,$Nint(A)$ is the largest nano open subset of $A$. The nano closure of $A$ is defined as the intersection of all nano closed sets containing $A$ and it is denoted by $Ncl(A)$. i.e.,$Ncl(A)$ is the smallest nano closed set containing $A$.*

**Definition 2.6** ([8]). *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then $A$ is said to be*

*(1). Nano semi open if $A \subseteq Ncl(NInt(A))$*

*(2). Nano pre open if $A \subseteq NInt(Ncl(A))$*

*(3). Nano $\alpha$ open if $A \subseteq NInt(Ncl(NInt(A)))$*

*(4). Nano regular open if $A = NInt(Ncl(A))$ $NSO(U, X), NPO(U, X)$, $N\alpha O(U, X)$ and $NRO(U, X)$ respectively, denote the families of all Nano semi open, nano pre open and nano $\alpha$-open and nano regular open subsets of $U$.*

*Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$, $A$ is said to be nano semi-closed, nano pre closed, nano $\alpha$-closed and nano regular closed if its complement is respectively nano semi open, nano pre open, nano alpha open and nano regular open.*

**Definition 2.7** ([4]). *A subset $A$ of $(U, \tau_R(X))$ is called*

*(1). Nano b-closed set (briefly $Nb$-closed) if $Ncl(NInt(A)) \bigcap NInt(Ncl(A)) \subseteq A$.*

*(2). Nano generalized closed set (briefly $Ng$-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano open in $(U, \tau_R(X))$.*

*(3). Nano generalized b-closed set (briefly $Ngb$-closed) if $Nbcl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano open in $(U, \tau_R(X))$.*

*(4). Nano regular generalized b-closed set (briefly $Nrgb$-closed) if $Nbcl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano regular open in $(U, \tau_R(X))$.*

**Definition 2.8** ([12]). *Let $(U, \tau_R(X))$ be a nano topological space. A subset $A$ of $(U, \tau_R(X))$ is called a nano generalized star closed set (briefly $Ng^*$- closed), if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is nano g-open.*

**Definition 2.9** ([13]). *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then $A$ is said to be nano regular generalized star closed set (briefly $Nrg^*$-closed) if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is nano regular open in $(U, \tau_R(X))$. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$, $A$ is said to be nano regular generalized star open set (briefly $Nrg^*$-open) if its complement is $Nrg^*$-closed.*

**Definition 2.10** ([13]). *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then $A$ is said to be nano regular generalized star b-closed set (briefly $Nrg^*b$-closed) if $Nbcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $Nrg^*$-open in $(U, \tau_R(X))$. $Nrg^*O(U, X)$ denotes the family of all nano $rg^*$-open subsets of $U$.*

**Definition 2.11** ([7]). *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano continuous on $U$ if the inverse image of every nano open set in $V$ is nano open in $U$.*

## 3. Nano Regular Generalized Star $b$-Continuous in Nano Topological Spaces

**Definition 3.1.** *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano regular generalized star continuous (briefly $Nrg^*$-closed) if $f^{-1}(S)$ is $Nrg^*$-open (resp.$Nrg^*$-closed) in $(U, \tau_R(X))$for every nano open set (resp.Nano closed set) $S$ in $(V, \tau_{R'}(Y))$.*

**Definition 3.2.** *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is nano regular generalized star b continuous (briefly $Nrg^*b$-closed) if $f^{-1}(S)$ is $Nrg^*b$-open (resp.$Nrg^*b$-closed) in $(U, \tau_R(X))$for every nano open set (resp.Nano closed set) $S$ in $(V, \tau_{R'}(Y))$.*

**Theorem 3.3.** *Let $U$ and $V$ are any two nano topological spaces. If $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ $f$ is nano continuous function, then $f$ is $Nrg^*b$-continuous, but not conversely.*

*Proof.* Let S be any nano closed set in $(V, \tau_{R'}(Y))$, Since [13] every nano closed set is $Nrg^*b$-closed, we have $f^{-1}(S)$ is $Nrg^*b$-closed in $(U, \tau_R(X))$. Therefore, $f$ is $Nrg^*b$ continuous. □

The converse of the theorem need not be true as seen from the following example.

**Example 3.4.** *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{w\}, \{y, z\}\}$ and $X = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$. Define a mapping $f : U \to V$ as $f(a) = z; f(b) = w; f(c) = x; f(d) = y$, Then $f$ is $Nrg^*b$-continuous but not nano continuous, as the inverse image of a nano closed set $\{a, c, d\}$ in $V$ is $\{x, y, z\}$ is which is not nano closed in $U$.*

**Theorem 3.5.** *Let $U$ and $V$ are any two nano topological spaces. If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ $f$ is $Nrg^*b$-continuous function, then $f$ is $Ngb$-continuous, but not conversely.*

*Proof.* Let S be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(S)$ is $Nrg^*b$-closed in $(U, \tau_R(X))$ as f is $Nrg^*b$-continuous. Since [13] every $Nrg^*b$ closed set is $Ngb$-closed. we have, $f^{-1}(S)$ is $Ngb$-closed in $(U, \tau_R(X))$. Therefore, $f$ is $Ngb$ continuous. □

The converse of the theorem need not be true as seen from the following example.

**Example 3.6.** *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{w\}, \{y, z\}\}$ and $X = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$. Define a mapping $f : U \to V$ as $f(a) = y; f(b) = x; f(c) = z; f(d) = w$, Then $f$ is $Ngb$-continuous but not $Nrg^*b$-continuous, as the inverse image of a $Nrg^*b$-closed set $\{a, b, c\}$ in $V$ is $\{x, y, z\}$ is which is not $Nrg^*b$ closed in $U$.*

**Theorem 3.7.** *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano $g$-continuous function, then $f$ is $Nrg^*b$ continuous but not conversely.*

*Proof.* Let f be $Ng$-continuous function and S be an nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(S)$ is $Ng$-closed in $(U, \tau_R(X))$. Since [13] every $Ng$ closed set is $Nrg^*b$ closed, $f^{-1}(S)$ is $Nrg^*b$-closed set. Hence, f is $Nrg^*b$-continuous. □

**Theorem 3.8.** *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano $g^*$-continuous function, then $f$ is $Nrg^*b$ continuous but not conversely.*

*Proof.* Let f be $Ng^*$-continuous function and S be an nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(S)$ is $Ng^*$-closed in $(U, \tau_R(X))$. Since [13 every $Ng^*$ closed set is $Nrg^*b$ closed, $f^{-1}(S)$ is $Nrg^*b$-closed set. Hence, f is $Nrg^*b$-continuous. □

**Theorem 3.9.** *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano regular continuous function, then $f$ is $Nrg^*b$-continuous but not conversely.*

*Proof.* Let S be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(S)$ is $Nr$-closed in $(U, \tau_R(X))$ as f is nano regular-continuous. Since [13] every nano $r$-closed set is $Nrg^*b$-closed. we have, $f^{-1}(S)$ is $Nrg^*b$-closed in $(U, \tau_R(X))$. Therefore f is $Nrg^*b$-continuous. □

The converse of the above theorem need not be true as seen from the following example.

**Example 3.10.** *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{z\}, \{y, w\}\}$ and $X = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define a mapping $f : U \to V$ as $f(a) = x; f(b) = y; f(c) = z; f(d) = w$, Then $f$ is $Nrg^*b$-continuous but not nano regular continuous.*

**Theorem 3.11.** *Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano $g^*$-continuous function, then $f$ is $Ng$ continuous but not conversely.*

*Proof.* Let f be $Ng^*$-continuous function and S be an nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(S)$ is $Ng^*$-closed in $(U, \tau_R(X))$. Since [13] every $Ng^*$ closed set is $Ng$ closed, $f^{-1}(S)$ is $Ng$-closed set. Hence, f is $Ng$-continuous. □

**Theorem 3.12.** *A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano $rg^*b$-continuous iff the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed*

*Proof.* Let f be nano $rg^*b$-continuous and F be nano closed in $V$. i.e.,$V - F$ is nano open in $V$. Since f is nano $rg^*b$-continuous, $f^{-1}(V - F)$ is nano $rg^*b$-open in $U$. i.e.,$U - f^{-1}(V - F)$ is nano $rg^*b$-closed in $U$. i.e.,$U - f^{-1}(F)$ is nano $rg^*b$-open in $U$. Therefore $f^{-1}(F)$ is nano $rg^*b$-closed in $U$. Thus the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed in $U$, if f is nano $rg^*b$-continuous on $U$.

Conversely, let the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed in $U$. Let $G$ be nano open in $V$, then $V - G$ is nano closed in $V$. Then $f^{-1}(V - G)$ is nano $rg^*b$-closed in $U$. i.e.,$U - f^{-1}(G)$ is nano $rg^*b$-closed in $U$. Therefore $f^{-1}(G)$ is nano $rg^*b$-open in $U$. By definition $f$ is nano $rg^*b$-continuous. □

**Theorem 3.13.** *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces and a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$. Then,*

*(1). Every nano semi continuous function is nano $rg^*b$-continuous.*

*(2). Every nano pre continuous function is nano $rg^*b$-continuous.*

*(3). Every nano $\alpha$ continuous function is nano $rg^*b$-continuous.*

*(4). Every nano regular continuous function is nano $rg^*b$-continuous.*

*Proof.*

(1). Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano continuous and $S$ be nano semi closed in $V$.Then $f^{-1}(S)$ is nano semi closed in $U$. Since every nano semi closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano semi closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

(2). Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano continuous and $S$ be nano pre closed in $V$. Then $f^{-1}(S)$ is nano pre closed in $U$. Since every nano pre closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano pre closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

(3). Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano continuous and $S$ be nano $\alpha$ closed in $V$. Then $f^{-1}(S)$ is nano $\alpha$ closed in $U$. Since every nano $\alpha$ closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano $\alpha$ closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

(4). Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano continuous and $S$ be nano regular closed in $V$. Then $f^{-1}(S)$ is nano regular closed in $U$. Since every nano regular closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano regular closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

□

**Remark 3.14.** *The converse of the above theorem need not be true which can be seen from the following examples.*

**Example 3.15.** *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $[\tau_R(X)]^c = \{U, \phi, \{b, c, d\}, \{d\}, \{a, d\}\}$. Then $V = \{x, y, z, w\}$, with $V/R^{'} = \{\{x\}, \{y, z\}, \{w\}\}$,and $Y = \{x, y\} \subseteq V$. Then, $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, z\}, \{x, y, z\}\}$ and $[\tau_{R'}(Y)]^c = \{V, \phi, \{y, z, w\}, \{x, w\}, \{w\}\}$*

*Define :* $f : U \to V$ *as* $f(a) = \{z\}, f(b) = \{y\}, f(c) = \{x\}, f(d) = \{w\}$. *Then* $f^{-1}(\{y,z,w\}) = \{a,b,d\}, f^{-1}(\{x,w\}) = \{c,d\}, f^{-1}(\{w\}) = \{d\}$. *Then* $f$ *is nano* $rg^*b$*-continuous since the inverse image of every nano closed set in* $V$ *is nano* $rg^*b$*-closed in* $U$. *But*

*(1).* $f$ *is not semi continuous, since* $f^{-1}(\{y,z,w\}) = \{a,b,d\}$ *and* $f^{-1}(\{x,w\}) = \{c,d\}$ *are not nano semi closed in* $U$ *where as* $\{y,z,w\}$ *and* $\{x,w\}$ *are nano closed in* $V$.

*(2).* $f$ *is not pre continuous, since* $f^{-1}(\{y,z\}) = \{a,b\}$ *and* $f^{-1}(\{w\}) = \{d\}$ *are not nano pre closed in* $U$ *where as* $\{y,z\}$ *and* $\{x,w\}$ *are nano closed in* $V$.

*(3).* $f$ *is not* $\alpha$ *continuous, since* $f^{-1}(\{y,z,w\}) = \{a,b,d\}$ *and* $f^{-1}(\{y,z\}) = \{a,b\}$ *are not nano* $\alpha$ *closed in* $U$ *where as* $\{y,z,w\}$ *and* $\{y,z\}$ *are nano closed in* $V$.

*(4).* $f$ *is not regular continuous, since* $f^{-1}(\{y,z,w\}) = \{a,b,d\}$ *and* $f^{-1}(\{y,z\}) = \{a,b\}$ *are not nano regular closed in* $U$ *where as* $\{y,z,w\}$ *and* $\{y,z\}$ *are nano closed in* $V$.

## 4. Nano Regular Generalized Star $b$-Open and Closed Functions

**Definition 4.1.** *A function* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *is said to be* $Nrg^*b$*-open (resp.*$Nrg^*b$*-closed) function if the image of every nano open (resp.nano closed) set in* $(U, \tau_R(X))$ *is* $Nrg^*b$*-open (resp.*$Nrg^*b$*-closed) in* $(V, \tau_{R'}(Y))$.

**Theorem 4.2.** *Let* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *be a function, if* $f$ *is an nano open function, then* $f$ *is* $Nrg^*b$*-open function.*

*Proof.* Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano open map and $S$ be a nano open set in $U$. Then $f(S)$ is nano open and hence $f(S)$ is $Nrg^*b$-open in $V$. Thus f is $Nrg^*b$-open. □

**Theorem 4.3.** *Let* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *be a function, if* $f$ *is an nano closed function, then* $f$ *is* $Nrg^*b$*-closed function.*

*Proof.* Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a nano closed map and $S$ be a nano closed set in $U$. Then $f(S)$ is nano closed and hence $f(S)$ is $Nrg^*b$-closed in $V$. Thus $f$ is $Nrg^*b$-closed. □

**Theorem 4.4.** *A function* $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *is* $Nrg^*b$*-closed if and only if for each subset* $B$ *of* $V$ *and for each nano open set* $G$ *containing* $f^{-1}(B)$ *there exists a* $Nrg^*b$*-open set* $F$ *of* $V$ *such that* $B \subseteq F$ *and* $f^{-1}(B) \subseteq G$.

*Proof.*

**Necessity :** Let $G$ be a nano open subset of $(U, \tau_R(X))$ and $B$ be a subset of $V$ such that $f^{-1}(B) \subseteq G$. Define, $F = V - f(U - G)$. Since, $f$ is $Nrg^*b$-closed. Then $F$ is $Nrg^*b$-open set containing $B$ such that $f^{-1}(F) \subseteq G$.

**Sufficiency :** Let $E$ be a nano closed subset of $(U, \tau_R(X)$. Then $f^{-1}(V - f(E)) \subseteq (U - E)$ and $(U - E)$ is nano open. By hypothesis, there is a $Nrg^*b$open-set $F$ of $(V, \tau_{R'}(y))$ such that $V - f(E) \subseteq F$ and $f^{-1}(B) \subseteq U - E$. Therefore $E \subseteq U - f^{-1}(F)$. Hence, $V - F \subseteq f(E) \subseteq f(U - f^{-1}(F)) \subseteq V - F$. Which implies that $f(E) = V - F$ and hence $f(E)$ is $Nrg^*b$-closed in $(V, \tau_{R'}(y))$. Therefore, $f$ is $Nrg^*b$-closed function. □

**Theorem 4.5.** *If a function* $f : U \to V$ *is nano closed and a map* $g : V \to W$ *is* $Nrg^*b$*-closed then their composition* $gof : U \to W$ *is* $Nrg^*b$*-closed.*

*Proof.* Let $H$ be a nano closed set is $U$. Then $f(H)$ is nano closed in $V$ and $(gof)(H) = g(f(H))$ is $Nrg^*b$-closed, as $g$ is $Nrg^*b$-closed. Hence, $gof$ is $Nrg^*b$-closed. □

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