



# On Edge Regular Antipodal Fuzzy Graphs

Research Article

K.Radha<sup>1</sup> and N.Kumaravel<sup>2\*</sup>

1 P.G.Department of Mathematics, Periyar E.V.R. College, Tiruchirappalli, Tamil Nadu, India.

2 Department of Mathematics, K S R Institute for Engineering and Technology, Namakkal, Tamil Nadu, India.

**Abstract:** In this paper, some properties of edge regular antipodal fuzzy graphs are studied. Antipodal fuzzy graph of an edge regular fuzzy graph need not be edge regular. Conditions under which it is edge regular are provided.

**MSC:** 03E72, 05C72.

**Keywords:** Strong fuzzy graph, complete fuzzy graph, edge regular fuzzy graph, totally edge regular fuzzy graph, antipodal fuzzy graph.

© JS Publication.

## 1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [11]. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and bang have also introduced various connectedness concepts in fuzzy graphs [12]. In crisp graph theory the concept of antipodal graph of a given graph  $G$  was introduced by Smith. A.Nagoor Gani and J.Malarvizhi (2010) introduced the concept of antipodal fuzzy graph [2]. K.Radha and N.Kumaravel (2014) introduced the concept of edge regular fuzzy graphs [9]. In this paper, we study about edge regular property of antipodal fuzzy graphs.

First we go through some basic definitions in the next section, which are represented in [1, 2, 4, 5, 8, 9].

## 2. Basic Concepts

Let  $V$  be a non-empty finite set and  $E \subseteq V \times V$ . A fuzzy graph  $G : (\sigma, \mu)$  is a pair of functions  $\sigma : V \rightarrow [0, 1]$  and  $\mu : E \rightarrow [0, 1]$  such that  $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$  for all  $x, y \in V$ . The order and size of a fuzzy graph  $G : (\sigma, \mu)$  are defined by  $O(G) = \sum_{x \in V} \sigma(x)$  and  $S(G) = \sum_{xy \in E} \mu(xy)$ . A fuzzy Graph  $G : (\sigma, \mu)$  is strong, if  $\mu(xy) = \sigma(x) \wedge \sigma(y)$  for all  $xy \in E$ . A fuzzy Graph  $G : (\sigma, \mu)$  is complete, if  $\mu(xy) = \sigma(x) \wedge \sigma(y)$  for all  $x, y \in V$ . The underlying crisp graph is denoted by  $G^* : (V, E)$ . The degree of a vertex  $x$  is  $d_G(x) = \sum_{x \neq y} \mu(xy)$ . If each vertex in  $G$  has same degree  $k$ , then  $G$  is said to be a regular fuzzy graph or  $k$ -regular fuzzy graph. The degree of an edge  $e = uv \in E$  in  $G^*$  is defined by  $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$ . If each edge in  $G^*$  has same degree, then  $G^*$  is said to be edge regular. The degree of an edge  $xy \in E$  is  $d_G(xy) = \sum_{x \neq z} \mu(xz) + \sum_{z \neq y} \mu(z y) - 2\mu(xy)$ . If each edge in  $G$  has same degree  $k$ , then  $G$  is said to be an edge regular fuzzy graph or  $k$ -edge regular fuzzy graph.

\* E-mail: [kumaramaths@gmail.com](mailto:kumaramaths@gmail.com)

The  $\mu$ -distance  $\delta(u, v)$  is the smallest  $\mu$ -length of any  $u-v$  path, where the  $\mu$ -length of a path  $\rho : u_0, u_1, u_2, \dots, u_n$  is  $\ell(\rho) = \sum_{i=1}^n \frac{1}{\mu(u_{i-1}, u_i)}$ . The eccentricity of a node  $v$  is defined as  $e(v) = \max_u(\delta(u, v))$ . The diameter  $diam(G) = \vee\{e(v) | v \in V\}$ , radius  $r(G) = \wedge\{e(v) | v \in V\}$ . A node whose eccentricity is minimum in a connected fuzzy graph is called a central node.

A connected fuzzy graph is called self-centered if each node is a central node.

Let  $G : (\sigma, \mu)$  be a fuzzy graph with the underlying set  $V$ . An antipodal fuzzy graph  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is defined as follows: The node set of  $G$  is taken as the node set of  $A(G)$  also. Two nodes in  $A(G)$  are made as neighbors if the  $\mu$ -distance between them is  $diam(G)$ . i.e.,  $\sigma_{A(G)}(u) = \sigma(u)$  for all  $u \in V$  and

$$\mu_{A(G)}(uv) = \begin{cases} \mu(uv), & \text{if } u \text{ and } v \text{ are neighbors in } A(G) \ \& \ uv \in E \\ \sigma(u) \wedge \sigma(v), & \text{if } u \text{ and } v \text{ are neighbors in } A(G) \ \& \ uv \notin E, \text{ for all } u, v \in V. \\ 0, & \text{otherwise.} \end{cases}$$

Here  $\delta(a, b) = 1.4$ ,  $\delta(a, c) = 3.9$ ,  $\delta(a, d) = 3.1$ ,  $\delta(b, c) = 2.5$ ,  $\delta(b, d) = 1.7$  and  $\delta(c, d) = 2$ . Therefore  $e(a) = 3.9$ ,  $e(b) = 2.5$ ,

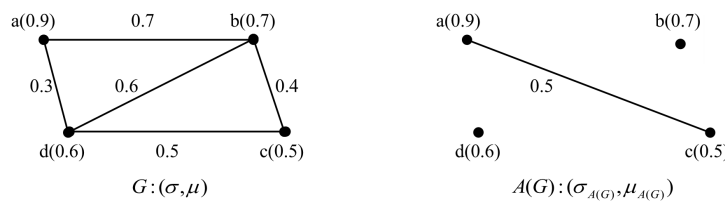


Figure 1.

$e(c) = 3.9$  and  $e(d) = 3.1$ . Therefore  $diam(G) = 3.9$  and  $r(G) = 2.5$ . Therefore  $diam(G) = 3.9 = \delta(a, c)$ .

**Theorem 2.1** ([8]). *Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . If  $\mu$  is a constant function, then  $G$  is edge regular if and only if  $G^*$  is edge regular.*

**Theorem 2.2** ([10]). *Let  $G : (\sigma, \mu)$  be a fuzzy graph on an odd cycle  $G^* : (V, E)$ . Then  $G$  is edge regular if and only if  $\mu$  is a constant function.*

**Theorem 2.3** ([10]). *Let  $G : (\sigma, \mu)$  be a fuzzy graph on an even cycle  $G^* : (V, E)$  with  $n$  vertices and let  $n \not\equiv 0, (mod 4)$ . Then  $G$  is an edge regular fuzzy graph if and only if  $\mu$  is a constant function.*

**Theorem 2.4** ([10]). *Let  $G : (\sigma, \mu)$  be a fuzzy graph on an even cycle  $G^* : (V, E)$  with  $n$  vertices and let  $n \equiv 0, (mod 4)$ . Then  $G$  is a  $k$ -edge regular fuzzy graph if and only if  $\mu$  assumes exactly four values  $r, s, t$  and  $l$  such that consecutive adjacent edges receives these values in cyclic order with  $r + t = k$  and  $s + l = k$ .*

### 3. Degree of an Edge in Antipodal Fuzzy Graph

By definition,

$$d_{A(G)}(uv) = \sum_{w \in E(A(G)), w \neq v} \mu_{A(G)}(uw) + \sum_{w \in E(A(G)), w \neq u} \mu_{A(G)}(wv), \forall uv \in E(A(G)).$$

For all  $uv \in E(A(G))$  and when  $uv \in E$ ,

$$d_{A(G)}(uv) = \sum_{\substack{\delta(u, w) = diam(G) \\ uv \in E, w \neq v}} \mu(uw) + \sum_{\substack{\delta(u, w) = diam(G) \\ uv \notin E}} \sigma(u) \wedge \sigma(w) + \sum_{\substack{\delta(w, v) = diam(G) \\ uv \in E, w \neq u}} \mu(wv) + \sum_{\substack{\delta(w, v) = diam(G) \\ uv \notin E}} \sigma(w) \wedge \sigma(v).$$

For all  $uv \in E(A(G))$  and when  $uv \notin E$ ,

$$d_{A(G)}(uv) = \sum_{\substack{\delta(u,w) = \text{diam}(G) \\ uv \in E}} \mu(uw) + \sum_{\substack{\delta(u,w) = \text{diam}(G) \\ uv \notin E, w \neq v}} \sigma(u) \wedge \sigma(w) + \sum_{\substack{\delta(w,v) = \text{diam}(G) \\ uv \in E}} \mu(wv) + \sum_{\substack{\delta(w,v) = \text{diam}(G) \\ uv \notin E, w \neq u}} \sigma(w) \wedge \sigma(v).$$

### 4. Edge Regular Property of Antipodal Fuzzy Graph of Fuzzy Graph

**Remark 4.1.** If  $G : (\sigma, \mu)$  is an edge regular fuzzy graph, then  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  need not be edge regular fuzzy graph. For example, in Figure 2,  $G : (\sigma, \mu)$  is 0.8-edge regular fuzzy graph, but  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is not an edge regular fuzzy graph.

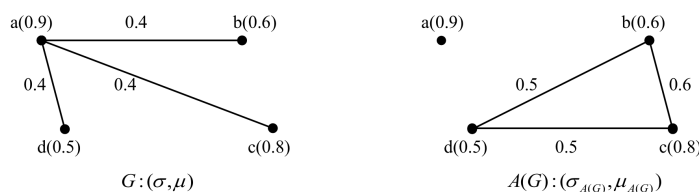


Figure 2.

Here  $\delta(a, b) = 2.5$ ,  $\delta(a, c) = 2.5$ ,  $\delta(a, d) = 2.5$ ,  $\delta(b, c) = 5$ ,  $\delta(b, d) = 5$  and  $\delta(c, d) = 5$ .

$$\begin{aligned} \therefore e(a) &= 2.5, e(b) = 5, e(c) = 5 \text{ and } e(d) = 5. \\ \therefore \text{diam}(G) &= 5 \text{ and } r(G) = 2.5. \\ \therefore \text{diam}(G) &= 5 = \delta(b, c) = \delta(b, d) = \delta(c, d) \end{aligned}$$

**Remark 4.2.** If  $L(G) : (\omega, \lambda)$  is an edge regular fuzzy graph, then  $G : (\sigma, \mu)$  need not be edge regular fuzzy graph. For example, in Figure 3,  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is 0.8-edge regular fuzzy graph. But  $G : (\sigma, \mu)$  is not an edge regular fuzzy graph.

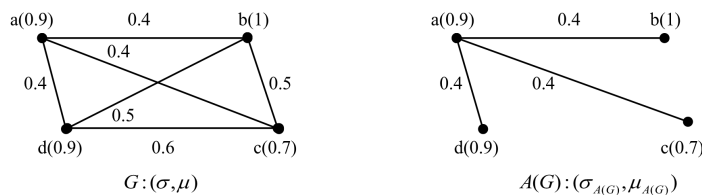


Figure 3.

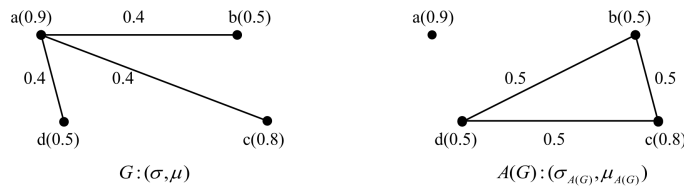
Here  $\delta(a, b) = 2.5$ ,  $\delta(a, c) = 2.5$ ,  $\delta(a, d) = 2.5$ ,  $\delta(b, c) = 2$ ,  $\delta(b, d) = 2$  and  $\delta(c, d) = 1.7$ .

$$\begin{aligned} \therefore e(a) &= 2.5, e(b) = 2.5, e(c) = 2.5 \text{ and } e(d) = 2.5. \\ \therefore \text{diam}(G) &= 2.5 \text{ and } r(G) = 2.5. \\ \therefore \text{diam}(G) &= 2.5 = \delta(a, b) = \delta(a, c) = \delta(a, d) \end{aligned}$$

**Theorem 4.3.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on a complete graph  $G^* : (V, E)$  such that  $\mu$  is a constant function. Then  $A(G)$  is an edge regular fuzzy graph.

*Proof.* Since  $G^*$  is complete and  $\mu$  is a constant function,  $G^*$  is edge regular. By Theorem 2.1,  $G$  is edge regular. Let  $\mu(e) = c$ , for all  $e \in E$ , where  $c$  is a constant. For any pair of distinct vertices  $u, v \in V$ , their  $\mu$ - distance  $\delta(u, v)$  is  $\frac{1}{c}$ . Then eccentricity of a vertex  $v \in V$  is  $e(v) = \max_u(\delta(u, v)) = \frac{1}{c}$  and the diameter  $diam(G) = \vee\{e(v) | v \in V\} = \frac{1}{c}$ . It follows that  $u$  and  $v$  are neighbors in  $A(G)$ , for all  $u, v \in V$ . Therefore  $\mu_{A(G)}(uv) = \mu(uv) = c$ , for all  $u, v \in V$ , since  $G$  is edge regular,  $A(G)$  is an edge regular fuzzy graph.  $\square$

**Remark 4.4.** The converse of above Theorem 4.3 need not be true. In the following figures,  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is 1.0-edge regular fuzzy graph. But  $G^* : (V, E)$  is not a complete graph and  $\mu$  is not a constant function.



**Figure 4.**

Here  $\delta(a, b) = 2.5$ ,  $\delta(a, c) = 2.5$ ,  $\delta(a, d) = 2.5$ ,  $\delta(b, c) = 5$ ,  $\delta(b, d) = 5$  and  $\delta(c, d) = 5$ .

$$\begin{aligned} \therefore e(a) &= 2.5, e(b) = 5, e(c) = 5 \text{ and } e(d) = 5. \\ \therefore diam(G) &= 5 \text{ and } r(G) = 2.5. \\ \therefore diam(G) &= 5 = \delta(b, c) = \delta(b, d) = \delta(c, d) \end{aligned}$$

In Figure 3,  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is 0.8-edge regular fuzzy graph. But  $\mu$  is not a constant function.

**Remark 4.5.** If  $G : (\sigma, \mu)$  is an edge regular fuzzy graph with  $\mu$  is a constant function, then  $A(G)$  need not be an edge regular fuzzy graph. In Figure 2,  $G : (\sigma, \mu)$  is 0.8-edge regular fuzzy graph with  $\mu = 0.4$ , for all  $uv \in E$ , but  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is not an edge regular fuzzy graph.

**Theorem 4.6.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on a complete graph  $G^* : (V, E)$  such that  $\mu$  is a constant function. Then  $A^n(G)$  is an edge regular fuzzy graph, where  $n$  is a positive integer.

*Proof.* By Theorem 4.3,  $G = A(G) = A^2(G) = A^3(G) = \dots = A^n(G)$ , it follows that  $A^n(G)$  is an edge regular fuzzy graph, where  $n$  is a positive integer.  $\square$

**Theorem 4.7.** If  $G : (\sigma, \mu)$  is a complete fuzzy graph such that  $\sigma$  is a constant function, then  $A(G)$  is an edge regular fuzzy graph.

*Proof.* Given  $G$  is a complete and  $\sigma$  is a constant function,  $\mu$  is also a constant function. Hence the result follows from Theorem 4.3.  $\square$

## 5. Edge Regular Property of Antipodal Fuzzy Graph of Fuzzy Graph on a Cycle

**Theorem 5.1.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on an odd cycle  $G^* : (V, E)$  such that  $\sigma$  is a constant function. If  $G$  is an edge regular fuzzy graph, then  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is an edge regular fuzzy graph.

*Proof.* Let  $G$  be an edge regular fuzzy graph on an odd cycle  $v_1v_2v_3 \dots v_nv_1$ , where  $n$  is odd. By Theorem 2.2,  $\mu$  is a constant function. Let  $\mu(e) = c$ , for all  $e \in E$ , where  $c$  is a constant. For any  $j$ , the distance from  $v_j$  to the vertex farthest from  $v_j$  in  $G^*$  is  $\frac{n-1}{2}$ . Since  $\mu$  is a constant function, the  $\mu$ -distance from  $v_j$  to the vertex farthest from  $v_j$  in  $G$  is  $(\frac{n-1}{2})(\frac{1}{c})$ . Therefore  $e(v_j) = \max_{v_i}(\delta(v_i, v_j)) = (\frac{n-1}{2})(\frac{1}{c})$ , for all  $v_j \in V$ . Thus  $diam(G) = \vee\{e(v_j) | v_j \in V\} = \frac{n-1}{2c} = e(v_i)$ , for all  $i = 1, 2, \dots, n$ .

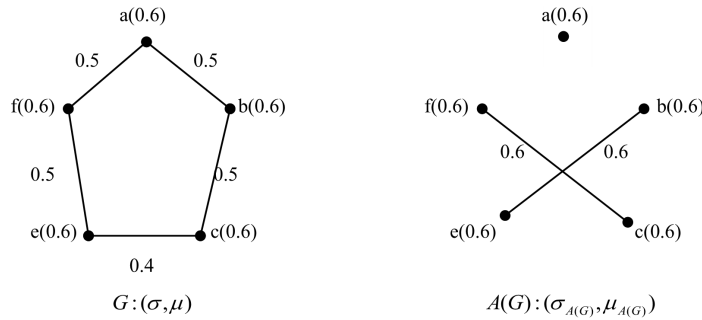
Now, if  $i \leq \frac{n+1}{2}$ , then the vertex farthest from  $v_i$  is  $v_{\frac{n-1}{2}+i}$ . If  $i > \frac{n+1}{2}$ , then the vertex farthest from  $v_i$  is  $v_{i-\frac{n+1}{2}}$ . Hence

$$diam(G) = e(v_i) = \begin{cases} \delta(v_i, v_{\frac{n-1}{2}+i}), & \text{if } i \leq \frac{n+1}{2} \\ \delta(v_i, v_{i-\frac{n+1}{2}}), & \text{if } i > \frac{n+1}{2} \end{cases}$$

In  $A(G)$ ,  $v_1$  is adjacent to  $v_{\frac{n+1}{2}}$ ,  $v_2$  is adjacent to  $v_{\frac{n+3}{2}}, \dots, v_{\frac{n-1}{2}}$  is adjacent to  $v_{n-1}$ ,  $v_{\frac{n+1}{2}}$  is adjacent to  $v_n$ ,  $v_{\frac{n+3}{2}}$  is adjacent to  $v_1$ ,  $v_{\frac{n+5}{2}}$  is adjacent to  $v_2$ ,  $v_{\frac{n+7}{2}}$  is adjacent to  $v_3, \dots, v_{n-1}$  is adjacent to  $v_{\frac{n-3}{2}}$ ,  $v_n$  is adjacent to  $v_{\frac{n-1}{2}}$ . Therefore  $A(G)$  is a fuzzy graph on an odd cycle  $v_1v_{\frac{n+1}{2}}v_nv_{\frac{n-1}{2}}v_{n-1}v_{\frac{n-3}{2}} \dots v_{\frac{n+5}{2}}v_2v_{\frac{n+3}{2}}v_1$ .

None of the edges in  $A(G)$  is in  $G$ . Let  $\sigma(v_i) = c_1, \forall v_i \in V$ , where  $c_1$  is a constant. Then  $\mu_{A(G)}(uv) = \sigma(u) \wedge \sigma(v) = c_1$ , for every edge  $uv$  in  $A(G)$ . Hence by Theorem 2.2,  $A(G)$  is an edge regular fuzzy graph.  $\square$

**Remark 5.2.** The converse of above theorem need not be true. For example, in Figure 5,  $A(G)$  is 0-edge regular fuzzy graph. But,  $G$  is not an edge regular.



**Figure 5.**

Here  $e(a) = 4$  and  $e(b) = e(c) = e(d) = e(e) = 4.5$ . Therefore  $diam(G) = 4.5$  and  $diam(G) = 4.5 = \delta(b, e) = \delta(c, f)$ .

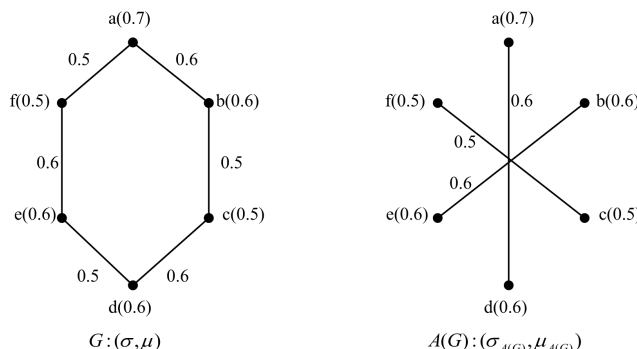
**Theorem 5.3.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on an even cycle  $G^* : (V, E)$  with  $n \not\equiv 0(mod 4)$ , where  $|V| = n$ . If  $G$  is an edge regular fuzzy graph, then  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is a zero-edge regular fuzzy graph.

*Proof.* Let  $G$  be an edge regular fuzzy graph on an even cycle  $v_1v_2 \dots v_nv_1$  with  $n \not\equiv 0(mod 4)$ . By Theorem 2.3,  $\mu$  is a constant function. Let  $\mu(e) = c$ , for all  $e \in E$ , where  $c$  is a constant. For any  $j$ , the distance from  $v_j$  to the vertex farthest from  $v_j$  in  $G^*$  is  $\frac{n}{2}$ . Since  $\mu$  is a constant function, the  $\mu$ -distance from  $v_j$  to the vertex farthest from  $v_j$  in  $G$  is  $(\frac{n}{2})(\frac{1}{c})$ . Therefore  $e(v_j) = \max_{v_i}(\delta(v_i, v_j)) = \frac{n}{2c}$ , for all  $j = 1, 2, \dots, n$ . Thus  $diam(G) = \vee\{e(v_j) | v_j \in V\} = \frac{n}{2c} = e(v_j)$ , for all  $j = 1, 2, \dots, n$ . Now

$$diam(G) = e(v_i) = \begin{cases} \delta(v_i, v_{i+\frac{n}{2}}), & \text{if } i \leq \frac{n}{2} \\ \delta(v_i, v_{i-\frac{n}{2}}), & \text{if } i > \frac{n}{2} \end{cases}$$

Therefore  $v_1v_{1+\frac{n}{2}}, v_2v_{2+\frac{n}{2}}, \dots, v_{\frac{n}{2}}v_n$  are the only edges in  $A(G)$ . Therefore every pair of distinct edges are non-adjacent in  $A(G)$ . Hence  $A(G)$  is a zero-edge regular fuzzy graph.  $\square$

**Remark 5.4.** The converse of above theorem need not be true. For example, in Figure 6,  $A(G)$  is 0-edge regular fuzzy graph. But,  $G$  is not an edge regular.



**Figure 6.**

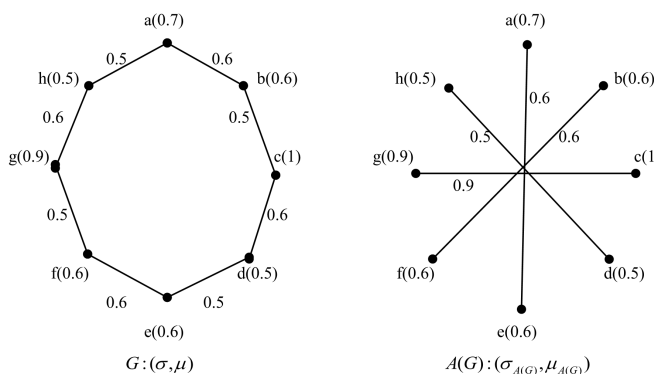
**Theorem 5.5.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on an even cycle  $G^* : (V, E)$  with  $n \equiv 0 \pmod{4}$  and  $\frac{n}{2} \equiv 0 \pmod{4}$ , where  $|V| = n$ . If  $G$  is an edge regular fuzzy graph, then  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is a zero-edge regular fuzzy graph.

*Proof.* Let  $G$  be an edge regular fuzzy graph on an even cycle  $v_1v_2, \dots, v_nv_1$  with  $n \equiv 0 \pmod{4}$  and  $\frac{n}{2} \equiv 0 \pmod{4}$ . By Theorem 2.4,  $\mu$  assumes exactly four values  $r, s, t$  and  $l$  such that consecutive adjacent edges receives these values in cyclic order with  $r + t = k$  and  $s + l = k$ . For any  $j$ , the distance from  $v_j$  to the vertex farthest from  $v_j$  in  $G^*$  is  $\frac{n}{2}$ . Since membership values of the edges are  $r, s, t$  and  $l$  in cyclic order and  $\frac{n}{2} \equiv 0 \pmod{4}$ , the  $\mu$ -distance from  $v_j$  to the vertex farthest from  $v_j$  in  $G$  is  $\binom{n}{4} (\frac{1}{r} + \frac{1}{s} + \frac{1}{t} + \frac{1}{l})$ . Thus  $diam(G) = \vee \{e(v_j) | v_j \in V\} = \binom{n}{4} (\frac{1}{r} + \frac{1}{s} + \frac{1}{t} + \frac{1}{l}) = e(v_j)$ , for all  $j = 1, 2, \dots, n$ . Now

$$diam(G) = e(v_i) = \begin{cases} \delta(v_i, v_{i+\frac{n}{2}}), & \text{if } i \leq \frac{n}{2} \\ \delta(v_i, v_{i-\frac{n}{2}}), & \text{if } i > \frac{n}{2} \end{cases}$$

Therefore  $v_1v_{1+\frac{n}{2}}, v_2v_{2+\frac{n}{2}}, \dots, v_{\frac{n}{2}}v_n$  are the only edges in  $A(G)$ . Therefore every pair of distinct edges are non-adjacent in  $A(G)$ . Hence  $A(G)$  is a zero-edge regular fuzzy graph. □

**Remark 5.6.** The converse of above theorem need not be true. For example, in Figure 7 and 8,  $A(G)$  is 0-edge regular fuzzy graph. But,  $G$  is not an edge regular.



**Figure 7.**

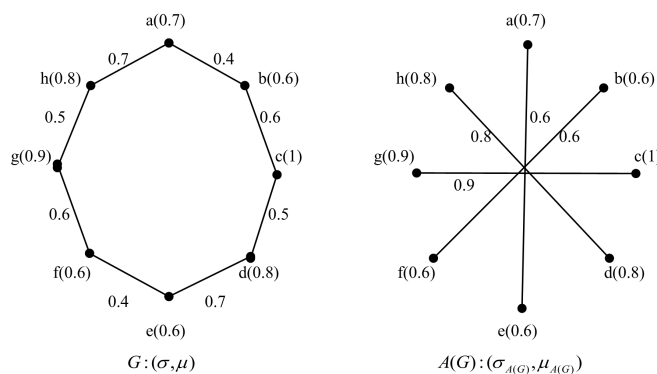


Figure 8.

**Remark 5.7.** When  $n \equiv 0 \pmod{4}$  but  $\frac{n}{2} \not\equiv 0 \pmod{4}$ ,  $\text{diam}(G)$  need not be equal to  $e(v_j)$ , for all  $j = 1, 2, \dots, n$ . Consider  $G^* : (V, E)$ , where  $V = \{a, b, c, d, e, f, g, h, i, j, k, l\}$  and  $E = \{ab, bc, cd, de, ef, fg, gh, hi, ij, jk, kl, la\}$ . Define  $G : (\sigma, \mu)$  by  $\sigma(a) = 0.8, \sigma(b) = 0.7, \sigma(c) = 0.5, \sigma(d) = 0.4, \sigma(e) = 0.5, \sigma(f) = 0.6, \sigma(g) = 1, \sigma(h) = 0.8, \sigma(i) = 0.5, \sigma(j) = 0.9, \sigma(k) = 0.6, \sigma(l) = 0.7$  and  $\mu(ab) = 0.3, \mu(bc) = 0.5, \mu(cd) = 0.4, \mu(de) = 0.2, \mu(ef) = 0.3, \mu(fg) = 0.5, \mu(gh) = 0.4, \mu(hi) = 0.2, \mu(ij) = 0.3, \mu(jk) = 0.5, \mu(kl) = 0.4, \mu(la) = 0.2$ . Here  $\text{diam}(G) = 18.1 \neq 17.3 = e(b)$ .

## 6. Properties of Edge Regular Antipodal Fuzzy Graphs

**Theorem 6.1.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on a complete graph  $G^* : (V, E)$  such that  $\mu(e) = c, \forall e \in E$ , where  $c$  is a constant. Then the size of  $A(G)$  is  $\frac{cn(n-1)}{2}$ , where  $|V| = n$ .

*Proof.* In  $G^*$ , there are exactly  $\frac{n(n-1)}{2}$  edges, where  $|V| = n$ . Since  $\mu(e) = c, \forall e \in E$ , where  $c$  is a constant, the size of  $G$  is  $\frac{cn(n-1)}{2}$ . By Proof of Theorem 4.3,  $A(G)$  and  $G$  have same number of edges and same membership function. Hence the size of  $A(G)$  is  $\frac{cn(n-1)}{2}$ , where  $|V| = n$ . □

**Theorem 6.2.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on an odd cycle  $G^* : (V, E)$  such that  $\sigma = c$ , where  $c$  is a constant. If  $G$  is an edge regular fuzzy graph, then the size of  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is  $nc$ , where  $|V| = n$ .

*Proof.* By Theorem 5.1,  $A(G)$  is a fuzzy graph on an odd cycle with  $n$  vertices and  $\mu_{A(G)}(e) = c$ , for every edge in  $A(G)$ . Therefore the size of  $A(G) : (\sigma_{A(G)}, \mu_{A(G)})$  is  $nc$ , where  $|V| = n$ . □

## References

- [1] S.Arumugam and S.Velammal, *Edge Domination in Graphs*, Taiwanese Journal of Mathematics, 2(2)(1998), 173-179.
- [2] A.Nagoor Gani and J.Malarvizhi, *On Antipodal Fuzzy Graph*, International Journal of Applied Mathematical Sciences, 4(43)(2010), 2145-2155.
- [3] A.Nagoor Gani and J.Malarvizhi, *Isomorphism on Fuzzy Graphs*, International Journal of Computational and Mathematical Sciences, 2(4)(2008), 190-196.
- [4] A.Nagoorgani and J.Malarvizhi, *Properties of  $\mu$ -Complement of a Fuzzy Graph*, International Journal of Algorithms, Computing and Mathematics, 2(3)(2009), 73-83.
- [5] A.Nagoor Gani and K.Radha, *On Regular Fuzzy Graphs*, Journal of Physical Sciences, 12(2008), 33-40.

- [6] A.Nagoor Gani and K.Radha, *Some Sequences in Fuzzy Graphs*, Far East Journal of Applied Mathematics, 31(3)(2008), 321-335.
- [7] A.NagoorGani and K.Radha, *Regular Property of Fuzzy Graphs*, Bulletin of Pure and Applied Sciences, 27E(2)(2008), 411-419.
- [8] K.Radha and N.Kumaravel, *On Edge Regular Fuzzy Graphs*, International Journal of Mathematical Archive, 5(9)(2014), 100-112.
- [9] K.Radha and N.Kumaravel, *Some Properties of Edge Regular Fuzzy Graphs*, Jamal Academic Research Journal (JARJ), Special issue, (2014), 121-127.
- [10] K.Radha and N.Kumaravel, *The Edge Degree and The Edge Regular Properties of Truncations of Fuzzy Graphs*, Bulletin of Mathematics and Statistics Research (BOMSR), 4(3)(2016), 7-16.
- [11] A.Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, (editors), *Fuzzy sets and their applications to cognitive and decision process*, Academic press, New York, (1975), 77-95.
- [12] R.T.Yeh and S.Y.Bang, Fuzzy relations, fuzzy graphs, and their applications to clustering analysis, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, (editors), *Fuzzy sets and their applications to cognitive and decision process*, Academic press, New York, (1975), 125-149.