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On Non-Neighbor Zagreb Indices and Non-Neighbor Harmonic Index

Research Article

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Abstract: A graph can be represented by a matrix, a sequence, a polynomial and a number which represents the whole graph. The representation of the graph by a unique number is called topological index. Computation of topological indices is a recent research problem in mathematical and computational chemistry. There are some special classes of topological indices such as distance based, degree based and counting related topological indices which have their own chemical significance. In this paper we introduce the computation of topological indices called Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index based on the non-neighbors of the vertices of a graph.

MSC: 05C12, 92E10.

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1. Introduction

Chemical Graph Theory is a branch of Mathematical Chemistry which applies graph theory to the mathematical model of chemical phenomenon. A representation of an object giving information only about the number of elements comprising it and their connectivity is named as topological representation of an object. A topological representation of a molecule is called molecular graph. A molecular graph or chemical graph is a representation of the structural formula of a chemical compound in terms of graph theory. A molecular graph [10] is a collection of points representing the atoms in the molecule and set of lines representing the covalent bonds. These points are named vertices and the lines are named edges in graph theory language. A hydrogen-depleted molecular graph or hydrogen-suppressed molecular graph is the molecular graphs with hydrogen atoms deleted. The characterization of a molecule by an associated graph leads to a large number of powerful and useful discriminators called topological indices. In chemical graph theory a topological index is a numerical parameter mathematically derived from hydrogen-suppressed molecular graphs. A topological index is a numerical descriptor of a molecule, based on a certain topological feature of the corresponding molecular graph such as distance based, degree based [8] and both degree and distance based.

In this paper we are concerned with simple graphs, having no directed or weighted edges, and no self loops. A graph G is an ordered pair of two sets V and E. The set $V = V(G)$ is a finite non empty set and $E = E(G)$ is a binary relation defined

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on V. A graph can be visualized by representing the elements of V by vertices and joining the pair of vertices u, v by an edge if and only if $uv \in E(G)$. The complement $\overline{G}$ of the graph G is the graph with vertex set $V(G)$, in which two vertices are adjacent if and only if they are not adjacent in G. The degree of the vertex $v \in V(G)$, written $d_G(v)$, is the number of first neighbors of v in the underlying graph G. The first and second Zagreb index are defined as

$$M_1 = M_1(G) = \sum_{v \in V(G)} d_G(v)^2$$

$$M_2 = M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v)$$

These topological indices were conceived in the 1970's [8, 9]. It was recognized that these terms increase with the increasing extent of branching of the carbon-atom skeleton and that these provide quantitative measures of molecular branching. Balaban included M_1 and M_2 among topological indices and named them Zagreb group indices. A similar formula to M_1 is the Modified Zagreb Index given by

$$M_X(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

The Zagreb indices belong to the oldest molecular structure descriptors, and their properties have been studied extensively [8, 9, 14, 15, 17, 18]. Recently, the concept of Zagreb coindices attracted the researchers in Mathematical Chemistry. In 2008, T. Doslic[5] put forward the first Zagreb coindex, defined as

$$\overline{M_1} = \overline{M_1}(G) = \sum_{uv \in \notin E(G)} [d_G(u) + d_G(v)]$$

The second Zagreb coindex is defined as

$$\overline{M_2} = \overline{M_2}(G) = \sum_{uv \in \notin E(G)} [d_G(u) d_G(v)]$$

The Zagreb coindices were recently studied in detail [1, 2, 6, 13–15, 18] and the relations between Zagreb indices and coindices were also examined [1, 7, 9, 13, 17]. In the 1980's, Siemion Fajtlowicz created a vertex-degree-based quantity which was re-introduced by Zhang in 2012 called Harmonic Index [11]. It is defined as

$$H(G) = \sum_{uv \in E(G)} \left[\frac{2}{d_G(u) + d_G(v)}\right]$$

In this paper we have introduced the computation of new topological indices called Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index based on the non-neighbors of the vertices of a graph. In the first and second sections, these new topological indices for some standard graphs and other special type of graphs is computed and given respectively. In the last section, the relation between these non-neighbor topological indices with the older Zagreb indices, coindices and Harmonic index is given in detail. For terms and definition not given in this paper refer to [3, 4, 9, 10, 12, 16].

2. Main Results

In this section, formula for the computation of topological indices called Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index based on the non-neighbors of the vertices of a graph is given. The Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index for some standard graphs is computed and given. The vertices that are not adjacent to a vertex $v \in V(G)$ are called non-neighbors of the vertex v. In this paper we define $\overline{d_G(v)}$ as the number of the non-neighbors of the vertex $v \in V(G)$, where $\overline{d_G(v)} = n-1-d_G(v)$. Hence

The ***Non-Neighbor First Zagreb Index*** is defined as

$$\overline{M_1(G)} = \sum_{uv \in E(G)} \left(\overline{d_G(u)} + \overline{d_G(v)} \right)$$

The ***Non-Neighbor Second Zagreb Index*** is defined as

$$\overline{M_2(G)} = \sum_{uv \in E(G)} \left(\overline{d_G(u)} \,.\, \overline{d_G(v)} \right)$$

The ***Non-Neighbor Harmonic Index*** is defined as

$$\overline{H(G)} = \sum_{uv \in E(G)} \left(\frac{2}{\overline{d_G(u)} + \overline{d_G(v)}} \right)$$

Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index for the Complete graph K_n is zero as each of its vertex is adjacent to all the other vertices (i.e) each of its vertex has no non-neighbors.

Theorem 2.1. *For a cycle* $C_n\,(n \geq 4)$, $\overline{M_1(C_n)} = 2n(n-3)$, $\overline{M_2(C_n)} = n(n-3)^2$, $\overline{H(C_n)} = \frac{n}{n-3}$.

Proof. A cycle C_n has n vertices each with degree 2. The number of non-neighbors of each vertex $v \in V(G)$ is $(n-3)$. Since there are n edges. The Non-Neighbor First Zagreb Index of a cycle $C_n\,(n \geq 4) = n[(n-3)+(n-3)]$. Hence $\overline{M_1(C_n)} = 2n(n-3)$. The Non-Neighbor Second Zagreb Index of a cycle $C_n\,(n \geq 4) = n[(n-3) \times (n-3)]$. Hence $\overline{M_2(C_n)} = n(n-3)^2$. The Non-Neighbor Harmonic Index of a cycle $C_n\,(n \geq 4) = n\left[\frac{2}{(n-3)+(n-3)}\right]$. Hence $\overline{H(C_n)} = \frac{n}{n-3}$. □

Theorem 2.2. *For a path* $P_n(n \geq 3)$, $\overline{M_1(P_n)} = 2(n-2)^2$; $\overline{M_2(P_n)} = (n-3)\left(n^2-4n+5\right)$ *and for a path* $P_n(n \geq 4)$, $\overline{H(P_n)} = \frac{4}{2n-5} + 1$.

Proof. A path P_n with n vertices has two pendant vertices and remaining are interior vertices each with degree 2. The number of non-neighbors of the pendant vertex is $(n-2)$. The number of non-neighbors of the interior vertex is $(n-3)$. There are totally $(n-1)$ edges in which 2 are corner edges and $(n-3)$ are interior edges. Hence, the Non-Neighbor First Zagreb Index of a path,

$$\begin{aligned} P_n\,(n \geq 3) &= 2\,[(n-2)+(n-3)] + (n-3)\,[(n-3)+(n-3)] \\ &= 2\,[2n-5] + (n-3)\,[2n-6]\,. \\ \text{Hence } \overline{M_1(P_n)} &= 2(n-2)^2. \end{aligned}$$

The Non-Neighbor Second Zagreb Index of a path,

$$\begin{aligned} P_n\,(n \geq 3) &= 2\,[(n-2) \times (n-3)] + (n-3)\,[(n-3) \times (n-3)] \\ &= 2\,(n-2)\,(n-3) + (n-3)^3. \\ \text{Hence } \overline{M_2(P_n)} &= (n-3)\left(n^2-4n+5\right). \end{aligned}$$

The Non-Neighbor Harmonic Index of a path,

$$P_n\,(n \geq 4) = 2\left[\frac{2}{(n-2)+(n-3)}\right] + (n-3)\left[\frac{2}{(n-3)+(n-3)}\right].$$

Hence $\overline{H\,(P_n)} = \frac{4}{2n-5} + 1$. □

Remark 2.3. *The Non-Neighbor Harmonic Index of a path $P_3 = 4$.*

Remark 2.4. *The path P_n refers to the molecular graph of the saturated acyclic hydrocarbons called alkanes which are the organic compounds [16, 17] in Chemistry. The Non-Neighbor First Zagreb Index of propane is 2 and that of butane is 8. The Non-Neighbor Second Zagreb Index of propane is 0 and that of butane is 5. The Non-Neighbor Harmonic Zagreb Index of propane is 4 and that of butane is 2.33. The Non-Neighbor First, Second Zagreb Indices and Harmonic Index of higher order alkanes can also be easily found.*

Theorem 2.5. *For a complete bipartite graph $K_{m,n}$, $\overline{M_1\,(K_{m,n})} = mn\,[m+n-2]$; $\overline{M_2\,(K_{m,n})} = mn\,[mn-m-n+1]$; $\overline{H\,(K_{m,n})} = \frac{2mn}{m+n-2}$.*

Proof. A complete bipartite graph has two set of vertices, V_1 with m vertices and V_2 with n vertices. Every vertex of V_1 is adjacent to all vertices of V_2 but no vertex within V_1 is adjacent. The non-neighbors of $v \in V_1$ is $V_1 - \{v\}$. The number of non-neighbors of $v \in V_1$ is $(m-1)$. Similarly the number of non-neighbors of $v \in V_2$ is $(n-1)$. There are totally mn edges. Hence,

The Non-Neighbor First Zagreb Index of a bipartite graph

$$K_{m,n} = mn\,[(m-1)+(n-1)]\,.\ \text{Hence}\ \ \overline{M_1\,(K_{m,n})} = mn\,[m+n-2]\,.$$

The Non-Neighbor Second Zagreb Index of a bipartite graph

$$K_{m,n} = mn\,[(m-1)\times(n-1)]\,.\ \text{Hence}\ \ \overline{M_2\,(K_{m,n})} = mn\,[mn-m-n+1]\,.$$

The Non-Neighbor Harmonic Index of a bipartite graph

$$K_{m,n} = mn\left[\frac{2}{(m-1)+(n-1)}\right].\ \text{Hence}\ \ \overline{H\,(K_{m,n})} = \frac{2mn}{m+n-2}.$$

□

Corollary 2.6. *For a k-regular bipartite graph $K_{k,k}$, $\overline{M_1\,(K_{k,k})} = 2k^2\,(k-1)$; $\overline{M_2\,(K_{k,k})} = k^2(k-1)^2$; $\overline{H\,(K_{k,k})} = \frac{k^2}{k-1}$.*

Corollary 2.7. *For a star graph $S_k\,(k \geq 2)$, $\overline{M_1\,(S_k)} = k(k-1)$; $\overline{M_2\,(S_k)} = 0$; $\overline{H(S_k)} = \frac{2k}{k-1}$.*

Theorem 2.8. *For a wheel graph $W_n(n \geq 4)$, $\overline{M_1\,(W_n)} = 3n(n-3)$. $\overline{M_2\,(W_n)} = n(n-3)^2$; $\overline{H(W_n)} = \frac{3n}{n-3}$.*

Proof. A n-wheel graph W_n with $n+1$ vertices is formed by connecting a single vertex to all vertices of a cycle of length n. The vertex in the center has no non-neighbors. The remaining n vertices on the cycle each has $(n-3)$ non neighbors. Totally there are $2n$ edges in which n edges are in the cycle and n edges between the central vertex and the cycle. Hence the Non-Neighbor First Zagreb Index of a wheel,

$$\begin{aligned} W_n\,(n \geq 4) &= n\,[(n-3)+(n-3)] + n\,[0+(n-3)] \\ &= n\,[2(n-3)] + n(n-3) \end{aligned}$$

$$\text{Hence}\ \ \overline{M_1\,(W_n)} = 3n(n-3).$$

The Non-Neighbor Second Zagreb Index of a wheel,

$$W_n\,(n \geq 4) = n\left[(n-3)\times(n-3)\right] + n\left[0\times(n-3)\right]. \text{ Hence } \overline{M_2\,(W_n)} = n(n-3)^2.$$

The Non-Neighbor Harmonic Index of a wheel,

$$\begin{aligned} W_n\,(n \geq 4) &= n\left[\frac{2}{(n-3)+(n-3)}\right] + n\left[\frac{2}{0+(n-3)}\right] \\ &= n\left[\frac{2}{2\,(n-3)}\right] + n\left[\frac{2}{(n-3)}\right] \\ Hence\ \ \overline{H(W_n)} &= \frac{3n}{n-3}. \end{aligned}$$

□

3. More Results on Special Types of Graphs

In this section, the definition for Equally Neighboring graph and other special types of graphs is given. The Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index for Equally Neighboring graph and other special types of graphs is computed and given.

Definition 3.1 (Equally Neighboring Graph)**.** *The graphs for which* $d_G\,(v) = \overline{d_G(v)}\ \ \forall\ v \in V(G)$ *is called* ***Equally Neighboring graph****.* C_5 *is an equally neighboring graph which is shown in Figure 1.*

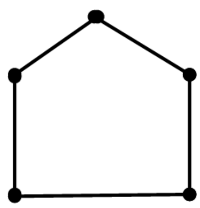

Figure 1.

Theorem 3.2. *For Equally Neighboring graph* G *,* $M_1\,(G) = \overline{M_1\,(G)}$*;* $M_2\,(G) = \overline{M_2\,(G)}$*;* $H\,(G) = \overline{H\,(G)}$*.*

Corollary 3.3. *For Equally Neighboring graph* C_5*,* $M_1\,(C_5) = \overline{M_1\,(C_5)} = 20$*;* $M_2\,(C_5) = \overline{M_2\,(C_5)} = 20$*;* $H\,(C_5) = \overline{H\,(C_5)} = 2.5$.

Corollary 3.4. *For a 2n-regular graph G with* $4n+1$ *vertices,* $M_1\,(G) = \overline{M_1\,(G)} = 4n^2(4n+1)$*;* $M_2\,(G) = \overline{M_2\,(G)} = 4n^3(4n+1)$*;* $H\,(G) = \overline{H\,(G)} = \frac{4n+1}{2}$*.*

Proof. $2n$-regular graph G with $4n+1$ vertices is a equally neighboring graph with $d_G\,(v) = \overline{d_G(v)}\ \ \forall\ v \in V(G)$. Here the total number of edges is $n\,(4n+1)$. Hence

$$\begin{aligned} M_1\,(G) = \overline{M_1\,(G)} &= n\,(4n+1)\,[2n+2n] \\ &= 4n^2(4n+1) \\ M_2\,(G) = \overline{M_2\,(G)} &= n\,(4n+1)\,[2n\times 2n] \\ &= 4n^3(4n+1) \end{aligned}$$

$$H(G) = \overline{H(G)} = n(4n+1)\left[\frac{2}{2n+2n}\right]$$
$$= \frac{4n+1}{2}$$

□

Definition 3.5 (Sunlet Graph)**.** *The n-sunlet graph G on $2n$ vertices is obtained by attaching n pendant edges to the cycle $C_n\,(n = 3)$ [19]. The 3-sunlet graph called the net graph is shown in Figure 2.*

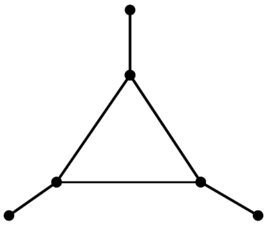

Figure 2.

Definition 3.6 (Complete n-sun Graph)**.** *A n-sun graph [12] is a chordal graph G with $2n$ vertices, $n = 3$, whose vertex set is partitioned into two sets $W = \{w_0, w_1, \ldots, w_{n-1}\}$ and $U = \{u_0, u_1, \ldots, u_{n-1}\}$, such that $U = \{u_0, u_1, \ldots, u_{n-1}\}$ induces a cycle, W is a stable set and for all $i \in \{0, 1, 2, \ldots, n-1\}$ w_i adjacent to exactly u_i and u_{i+1}. A complete n-sun graph is a n-sun graph where $G[U]$ is complete. A complete 4-sun graph is shown in Figure 3.*

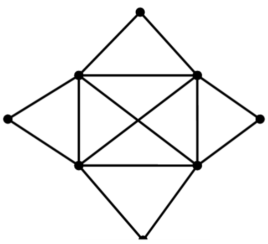

Figure 3.

Definition 3.7 (Fan Graph)**.** *The fan graph $f_n\,(n = 2)$, [4] is obtained by joining all vertices of a path P_n to a further vertex, called the center, by edges. It is the graph join $P_n + K_1$. The fan f_4 is shown in Figure 4.*

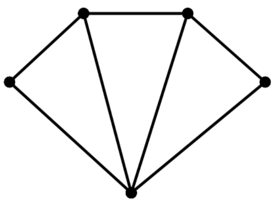

Figure 4.

Definition 3.8 (Friendship Graph)**.** *The friendship graph F_n [20] is constructed by joining n copies of cycle C_3 with a common vertex. The friendship graph F_4 is shown in the Figure 5.*

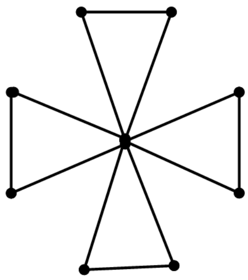

Figure 5.

Definition 3.9 (Helm)**.** *The helm graph H_n $(n = 3)$ [21] is the graph obtained from an n-wheel graph by adjoining a pendant edge to each vertex of the cycle. The helm graph H_3 is shown in the Figure 6.*

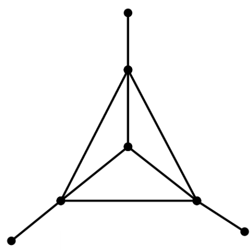

Figure 6.

Definition 3.10 (Perfect Binary Tree)**.** *A perfect binary tree [22] is a binary tree in which all interior vertices have two children and all leaves have the same depth or same level. A perfect binary tree of height h has $2^{h+1} - 1$ vertices. A perfect binary tree of height 3 is shown in the Figure 7.*

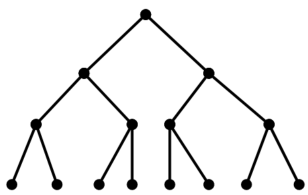

Figure 7.

Theorem 3.11. *For a sunlet graph G obtained from cycle C_n $(n = 3)$, $\overline{M_1(G)} = 2n(4n-7)$; $\overline{M_2(G)} = 4n(n-2)(2n-3)$; $\overline{H(G)} = \frac{n}{2}\frac{(4n-7)}{(n-2)(2n-3)}$.*

Proof. The n pendant vertices attached to C_n each has degree 1. The number of non-neighbors of each pendant vertex is $(2n-2)$. The remaining vertices lying in the cycle C_n, each has degree 3. The number of non-neighbors of the vertex $v \in V(C_n)$ is $(2n-4)$. Since there are $2n$ edges.

The Non-Neighbor First Zagreb Index of a sunlet graph G

$$= n\left[(2n-2)+(2n-4)\right]+n\left[(2n-4)+(2n-4)\right]. \text{ Hence } \overline{M_1(G)} = 2n(4n-7).$$

The Non-Neighbor Second Zagreb Index of a sunlet graph G

$$= n\left[(2n-2)(2n-4)\right]+\ n\left[(2n-4)(2n-4)\right]$$

$$= n(2n-4)(4n-6)$$

$$\text{Hence } \overline{M_2(G)} = 4n(n-2)(2n-3).$$

The Non-Neighbor Harmonic Index of a sunlet graph G

$$= n\left[\frac{2}{(2n-2)+(2n-4)}\right]+n\left[\frac{2}{(2n-4)+(2n-4)}\right]. \text{ Hence } \overline{H(G)} = \frac{n}{2}\frac{(4n-7)}{(n-2)(2n-3)}.$$

□

Theorem 3.12. *For complete n-sun graph G obtained from complete graph K_n $(n=2)$, $\overline{M_1(G)} = n^3+3n^2-8n$; $\overline{M_2(G)} = \frac{n^4+3n^3-20n^2+20n}{2}$ and for $(n=3)$, $\overline{H(G)} = \frac{n}{2}\frac{(3n^2-11)}{(3n-5)(n-2)}$.*

Proof. The central vertices lying in the complete graph each has degree $n+1$. The non-neighbors of these central vertices is $(n-2)$. The vertex in the outer ring each has degree 2. The non-neighbors of these vertices is $(2n-3)$. There are nC_2 edges lying in the complete graph and $2n$ edges lying in the outer ring. Hence

The Non-Neighbor First Zagreb Index of a sun graph G

$$= 2n\left[(2n-3)+(n-2)\right]+\ nC_2\left[(n-2)+(n-2)\right]. \text{ Hence } \overline{M_1(G)} = n^3+3n^2-8n.$$

The Non-Neighbor Second Zagreb Index of a sun graph G

$$= 2n\left[(2n-3)(n-2)\right]+nC_2\left[(n-2)(n-2)\right]. \text{ Hence } \overline{M_2(G)} = \frac{n^4+3n^3-20n^2+20n}{2}.$$

The Non-Neighbor Harmonic Index of a sun graph G

$$= 2n\left[\frac{2}{(2n-3)+(n-2)}\right]+nC_2\left[\frac{2}{(n-2)+(n-2)}\right]. \text{ Hence } \overline{H(G)} = \frac{n}{2}\frac{(3n^2-11)}{(3n-5)(n-2)}.$$

□

Theorem 3.13. *For fan graph f_n $(n=4)$, $\overline{M_1(f_n)} = 3n^2-11n+10$; $\overline{M_2(f_n)} = (n-3)(n^2-4n+5)$; $\overline{H(f_n)} = \frac{4}{2n-5}+\frac{4}{n-2}+\frac{2(n-2)}{n-3}+1$.*

Proof. The center vertex has no non-neighbors. There are two corner vertices each of degree 2 and the number of non-neighbors of these vertices is $(n-2)$. The remaining $(n-2)$ vertices are each with degree 3. The number of non neighbors of these remaining vertices is $(n-3)$. There are totally $2n-1$ edges. Hence

The Non-Neighbor First Zagreb Index of a fan graph $f_n(n=4)$

$$= 2\left[(n-2)+(n-3)\right]+2\left[(n-2)\right]+(n-2)\left[(n-3)\right]+(n-3)\left[(n-3)+(n-3)\right]. \text{ Hence } \overline{M_1(f_n)} = 3n^2-11n+10.$$

The Non-Neighbor Second Zagreb Index of a fan graph f_n $(n=4)$

$$= 2\left[(n-2)(n-3)\right] + (n-3)\left[(n-3)(n-3)\right]. \text{ Hence } \overline{M_2\left(f_n\right)} = (n-3)\left(n^2-4n+5\right).$$

The Non-Neighbor Harmonic Index of a fan graph $f_n(n \geq 4)$

$$= 2\left[\frac{2}{(n-2)+(n-3)}\right] + 2\left[\frac{2}{(n-2)}\right] + (n-2)\left[\frac{2}{(n-3)}\right] + (n-3)\left[\frac{2}{(n-3)+(n-3)}\right].$$

Hence $\overline{H\left(f_n\right)} = \frac{4}{2n-5} + \frac{4}{n-2} + \frac{2(n-2)}{n-3} + 1$. □

Remark 3.14. *For the fan graph f_3, The Non-Neighbor First Zagreb Index $\overline{M_1\left(f_3\right)} = 4$ and the Non-Neighbor Second Zagreb Index $\overline{M_2\left(f_3\right)} = 0$.*

Theorem 3.15. *For friendship graph F_n $(n=2)$, $\overline{M_1\left(F_n\right)} = 8n(n-1)$; $\overline{M_2\left(F_n\right)} = 4n(n-1)^2$, $\overline{H(F_n)} = \frac{5n}{2(n-1)}$.*

Proof. The common vertex in the center has no non-neighbors. The remaining $2n$ vertices have degree 2. The number of non-neighbors of these remaining vertices is $(2n-2)$. There are totally $3n$ edges. Hence
The Non-Neighbor First Zagreb Index of a friendship graph F_n $(n=2)$

$$= n\left[(2n-2)+(2n-2)\right] + 2n(0+(2n-2)). \text{ Hence } \overline{M_1\left(F_n\right)} = 8n(n-1).$$

The Non-Neighbor Second Zagreb Index of a friendship graph F_n $(n=2)$

$$= n\left[(2n-2)(2n-2)\right] + 2n(0 \times (2n-2)) \text{ Hence } \overline{M_2\left(F_n\right)} = 4n(n-1)^2.$$

The Non-Neighbor Harmonic Index of a friendship graph F_n $(n=2)$

$$= n\left[\frac{2}{(2n-2)+(2n-2)}\right] + 2n\left[\frac{2}{0+(2n-2)}\right] \text{ Hence } \overline{H\left(F_n\right)} = \frac{5n}{2(n-1)}.$$

□

Theorem 3.16. *For helm graph H_n $(n=3)$, $\overline{M_1\left(H_n\right)} = n(11n-17)$; $\overline{M_2\left(H_n\right)} = 10n(n-2)(n-1)$; $\overline{H(H_n)} = \frac{2n}{3n-4} + \frac{2n}{4n-5} + \frac{2n}{4n-8}$.*

Proof. The helm graph H_n is the graph obtained from a wheel graph W_n by adjoining a pendant edge at each vertex of the cycle. The vertex at the center of the wheel has n pendant vertices as its non-neighbors. The pendant vertices have $(2n-1)$ non-neighbors. The vertex on the wheel has $(2n-4)$ non neighbors. There are totally $3n$ edges. Hence
The Non-Neighbor First Zagreb Index of a helm graph

$$\begin{aligned} H_n\,(n=3) &= n\left[n+(2n-4)\right] + n\left[(2n-4)+(2n-1)\right] + n[(2n-4)+(2n-4) \\ &= n\,(3n-4) + n\,(4n-5) + n\,(4n-8) \\ \textit{Hence } \overline{M_1\left(H_n\right)} &= n(11n-17). \end{aligned}$$

The Non-Neighbor Second Zagreb Index of a helm graph

$$H_n\,(n=3) = n\,[n.(2n-4)] + n\,[(2n-4)\,.(2n-1)] + n[(2n-4)\,.(2n-4)$$
$$= (2n-4)\,[n^2 + n\,(2n-1) + n\,(2n-4)$$
$$= (2n-4)\,(5n^2-5n)$$
$$\text{Hence } \overline{M_2\,(H_n)} = 10n\,(n-2)\,(n-1).$$

The Non-Neighbor Harmonic Index of a helm graph

$$H_n\;(n=3) = n\left[\frac{2}{n+(2n-4)}\right] + n\left[\frac{2}{(2n-4)+(2n-1)}\right] + n\left[\frac{2}{(2n-4)+(2n-4)}\right]$$
$$= n\left[\frac{2}{(3n-4)}\right] + n\left[\frac{2}{(4n-5)}\right] + n\left[\frac{2}{(4n-8)}\right]$$
$$\text{Hence } \overline{H(H_n)} = \frac{2n}{3n-4} + \frac{2n}{4n-5} + \frac{2n}{4n-8}.$$

□

Theorem 3.17. *For a perfect binary tree T of height h $(h=2)$, $\overline{M_1\,(T)} = 2^{2h+3} - 262^h + 22$; $\overline{M_2\,(T)} = 2^{3h+3} + 842^h - 442^{2h} - 60$; $\overline{H(T)} = \frac{2^2}{2^{h+2}-9} + \frac{2^{h+1}}{2^{h+2}-8} + \frac{2^{h+1}-8}{2^{h+2}-10}$.*

Proof. In a perfect binary tree the vertex at the beginning has degree 2. The non-neighbors of this vertex is $2^{h+1}-4$. The children vertices at the end each have degree 1. The number of non neighbors of these vertices is $2^{h+1}-3$. The remaining vertices each has degree 3. The number of non-neighbors of these remaining vertices is $2^{h+1}-5$. There are totally $2^{h+1}-2$ edges. Hence

The Non-Neighbor First Zagreb Index of a perfect binary tree T of height h, $(h=2)$

$$= 2\left[\left(2^{h+1}-4\right) + \left(2^{h+1}-5\right)\right] + 2^h\left[\left(2^{h+1}-5\right) + \left(2^{h+1}-3\right)\right] + (2^{h-}2^2)\left[(2^{h+2}-5) + (2^{h+2}-5)\right]$$
$$= 2\left[2^{h+2}-9\right] + 2^h\left[2^{h+2}-8\right] + (2^{h-}2^2)\left[2^{h+2}-10\right]$$
$$\text{Hence } \overline{M_1\,(T)} = 2^{2h+3} - 262^h + 22.$$

The Non-Neighbor Second Zagreb Index of a perfect binary tree T of height h, $(h=2)$

$$= 2\left[\left(2^{h+1}-4\right)\left(2^{h+1}-5\right)\right] + 2^h\left[\left(2^{h+1}-5\right)\left(2^{h+1}-3\right)\right] + (2^{h-}2^2)\left[(2^{h+2}-5)(2^{h+2}-5)\right]$$
$$\text{Hence } \overline{M_2\,(T)} = 2^{3h+3} + 842^h - 442^{2h} - 60.$$

The Non-Neighbor Second Zagreb Index of a perfect binary tree T of height h, $(h=2)$

$$= 2\left[\frac{2}{(2^{h+1}-4)+(2^{h+1}-5)}\right] + 2^h\left[\frac{2}{(2^{h+1}-5)+(2^{h+1}-3)}\right] + \left(2^h-2^2\right)\left[\frac{2}{(2^{h+2}-5)+(2^{h+2}-5)}\right]$$
$$\text{Hence } \overline{H(T)} = \frac{2^2}{2^{h+2}-9} + \frac{2^{h+1}}{2^{h+2}-8} + \frac{2^{h+1}-8}{2^{h+2}-10}.$$

□

4. Relation Between Non-Neighbor Zagreb Indices and Non-Neighbor Harmonic Index with the Zagreb Indices, Coindices and Harmonic Index

Theorem 4.1. *Let G be a graph with n vertices and m edges. Then* $\overline{M_1(G)} = 2m(n-1) - M_X(G)$.

Proof.

$$\begin{aligned}\overline{M_1(G)} &= \sum_{uv\in E(G)} \left(\overline{d_G(u)} + \overline{d_G(v)}\right) \\ &= \sum_{uv\in E(G)} (n-1-d_G(u)+n-1-d_G(v)) \\ &= \sum_{uv\in E(G)} (2n-2-(d_G(u)+d_G(?)) \\ &= \sum_{uv\in E(G)} (2n-2) + \sum_{uv\in E(G)} (d_G(u)+d_G(v)) \\ &= 2m(n-1) - M_X(G).\end{aligned}$$

□

Corollary 4.2. *For any simple graph G,* $\overline{M_1(G)} = \overline{M_1}(G) = \overline{M_1}\left(\overline{G}\right)$.

Theorem 4.3. *Let G be a graph with n vertices and m edges, then* $\overline{M_2(G)} = m(n-1)^2 - (n-1)M_1(G) + M_2(G)$.

Proof.

$$\begin{aligned}\overline{M_2(G)} &= \sum_{uv\in E(G)} \left(\overline{d_G(u)}.\overline{d_G(v)}\right) \\ &= \sum_{uv\in E(G)} (n-1-d_G(u))(n-1-d_G(v)) \\ &= \sum_{uv\in E(G)} \left((n-1)^2 - (n-1)[d_G(u)+d_G(v)] + (d_G(u)\,d_G(v))\right) \\ &= m(n-1)^2 - (n-1)\sum_{uv\in E(G)} (d_G(u)+d_G(v)) + \sum_{uv\in E(G)} (d_G(u)\,d_G(v)) \\ \overline{M_2(G)} &= m(n-1)^2 - (n-1)M_1(G) + M_2(G)\end{aligned}$$

□

Corollary 4.4. *For any simple graph G,* $\overline{M_2(G)} = \overline{M_2}\left(\overline{G}\right)$.

Theorem 4.5. *For any simple graph G,* $\frac{\overline{H(G)}}{H(G)} = \frac{M_X(G)}{\overline{M_1(G)}}$.

Proof.

$$\begin{aligned}\frac{\overline{H(G)}}{H(G)} &= \frac{\sum_{uv\in E(G)}\left(\frac{2}{\overline{d_G(u)}+\overline{d_G(v)}}\right)}{\sum_{uv\in E(G)}\left(\frac{2}{d_G(u)+d_G(v)}\right)} \\ &= \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{\overline{d_G(u)}+\overline{d_G(v)}} \\ &= \frac{\sum_{uv\in E(G)}(d_G(u)+d_G(v))}{\sum_{uv\in E(G)}\left(\overline{d_G(u)}+\overline{d_G(v)}\right)} \\ &= \frac{M_X(G)}{\overline{M_1(G)}}\end{aligned}$$

□

5. Conclusion

In this paper we have introduced the computation of new graph theoretical topological indices based on non-neighbors of all the vertices of the graph G. We have given the formula for the computation of topological indices called Non-Neighbor First Zagreb Index, Non-Neighbor Second Zagreb Index and Non-Neighbor Harmonic Index. These topological indices have been computed for some standard graphs. The definition of equally neighboring graph and results on these graphs has been discussed. These topological indices have been computed for some other special types of graphs. Also the relationship between these non-neighbor topological indices with the older Zagreb Indices, coindices and Harmonic Index has been studied in detail. It has been noted that Non-Neighbor First Zagreb Index coincides with the First Zagreb coindex. But Non-Neighbor Second Zagreb Index does not coincide with the second Zagreb coindex. Hence Non-Neighbor First Zagreb Index provides an alternating formula for the First Zagreb coindex.

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