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Common Fixed Point Theorem in Intuitionistic Fuzzy 3-Metric Spaces

Research Article

V.Malliga Devi[1]*, R.Mohan Raj[1], M.Jeyaraman[2] and M.Kayalvizhi[2]

1 Department of Mathematics, University V.O.C College of Engineering, Anna University, Tirunelveli Region, Tuticorin, India.

2 PG and Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai, India.

Abstract: Urmila Mishra [12] proved a common fixed point theorem in fuzzy metric space by introducing reciprocal continuity. In this paper, we have extended and generalize [12] the above result for intuitionistic fuzzy 3-metric spaces.

MSC: 47H10, 54H25.

Keywords: Metric Space, Common Fixed Point, Intuitionistic Fuzzy 2-Metric Space, Intuitionistic Fuzzy 3-Metric Space.

1. Introduction

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [13]. In 2004, Park [8] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] Using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to Kramosil and Michalek [4]. Further, Alaca et al. [1] proved intuitionistic fuzzy Banach and intuitionistic fuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given in [8]. The idea of fuzzy 2-metric space and fuzzy 3-metric space was used by Sushil Sharma [11] and obtained some fruitful results. Motivated by Sushil Sharma [11], we prove some common fixed point theorem in fuzzy 2-metric space and fuzzy 3-metric space by employing the notion of reciprocal continuity, of which we can widen the scope of many interesting fixed point theorems in fuzzy metric space. Urmila Mishra [12] proved a common fixed point theorem in fuzzy metric space by introducing reciprocal continuity. In this paper, we have extended and generalize [12] the above result in intuitionistic fuzzy 3-metric spaces.

2. Preliminaries

Definition 2.1. *A 5-tuple $(X, \mathcal{M}, N, *, \diamondsuit)$ is called a generalized intuitionistic fuzzy 2- metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and $\mathcal{M}$, $\mathcal{N}$ are fuzzy sets on $X^3 \times [0, \infty]$ satisfying the following conditions:*

For each $x, y, z, u \in X$ and $t_1,\ t_2,\ t_3 > 0$.

* E-mail: sundersaimd@gmail.com

(a). $\mathcal{M}(x,\ y,\ z,\ t)+\mathcal{N}(x,\ y,\ z,\ t)\leq 1$,

(b). $\mathcal{M}(x,\ y,\ z,\ 0)=0$,

(c). $\mathcal{M}(x,\ y,\ z,\ t)=1$ *for all* $x,\ y$ *in* X *and* $t>0$ *if and only if* $x=y$,

(d). $\mathcal{M}(x,\ y,\ z,\ t)=\ \mathcal{M}\ (p\{x,\ y,\ z\},\ t)$, *where* p *is a permutation function,*

(e). $\mathcal{M}(x,\ y,\ z,\ t_1+t_2+t_3)\geq \mathcal{M}(x,\ y,\ u,\ t_1)*\mathcal{M}(x,\ u,\ z,\ t_2)*\mathcal{M}(u,\ y,\ z,\ t_3)$

(f). $\mathcal{M}(x,\ y,\ z,\ .):[\ 0,\ \infty)\rightarrow[0,\ 1]$ *is left continuous,*

(g). $\mathcal{N}(x,\ y,\ z,\ t)<0$,

(h). $\mathcal{N}(x,\ y,\ z,\ \ 0)=\ 0$ *iff* $x=y=z$,

(i). $\mathcal{N}(x,\ y,\ z,\ t)=\mathcal{N}(\ p\{x,\ y,\ z\},\ t)$, *where* p *is a permutation function,*

(j). $\mathcal{N}(x,\ y,\ z,\ t_1+t_2+t_3)\leq \mathcal{N}(x,\ y,\ u,\ t_1)\diamondsuit\mathcal{N}(x,\ u,\ z,\ t_2)\diamondsuit\mathcal{N}(u,\ y,\ z,\ t_3)$

(k). $\mathcal{N}(x,\ y,\ z,\ .):[\ 0,\ \infty)\rightarrow[0,\ 1]$ *is right continuous.*

Definition 2.2. *A sequence* $\{x_n\}$ *in a intuitionistic fuzzy 2-metric space* $(X,\mathcal{M},\mathcal{N},\ *,\ \diamondsuit)$ *is said to be converge to* x *in* X *iff* $\lim_{n\to\infty}\mathcal{M}(x_n,\ x,\ a,\ t)=1$ *and* $\lim_{n\to\infty}\mathcal{N}(x_n,\ x,\ a,\ t)=0$, *for all* $a\in X$ *and* $t>0$.

Definition 2.3. *Let* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *be a intuitionistic fuzzy 2-metric space. A sequence* $\{x_n\}$ *in* X *is called Cauchy sequence, iff* $\lim_{n\to\infty}\mathcal{M}(x_{n+p},\ x_p,\ a,\ t)=1$ *and* $\lim_{n\to\infty}\mathcal{N}(x_{n+p},\ x_p,\ a,\ t)=0$, *for all* $a\in X$ *and* $p>0,\ t>0$.

Definition 2.4. *A intuitionistic fuzzy 2-metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to be complete iff every Cauchy sequence in X is convergent in X.*

Definition 2.5. *The 5-tuple* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is called a intuitionistic fuzzy 3-metric space if X is an arbitrary set, * is a continuous t-norm and* $\diamondsuit$ *is a continuous t-conorm and* $\mathcal{M},\ \mathcal{N}$ *is a fuzzy set in* $X^4\times[0,\ \infty]$ *satisfying the following conditions, for all* $x,\ y,\ z,\ w,\ u\in X$ *and* $t_1,\ t_2,\ t_3,\ t_4>0$.

(a). $\mathcal{M}(x,\ y,\ z,\ w,\ 0)+\mathcal{N}(x,\ y,\ z,\ w,\ 0)\leq 1$,

(b). $\mathcal{M}(x,\ y,\ z,\ w,\ 0)=0$,

(c). $\mathcal{M}(x,\ y,\ z,\ w,\ t)=1$, *for all* $t>0$.

(d). $\mathcal{M}(x,\ y,\ z,\ w,\ t)=\mathcal{M}(p(x,\ y,\ z,\ w),\ t)$, *where* p *is a Permutation function,*

(e). $\mathcal{M}(x,\ y,\ z,\ w,\ t_1+t_2+t_3+t_4)\geq\mathcal{M}(x,\ y,\ z,\ u,\ t_1)*\mathcal{M}(x,\ y,\ u,\ w,\ t_2)*\mathcal{M}(x,\ u,\ z,\ w,\ t_3)*\mathcal{M}(x,\ u,\ z,\ w,\ t_4)$,

(f). $\mathcal{M}(x,\ y,\ z,\ w,\ .):[0,\ \infty)\rightarrow[0,\ 1]$ *is left continuous ,*

(g). $\mathcal{N}(x,\ y,\ z,\ w,\ 0)=1$,

(h). $\mathcal{N}(x,\ y,\ z,\ w,\ t)=0$, *for all* $t>0$

(i). $\mathcal{N}(x,\ y,\ z,\ w,\ t)=\mathcal{N}(p(x,\ y,\ z,\ w),\ t)$, *where* p *is a permutation function,*

(j). $\mathcal{N}(x,\ y,\ z,\ w,\ t_1+t_2+t_3+t_4)\ \leq\ \mathcal{N}(x,\ y,\ z,\ u,\ t_1)\mathcal{N}(x,\ y,\ u,\ w,\ t_2)\mathcal{N}(x,\ u,\ z,\ w,\ t_3)\mathcal{N}(x,\ u,\ z,\ w,\ t_4)$,

(k). $\mathcal{N}(x,\ y,\ z,\ w,\ .) : [0,\ \infty) \to [0,\ 1]$ *is right continuous.*

Definition 2.6. *A sequence* $\{x_n\}$ *in a intuitionistic fuzzy 3-metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to converge to x in X iff* $\lim_{n\to\infty} \mathcal{M}(x_n,\ x,\ a,\ b,\ t) = 1$ *and* $\lim_{n\to\infty} \mathcal{N}(x_n,\ x,\ a,\ b, t) = 0$*, for all* $a, b \in X$*,* $t > 0$*.*

Definition 2.7. *Let* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *be a intuitionistic fuzzy 3-metric space. A sequence* $\{x_n\}$ *in X is called Cauchy sequence iff* $\lim_{n\to\infty} \mathcal{M}(x_{n+p},\ x_n,\ a,\ b,\ \ t) = 1$ *and* $\lim_{n\to\infty} \mathcal{N}(x_{n+p},\ x_n,\ a,\ b, t) = 0$*, for all* $a, b \in X$*,* $p > 0$ *and* $t > 0$*.*

3. Intuitionistic Fuzzy 3-Metric Spaces

Theorem 3.1. *Let A, B, C, R, S and T be self maps on a complete intuitionistic fuzzy 3-metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$*, where * is a continuous t-norm,* $\diamondsuit$ *is a continuous t-conorm satisfying*

(1). $AX \subseteq TX$*,* $BX \subseteq SX$*,* $CX \subseteq RX$*.*

(2). $(B,\ S)$ *and* $(C,\ T)$ *are weakly compatible and reciprocally continuous,*

(3). For each $x,\ y,\ z \in X$ *and* $t > 0$*,* $\mathcal{M}(Ax,\ By,\ Cz,\ u,\ t) \geq \phi(\mathcal{M}(Rx,\ Sy,\ Tz,\ u,\ t))$*,* $\mathcal{N}(Ax,\ By,\ Cz,\ u,\ t) \leq \psi(\mathcal{N}(Rx,\ Sy,\ Tz,\ u,\ t))$*,*

Where $\phi,\ \psi : [0,\ 1] \to [0,\ 1]$ *is a continuous function such that* $\phi(1) = 1$*,* $\phi(0) = 0$*,* $\psi(0) = 0$ *and* $\psi(1) = 1$ *and* $\phi(a) > a$ *and* $\psi(a) < a$ *for each* $0 < a < 1$*. If* $(A,\ R)$ *is semi compatible and reciprocally continuous, then A, B, C, R, S and T have a common fixed point.*

Proof. Suppose $x_0 \in X$ be an arbitrary point. Then there exists $x_1,\ x_2,\ x_3 \in X$ such that $Ax_0 = Tx_1$, $Bx_1 = Sx_2$ and $Cx_2 = Rx_3$. Thus we can form sequences $\{z_n\}$, $\{y_n\}$ and $\{x_n\}$ in X such that $y_{2n+1} = Ax_{2n} = Tx_{2n+1}$, $y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$, $y_{2n+3} = Cx_{2n+2} = Rx_{2n+3}$, for $n = 0,\ 1,\ \ldots$

$$\begin{aligned}
\mathcal{M}(y_{2n+1},\ y_{2n+2},\ y_{2n+3},\ u,\ t) &= \mathcal{M}(Ax_{2n},\ Bx_{2n+1},\ Cx_{2n+2},\ u,\ t)\\
&\geq \phi(\mathcal{M}(Rx_{2n},\ Sx_{2n+1},\ Tx_{2n+2},\ u,\ t))\\
&> \phi(\mathcal{M}(y_{2n},\ y_{2n+1},\ y_{2n+2},\ u,\ t))\\
\mathcal{N}(y_{2n+1},\ y_{2n+2},\ y_{2n+3},\ u,\ t) &= \mathcal{N}(Ax_{2n},\ Bx_{2n+1},\ Cx_{2n+2},\ u,\ t)\\
&\leq \psi(\mathcal{N}(Rx_{2n},\ Sx_{2n+1},\ Tx_{2n+2},\ u,\ t))\\
&< \psi(N(y_{2n},\ y_{2n+1},\ y_{2n+2},\ u,\ t)).
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\mathcal{M}(y_{2n+3},\ y_{2n+4},\ y_{2n+5},\ u,\ t) > \phi(\mathcal{M}(y_{2n+2},\ y_{2n+3},\ y_{2n+4},\ u,\ t)) \text{ and}\\
&\mathcal{N}(y_{2n+3},\ y_{2n+4},\ y_{2n+5},\ u,\ t) < \psi(\mathcal{N}(y_{2n+2},\ y_{2n+3},\ y_{2n+4},\ u,\ t)).
\end{aligned}$$

More generally,

$$\begin{aligned}
&\mathcal{M}(y_{n+1},\ y_n,\ y_{n-1},\ u,\ t) > \phi(\mathcal{M}(y_n,\ y_{n-1},\ y_{n-2},\ u,\ t)) \text{ and}\\
&\mathcal{N}(y_{n+1},\ y_n,\ y_{n-1},\ u,\ t) < \psi(\mathcal{N}(y_n,\ y_{n-1},\ y_{n-2},\ u,\ t)).
\end{aligned}$$

Therefore, $\{\mathcal{M}(y_{n+1}, y_n, y_{n-1}, u, t)\}$ is an increasing sequence and $\{\mathcal{N}(y_{n+1}, y_n, y_{n-1}, u, t)\}$ is a decreasing sequence of positive real numbers in $[0, 1]$ and tends to limit $\ell \leq 1$. We claim that $\ell = 1$. If $\ell < 1$, then $\mathcal{M}(y_{n+1}, y_n, y_{n-1}, u, t) > \phi(\mathcal{M}(y_n, y_{n-1}, y_{n-2}, u, t))$ and $\mathcal{N}(y_{n+1}, y_n, y_{n-1}, u, t) < \psi(\mathcal{N}(y_n, y_{n-1}, y_{n-2}, u, t))$. On letting $n \to \infty$, we get, $\lim_{n\to\infty} \mathcal{M}(y_{n+1}, y_n, y_{n-1}, u, t) > \lim_{n\to\infty} \mathcal{M}(y_n, y_{n_1}, y_{n-2}, u, t)$ and $\lim_{n\to\infty} \mathcal{N}(y_{n+1}, y_n, y_{n-1}, u, t) < \lim_{n\to\infty} \mathcal{N}(y_n, y_{n_1}, y_{n-2}, u, t)$ that is $\ell \geq \phi(1) > 1$ and $\ell \leq \psi(1) < 1$ a contradiction. Now for any positive integer p.

$$
\begin{aligned}
\mathcal{M}(y_n, y_{n+1}, y_{n-p}, u, t) \geq \mathcal{M}&\left(y_n, y_{n+1}, y_{n+2}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \mathcal{M}\left(y_{n+1}, y_{n+2}, y_{n+3}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \cdots * \mathcal{M}\left(y_{n+p-3}, y_{n+p-2}, y_{n+p-1}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \mathcal{M}\left(y_n, y_{n+1}, y_{n+2}, z, \frac{t}{(3(p-1)+1)}\right)\\
&* \mathcal{M}\left(y_{n+1}, y_{n+2}, y_{n+3}, z, \frac{t}{(3(p-1)+1)}\right)\\
&* \mathcal{M}\left(y_{n+2}, y_{n+3}, y_{n+4}, z, \frac{t}{(3(p-1)+1)}\right)\\
&* \cdots * \mathcal{M}\left(y_{n+p-2}, y_{n+p-1}, y_{n+p}, z, \frac{t}{(3(p-1)+1)}\right)\\
&* \mathcal{M}\left(y_{n+p-2}, y_{n+p-1}, y_{n+p}, z, \frac{t}{(3(p-1)+1)}\right) \text{ and}\\
\mathcal{N}(y_n, y_{n+1}, y_{n-p}, u, t) \leq \mathcal{N}&\left(y_n, y_{n+1}, y_{n+2}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \mathcal{N}\left(y_{n+1}, y_{n+2}, y_{n+3}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \ldots \Diamond \mathcal{N}\left(y_{n+p-3}, y_{n+p-2}, y_{n+p-1}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \mathcal{N}\left(y_n, y_{n+1}, y_{n+2}, z, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \mathcal{N}\left(y_{n+1}, y_{n+2}, y_{n+3}, z, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \mathcal{N}\left(y_{n+2}, y_{n+3}, y_{n+4}, z, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \ldots \Diamond \mathcal{N}\left(y_{n+p-2}, y_{n+p-1}, y_{n+p}, z, \frac{t}{(3(p-1)+1)}\right)\\
&\Diamond \mathcal{N}\left(y_{n+p-2}, y_{n+p-1}, y_{n+p}, z, \frac{t}{(3(p-1)+1)}\right)
\end{aligned}
$$

Taking limits

$$
\begin{aligned}
\mathcal{M}(y_n, y_{n+1}, y_{n-p}, u, t) \geq &\lim_{n\to\infty} \mathcal{M}\left(y_n, y_{n+1}, y_{n+2}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \lim_{n\to\infty} \mathcal{M}\left(y_{n+1}, y_{n+2}, y_{n+3}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \lim_{n\to\infty} \mathcal{M}\left(y_{n+2}, y_{n+3}, y_{n+4}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \cdots * \lim_{n\to\infty} \mathcal{M}\left(y_{n+p-3}, y_{n+p-2}, y_{n+p-1}, y_{n+p}, \frac{t}{(3(p-1)+1)}\right)\\
&* \lim_{n\to\infty} \mathcal{M}\left(y_n, y_{n+1}, y_{n+2}, z, \frac{t}{(3(p-1)+1)}\right)\\
&* \lim_{n\to\infty} \mathcal{M}\left(y_{n+1}, y_{n+2}, y_{n+3}, z, \frac{t}{(3(p-1)+1)}\right)\\
&* \lim_{n\to\infty} \mathcal{M}\left(y_{n+2}, y_{n+3}, y_{n+4}, z, \frac{t}{(3(p-1)+1)}\right)
\end{aligned}
$$

$$* \cdots * \lim_{n\to\infty} \mathcal{M}\left(y_{n+p-2},\ y_{n+p-1},\ y_{n+p},\ z,\ \frac{t}{(3(p-1)+1)}\right)$$
$$* \lim_{n\to\infty} \mathcal{M}\left(y_{n+p-2},\ y_{n+p-1},\ y_{n+p},\ z,\ \frac{t}{(3(p-1)+1)}\right) \text{ and}$$
$$\mathcal{N}(y_n,\ y_{n+1},\ y_{n-p},\ u,\ t) \leq \lim_{n\to\infty} \mathcal{N}\left(y_n,\ y_{n+1},\ y_{n+2},\ y_{n+p},\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+1},\ y_{n+2},\ y_{n+3},\ y_{n+p},\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+2},\ y_{n+3},\ y_{n+4},\ y_{n+p},\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \ldots \diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+p-3},\ y_{n+p-2},\ y_{n+p-1},\ y_{n+p},\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_n,\ y_{n+1},\ y_{n+2},\ z,\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+1},\ y_{n+2},\ y_{n+3},\ z,\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+2},\ y_{n+3},\ y_{n+4},\ z,\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \ldots \diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+p-2},\ y_{n+p-1},\ y_{n+p},\ z,\ \frac{t}{(3(p-1)+1)}\right)$$
$$\diamondsuit \lim_{n\to\infty} \mathcal{N}\left(y_{n+p-2},\ y_{n+p-1},\ y_{n+p},\ z,\ \frac{t}{(3(p-1)+1)}\right)$$

that is , $\lim_{n\to\infty} \mathcal{M}(y_n,\ y_{n+1},\ y_{n+p},\ u,\ t) \geq 1*1*1*\cdots*1 = 1$ and $\lim_{n\to\infty} \mathcal{N}(y_n,\ y_{n+1},\ y_{n+p},\ u,\ t) \leq 0 \diamondsuit 0 \diamondsuit 0 \diamondsuit \ldots \diamondsuit 0 = 0$. Which means $\{y_n\}$ is a Cauchy sequence in X. Since X is complete $y_n \to w$ in X. That is $\{Ax_{2n}\}$, $\{Tx_{2n+1}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n+2}\}$, $\{Cx_{2n+2}\}$, $\{Rx_{2n+2}\}$ also converge to w in X. That is $\lim_{n\to\infty} Rx_{2n} \to w$ and $\lim_{n\to\infty} Ax_{2n} \to w$. Since (A, R) is semi-compatible, $\lim_{n\to\infty} Ax_{2n} \to Rw$. Also (A, R) is reciprocal continuous also, therefore, $\lim_{n\to\infty} Ax_{2n} \to Aw$ combining this process, we get $Aw = Rw$. Now to prove that $Aw = w$, for if we consider that $Aw \neq w$. Then by the contractive condition,

$$\mathcal{M}(Aw,\ Bx_{2n+1},\ Cx_{2n+2},\ u,\ t) \geq \phi(\mathcal{M}(Rw,\ Sx_{2n+1},\ Tx_{2n+2},\ u,\ t)) \text{ and}$$
$$\mathcal{N}(Aw,\ Bx_{2n+1},\ Cx_{2n+2},\ u,\ t) \leq \psi(\mathcal{N}(Rw,\ Sx_{2n+1},\ Tx_{2n+2},\ u,\ t))$$

Letting n $\to \infty$, $\mathcal{M}(Aw,\ w, w,\ u,\ t) \geq \phi(\mathcal{M}(Rw,\ w, w,\ u,\ t)) > \mathcal{M}(Aw,\ w,\ w,\ u,\ t)$ and $\mathcal{N}(Aw,\ w, w,\ u,\ t) \leq \psi(\mathcal{N}(Rw,\ w,\ w,\ u,\ t)) < \mathcal{N}(Aw,\ w,\ w,\ u,\ t)$ a contradiction. Therefore $Aw = w = Rw$. Since (B, S) and (C, T) are weakly compatible and reciprocally continuous, as above, we get $Bw = w = Sw$ and $Cw = w = Tw$. Therefore A, B, C, R, S and T has a common fixed point. □

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