International Journal of Mathematics And its Applications
Volume 4, Issue 2–D (2016), 139–152.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Inventory Model for Deteriorating Items Involving Trade Credit Policy in Three-Echelon Supply Chain System

Research Article

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Abstract: In present study, we generalize order linked trade credit policy in three echelon supply chain system where manufacturer, distributor and retailer are involved. Here manufacturer provide delay period to distributors also distributor provide trade credit policy to his retailers. In this paper, we discuss a three echelon supply chain system as cost minimization to determine the system's optimal cycle time. We determine the optimal order time, order quantity and optimal payment time. To investigate the effect of changes in inventory parameter values on the optimal policy, a sensitivity analysis is conducted.

MSC: 90B05.

Keywords: Constant Demand, Deterioration, Order Linked Trade Credit,Three- Echelon Supply Chain.

1. Introduction

Traditionally, inventory models considered the different subsystem in the supply chain independently. With the recent advances in communication and information technologies, the integration of these function are a common phenomena. Moreover due to limited resources, increasing competition and market globalization, enterprises are force to develop supply chain that can respond quickly to customer need with minimum stock and minimum service level.

Deterioration plays a significant role in many inventory systems. Deterioration is defined as decay, dryness and spoilage. It is the process in which an item loses its utility and becomes useless. In the real life situation, it is too difficult to preserve highly volatile items like alcohol,liquid medicines, blood, etc., for all manufacturing sectors. These types of items may deteriorate over time. So decay or deterioration of physical goods in stock is a very realistic factor and there is a big need to consider this in inventory modeling. Palanivel and Uthayakumar [7] discussed with an economic production quantity model for deteriorating items with price and advertisement dependent demand. Also considered three types of continuous probabilistic deterioration functions to determine the total inventory cost.

In real life, a supplier/retailer allows their retailers/customers a delay of specified credit period for payment without penalty to stimulated the demand of the consumable products. No interest is charged if the amount is paid within the credit period. If not, the interest is charged on the outstanding amount. The credit financing produces two benefits to the supplier and the retailer [2] 1 it not only encourages customers to order more, but also attracts new customers 2 it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price

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reductions. On the other hand, the policy of granting credit terms adds not only an additional cost but also an additional dimension of default risk to the supplier/retailer.

At the end of the trade credit period, some retailers keep their profits for emergency or other use rather than paying paying off the loan while some retailers will pay off the amount owed to the supplier whenever they have money obtained from sales. That is, the retailer has two possible methods to pay off the loan based on his need. Narayan et al. [6] developed the integrated manufacturer- distributor - retailer models with different type of permissible delays in payments to determine the optimal replenishment time interval and replenishment frequency to reduce the total system costs to the all.

Thangam and Uthayakumar [9] implemented two different payment methods for the retailer to pay off the loan to the supplier under two- echelon trade credit scenario. Jui [4] derived a production model for the lot-size inventory system with finite production rate, taking into consideration the effect of decay and the condition of permissible delay in payments, in which the restrictive assumption of a permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank. Narayan et al. [5] presents the integrated manufacturer-distributor-retailer models with two-level permissible delays in payments (manufacturer offers the distributor and distributor, analogously, provides to the retailer) to determine the optimal replenishment time interval and replenishment frequency, with the aim of reducing the total system costs to all the parties. Teng [8] established an easy analytical closed-form solution to the problem. Also the theoretical results obtained here reveal the two managerial phenomena. Aggarwal [1] developed mathematical models in inventory control it is assumed that payment will be made to the supplier for the goods immediately after receiving the consignment. Yang [11] used a method of defuzzification which is called signed distance to find the estimation of annual demand in the fuzzy sense, and then the corresponding optimal n and Q are derived to maximize the total profit.

Tripathy [10] discussed with the development of an inventory model for Weibull deteriorating items with constant demand when delay in payments is allowed to the retailer to settle the account against the purchases made. Jaggi et al. [3] considered price-dependent demand and the possibility of higher interest earn rate than interest payable rate. Yang et al. [12] formulated three-echelon inventory model with defective rate and rework considerations under credit period situation.

2. Notations and Assumptions

The following notations and assumptions are used to the single channel three-echelon supply chain system with the trade credit consideration.

2.1. Notations

Manufacturer's Parameter

D	Demand rate
P	Annual production rate of manufacturer as $P > D$
A_m	Fixed production setup cost per lot size
h_m	Stock holding cost per unit per year(\$/unit/year)
τ_m	The transportation cost of a shipment from manufacturer to supplier
I_{m_1}	The inventory level that changes with time t during production period
I_{m_2}	The inventory level that changes with time t during non- production period
T	Common cycle time of production/ordering cycle
s_m	Selling price is determined by a mark-up over the unit production cost v i.e. $s = nv$, where n is mark-up

I_m	Annual interest rate for calculating the manufacturer's opportunity interest loss due to the delay payment
TC_m	Annual total relevant cost of the manufacturer

Distributor's Parameters

A_d	Distributor's ordering cost per shipment
h_d	Stock holding cost per unit per year (\$/unit/year)
τ_{d_1}	The transportation cost of receiving a shipment from manufacturer
τ_{d_2}	The transportation cost of the distributors of delivering a shipment to retailers
N	Distributors permissible delay period offered by manufacturer to distributor
n	Number of shipment per order from manufacturer to distributor $n \geq 1$
$I_d(t)$	The inventory level that changes with time t during the period T_3
T_3	The replenishment time interval and $T_3 = \frac{T}{n}$
I_o	Annual interest rate for calculating the distributor
s_d	Unit selling price item of good quality
Q_d	Shipment quantity from manufacturer to distributor in each shipment
TC_d	The annual total relevant cost of the distributor

Retailer's Parameter

A_r	The retailer ordering cost per contract
h_r	Stock holding cost per unit per year(\$/unit/year)
τ_r	The fixed transportation cost of receiving a shipment from distributor(\$/shipment)
Q_r	Shipment size from distributor to retailer in each shipment (unit)
s_r	Unit selling price per item of good quality
m	Number of shipment per order from distributor to retailer $m \geq 1$
w	Minimum order quantity at which the trade credit is permitted
M	Retailer's trade credit period offered by distributor linked to order quantity w
I_e	Interest earned per dollar per year
I_p	Interest payable per dollar per year
I_r	Inventory level that changes with time t during the period T_4
T_4	$\frac{T_3}{m} = \frac{T}{mn}$
TC_r	The annual total relevant cost of the retailer

2.2. Assumptions

1. The production rate per unit time P is variable, which is more than the demand rate.

2. Production cost $v(P) = C_{rw} + A_c + \frac{L}{P^g} + KP^h$ where C_{rw}, L and K-negative real numbers to be provide the best fit for the estimated unit cost function.C_{rw}, L and K represent the raw material cost, labour charges and a positive constant. Also g,h are chosen to provide the feasible solution to the model. We shall use $v$and $v(P)$interchangeable

in the rest of the paper.

3. The retailers ordering quantity from distributor has to be on Just in Time (JIT) basis that may require small and frequent replenishment basis and all shipments are of equal basis.

4. Manufacturer offers a certain permissible delay period to his distributor and the distributor offers a conditional trade credit to the retailers such as if $Q_r < w$, the trade credit is not permitted. Otherwise, fixed trade credit period M is permitted. Hence, if $Q_r < w$, pay $s_d Q_r$when order is received. If$Q_r \geq w$, pay $s_d Q_r M$ time periods after the order is received.

5. During the time the account is not settled, generated sales revenue is deposited in an interest bearing account. When $T \geq M$, the account is settled at $T = M$, the retailer starts paying for the higher interest charges on the items in stock. When $T \leq M$, the account is settled at $T = M$ and the retailer does not need to pay any interest charges.

6. The ordering cycle times are equal for both distribution canters and retailers that is same as the production cycle time of the manufacturer.

7. There is no replacement or repair of deteriorated items takes place in a given cycle.

8. The lead time is zero and shortages are not allowed.

9. Time horizon is infinite.

3. Model Formulation

In order not to allow any shortage, the production P is assumed to be higher than the demand rate of the product. Given that in each ordering cycle, the manufacturer delivers n shipments to the distributor and each shipment having Q_d units of products, the manufacturer uses a policy of producing nQ_d units with time dependent production rate in time T_1 shown in figures (1) and (2)

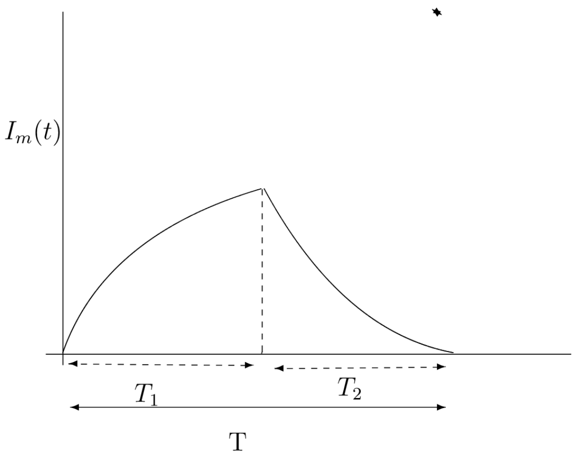

Figure 1. **(a) Manufacturer inventory level with respect to time**

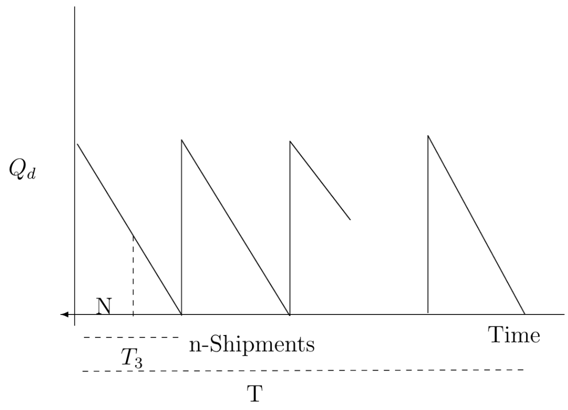

Figure 2. **(b) Distributor inventory level with respect to time**

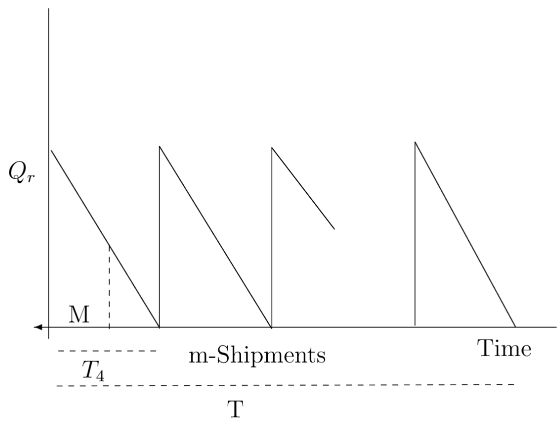

Figure 3. **(c) Retailers inventory level with respect to time**

Distributor again split the quantity Q_d into m-shipments and deliver Q_r units of the products to the m retailers in each shipment. So the inventory of the distribution center resembles a step function, each step having the height of quantity $Q_r(\frac{Q_d}{m})$ shown in figure(3).

3.1. Manufacturer's Model

A variable production rate starts at $t = 0$ and continuous up to $t = T_1$ where inventory level reaches the maximum level. Production then stops at $t = T_1$ and the inventory gradually depletes to zero at the end of cycle time $t = T$ due to consumption as shown in figure (1). Therefore, during the time interval $(0, T_1)$, the system is subject to the effect of production and demand and the time interval $(0, T_2)$, the system is subject to the effect of demand only. Then the change in inventory level can be described by the following differential equation

$$\frac{dI_{m1}(t)}{dt} + \theta I_{m1}(t) = P - D, 0 \leq t \leq T_1 \tag{1}$$

and

$$\frac{dI_{m2}(t)}{dt} = -D, 0 \leq t \leq T_2 \tag{2}$$

with boundary conditions $I_{m1} = 0$ and $I_{m2}(t) = 0$ solution of above equations are

$$I_{m1}(t) = \frac{P-D}{\theta}\left(1 - e^{\theta t}\right), 0 \leq t \leq T_1 \tag{3}$$

and

$$I_{m2}(t) = -Dt, 0 \leq t \leq T_2 \tag{4}$$

The individual costs are now evaluated before they are grouped together

1. Annual set-up cost $(SC_m) = \frac{A_m}{T}$
2. Annual transportation cost $(TC_m) = \frac{\tau_m n}{T}$
3. Annual stock holding cost $(HC_m) = \frac{h_m}{T}\left[\int_0^{T_1} I_{m1}(t)dt + \int_0^{T_2} I_{m2}(t)dt\right] = \frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1 + \frac{e^{\theta T_1}}{\theta} - \frac{1}{\theta}\right] - \left[\frac{DT_2^2}{2}\right]\right]$
4. Opportunity interest loss per unit time in n-shipments $IL_m = \frac{I_m s_m n}{T}\int_0^T D(t)dt = \frac{I_m s_m n}{T}\left(DN\right)$
5. Deterioration Cost$(DC_m) = p\left(PT_1 - DT\right)$ The annual total relevant cost of the manufacturer $TC_m = SC_m + TC_m + HC_m + IL_m + DC_m$

3.2. Distributor's Model

The level of inventory $I_d(t)$ gradually decreases to meet demands to retailers. It is shown in figure (2).Hence the variation of inventory with respect to time t can be described by the following differential equations:

$$\frac{dI_d(t)}{dt} + \theta I_d(t) = -D, where 0 \leq t \leq T_3 \tag{5}$$

with boundary condition $I_d(T_3) = 0$ consequently solution is given by,

$$I_d(t) = \frac{D}{\theta}\left(e^{\theta(T_3 - t)} - 1\right)$$

$T_3 = \frac{T}{n}$ and the order quantity is

$$Q_d = I_d(0) = \frac{D}{\theta}\left(e^{T_3} - 1\right) \tag{6}$$

The individual costs are now calculated before they are grouped together

1. Annual ordering cost $(OC_d) = \frac{nA_d}{T}$
2. Annual stock holding cost (excluding interest charges)

$$HC_d = \frac{nh_d}{T}\int_0^{T_3} I_d(t)dt = \frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta} - \frac{1}{\theta} - T_3\right]$$

3. The distributor incurs two annual shipment cost element one for receiving shipment from manufacturer and the other for delivering shipments to the retailers.
   1. The shipment cost for receiving $(TC_{d1}) = \tau_{d1}\frac{n}{T}$
   2. The shipment cost for delivering $TC_{d2} = \tau\frac{n}{T}$

4. Opportunity interest loss per unit time in mn-shipments

$$IL_d = \frac{I_o s_d mn}{T}\int_0^M D(t)dt = \frac{I_o s_d mn}{T}\left[DM\right]$$

5. Deterioration Costs$(DC_d) = p\left(Pt_1 - DT\right)$

6. Regarding interest earned and payable, we have following two possible cases based on the value of T_3 and N

Case-I When $N \leq T_3$

i. Interest earned per year in n shipments

$$\begin{aligned} IE_{d1} &= \frac{nI_e s_d}{T}\int_0^{T_3}(T_3 - t)D(t)dt \\ &= \frac{nI_e s_d D}{T}\left[\frac{T_3^2}{2}\right] \end{aligned}$$

ii. Interest payable per year

$$\begin{aligned} IP_{d1} &= \frac{nI_p s_m}{T}\int_N^{T_3} I_d(t)dt \\ &= \frac{nI_p s_m D}{T\theta}\left[N - T_3 + \frac{e^{\theta(T_3-N)}}{\theta} - \frac{1}{\theta}\right] \end{aligned}$$

Case II When $N \geq T_3$

i. Interest earned per year in n shipments

$$\begin{aligned} IE_{d2} &= \frac{nI_e s_d}{T}\left[\int_0^{T_3}(T_3 - t)D(t)dt + (N - T_3)\int_0^{T_3} D(t)dt\right] \\ &= \frac{nI_e s_d}{T}\left[D\left(\frac{T_3^2}{2}\right) + (N - T_3)DT_3\right] \end{aligned}$$

In this case, no interest charges are paid for the items kept in stock, i.e.$IP_{d2} = 0$. Therefore, the annual total relevant cost of the distributor is

$$TAC_d = \begin{cases} TAC_{d1} & \text{if} \quad N \leq T_3 \\ TAC_{d2} & \text{if} \quad N \geq T_3 \end{cases} \tag{7}$$

Where,

$$TAC_{d1} = OC_d + HC_d + TC_{d1} + TC_{d2} + DC_d + IL_d + IP_{d1} - IE_{d1}$$

$$TAC_{d2} = OC_d + HC_d + TC_{d1} + TC_{d2} + DC_d + IL_d + IP_{d2} - IE_{d2}$$

3.3. Retailer's Model

The level of inventory $I_r(t)$ gradually decreases to meet demands to customers. It is shown in figure (3). Hence the variation of inventory with respect to time t can be described by the following differential equations

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -D, 0 \leq t \leq T_4 \tag{8}$$

with boundary condition $I_r(T_4) = 0$ consequently solution is given by

$$I_r(t) = \frac{D}{\theta}\left[e^{\theta(T_4-t)} - 1\right] \tag{9}$$

$T_4 = \frac{T_3}{m}$ and $T_4 = \frac{T}{mn}$ and the order quantity is

$$Q_r = I_r(0) = \frac{D}{\theta}\left[e^{\theta T_4} - 1\right] \tag{10}$$

The individual costs are now calculated before they are grouped together

1. Annual ordering cost$(OC_r) = \frac{mnA_r}{T}$

2. Annual stock-holding cost (excluding interest charges)

$$HC_r = \frac{mnh_r}{T}\int_0^{T_4} I_r(t)dt = \frac{mnh_rD}{T\theta}\left[-\frac{1}{\theta} - T_4\right]$$

3. The transportation cost for receiving shipments from distributor $TC_r = \frac{\tau_r mn}{T}$

4. Deterioration costs $(DC_r) = p\left(Pt_1 - DT\right)$

5. Regarding interest earned and payable, we have following three cases based on the values of $Q_r, w, MandT_4$

   Case 1 $Q_r < w$

   According to assumption, the trade credit is not permitted. The retailer must pay $s_d.Q_r$ when the order is received. Therefore,

   i. In this case, no earned interest, i.e. $IE_{r1} = 0$

   ii. Interest payable for the items kept in stock per year in mn-shipments

$$\begin{aligned} IP_{r1} &= \frac{mnI_ps_d}{T}\int_0^{T_4} tD(t)dt \\ &= \frac{mnI_ps_d}{T}\frac{DT_4^2}{2} \end{aligned}$$

   Case 2 $Q_r > w$, the fixed credit period M is permitted. Therefore two cases arise

   Case 2.1 $M < T_4$

   i. Interest earned per year in mn - shipments

$$\begin{aligned} IE_{r2} &= \frac{mnI_es_r}{T}\int_0^{T_4} tD(t)dt \\ &= \frac{mnI_es_r}{T}\frac{DT_4^2}{2} \end{aligned}$$

   ii. Interest payable per year in mn-shipments

$$\begin{aligned} IP_{r2} &= \frac{mnI_ps_d}{T}\int_M^{T_4} tD(t)dt \\ &= \frac{mnI_ps_d}{T}\left[\frac{T_4^2}{2} - \frac{M^2}{2}\right] \end{aligned}$$

Case 2.2 $M \geq T_4$

i. Interest earned per year in mn-shipments

$$IE_{r3} = \frac{mnI_e s_r}{T}\left[\int_0^{T_4}(T_4 - t)D(t)dt + \int_0^{T_4}(M - T_4)Ddt\right]$$
$$= \frac{mnI_e s_r D}{T}\left[MT_4 - \frac{T_4^2}{2}\right]$$

ii. In this case, no interest charges are paid for the items kept in stock. i.e. $IP_{r3} = 0$. Therefore, the annual total relevant cost of the retailer is

$$TAC_r = \begin{cases} TAC_{r1} & \text{if} \quad Q_r < w \\ & whenQ_r > w \\ TAC_{r2} & \text{if} \quad M < T_4 \\ TAC_{r3} & \text{if} \quad M \geq T_4 \end{cases} \tag{11}$$

Where,

$$TAC_{r1} = OC_r + HC_r + TC_r + DC_r + IP_{r1} - IE_{r1}$$
$$TAC_{r2} = OC_r + HC_r + TC_r + DC_r + IP_{r2} - IE_{r2}$$
$$TAC_{r3} = OC_r + HC_r + TC_r + DC_r + IP_{r3} - IE_{r3}$$

Finally, the annual total cost of the entire supply chain TCS is composed of the manufacturer's annual cost TAC_m the distributor's annual cost TAC_d and the retailer's annual cost TAC_r. It is important to note that having cycle time T and permissible delay periods N and M incur different annual cost to the distributor and retailer. Hence, the annual total relevant cost of the entire system will also be different for different cases.

Case I $N \leq T_3$

$$TCS^{\alpha} = \begin{cases} TCS_1 & \text{if} \quad Q_r < w \\ & whenQ_r > w \\ TCS_2 & \text{if} \quad M < T_4 \\ TCS_3 & \text{if} \quad M \geq T_4 \end{cases} \tag{12}$$

where,

$$TCS_1 = TAC_m + TAC_{d1} + TAC_{r1}$$
$$TCS_2 = TAC_m + TAC_{d1} + TAC_{r2}$$
$$TCS_3 = TAC_m + TAC_{d1} + TAC_{r3}$$

Case II $N \geq T_3$

$$TCS^{\beta} = \begin{cases} TCS_4 & \text{if} \quad Q_r < w \\ & whenQ_r > w \\ TCS_5 & \text{if} \quad M < T_4 \\ TCS_6 & \text{if} \quad M \geq T_4 \end{cases} \tag{13}$$

where,

$$TCS_4 = TAC_m + TAC_{d2} + TAC_{r1}$$

$$TCS_5 = TAC_m + TAC_{d2} + TAC_{r2}$$

$$TCS_6 = TAC_m + TAC_{d2} + TAC_{r3}$$

This study develops an integrated production inventory model with a certain permissible delay in payment for distributor and retailers. An approximate models with a single manufacturer a single distributor and a single retailer is developed to derive the optimal production policy and lot size. Since $T_4 = \frac{T}{mn}$, $T_3 = \frac{T}{n}$. The problem can stated as an optimization problem and it can be formulated as:

$$Minimize : TCS(m, n, T) = TAC_m + TAC_d + TAC_r \tag{14}$$

$$subject : 0 \leq T, 0 \leq m, 0 \leq n$$

4. Algorithm

The optimization technique is used to minimize (14) to derive T as follows:

Step 1: The number of delivery per order m and n are an integer value, start by choosing an integer value of $m, n \geq 1$.

Step 2: Take the derivatives of $TCS(m, n, T)$ with respect to T and equate the result to zero.

$$TCS^{l}(m, n, T) = 0 \text{ and solving for T}$$

Step 3: Find those values of T from Step 2 for that

$$TCS^{ll}(m, n, T) > 0$$

Step 4: Using these values of T in (14) and find the minimum value of TCS.

Step 5: Repeat Step 2 and Step 3 for all possible values of m,n until the minimum $TCS(m^*, n^*, T^*)$ is found. The $TCS(m^*, n^*, T^*)$ values constitute the optimal solution that satisfy the condition mentioned in Step 3.

Step 6: Derive $Q_m^*, Q_d^*, Q_r^*, TAC_m^*, TC_d^*, TAC_r^*$.

5. Numerical Examples

Optimal production and replenishment policy to minimize the total system cost may be obtained by using the methodology proposed in the proceeding section. The following numerical examples are illustrated the model. The values of parameters adopted in this study are

$A_m = 500, P = 144, D = 100, \theta = 0.05, T_1 = 0.6332, A_d = 300, A_r = 100, T_m = 300, T_{d1} = 70, T_{d2} = 150, T_r = 50, I_e = 0.2, I_p = 0.3, I_m = 0.1, I_o = 0.15, s_m = 8, s_d = 10, T = 0.8904, s_r = 12, T_2 = 0.453, h_m = 2, h_d = 3, h_r = 5, k = 3, M = 2N = 3, n = 1m = 2, w = 25, p = 0.04, \rho = 0.01, T_3 = 0.10$. The computational results are shown below as

Changing Parameters	Values	T_i	T	Q_r	TCS
n	1	1.88			
m	2	2.82			
		4.70	4.70	30	560
		2.35			
n	2	3.6			
m	3	5.4			
		4.5	9.02	15	650
		1.5			
n	1	2.16			
m	3	3.24			
		5.40	5.40	21	580
		1.8			

Table 1. **When w= 25;i=1,2,3,4**

Changing Parameters	Values	T_i	T	Q_r	TCS
n	3	3.08			
m	1	4.62			
		2.8	7.7	20	710
		2.8			
n	4	4.64			
m	2	6.96			
		2.9	11.6	35	675
		1.45			
n	4	4.08			
m	1	6.13			
		2.5	10.22	37	658
		2.5			

Table 2. **When w= 30; i=1,2,3,4**

The results are satisfied the conditions $Q_r > w, N \leq T_3$ and $M \geq T_4$.

6. Conclusion

In supply chain management the costs reduce by using various methods is the major focus. In order to decrease the joint total cost, the manufacturer, distributor and retailer are willing to invest in reducing the different costs. we developed the integrated manufacturer-distributor-retailer models with different type of permissible delay in payments to determine the optimal replenishment time interval and frequency to reduce the total system costs.we have provided an example, discussed all the cases and found the results that satisfied the conditions.

Acknowledgement

This research was fully supported by National Board for Higher Mathematics, Government of India under the scheme of NBHM research project with 2/48(9)/2013/NBHM(R.P)/R&D II /Dated 16.01.2014.

Appendix I

$$TAC_m = \frac{A_m}{T} + \frac{\tau_m n}{T} + \frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1 + \frac{e^{\theta T_1}}{\theta} - \frac{1}{\theta}\right] - \left[\frac{DT_2^2}{2}\right]\right] + \frac{I_m s_m n}{T}\left(DN\right) + p\left(PT_1 - DT\right) \tag{15}$$

$$\begin{aligned}
TAC_{d1} &= \frac{nA_d}{T} + \frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta} - \frac{1}{\theta} - T_3\right] + \tau_{d1}\frac{n}{T} + \tau\frac{n}{T} + \frac{I_o s_d mn}{T}\left[DM\right] \\
&\quad + p\left(Pt_1 - DT\right) + \frac{nI_p s_m D}{T\theta}\left[N - T_3 + \frac{e^{\theta(T_3-N)}}{\theta} - \frac{1}{\theta}\right] - \frac{nI_e s_d D}{T}\left[\frac{T_3^2}{2}\right] \\
TAC_{d2} &= \frac{nA_d}{T} + \frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta} - \frac{1}{\theta} - T_3\right] + \tau_{d1}\frac{n}{T} + \tau\frac{n}{T} + \frac{I_o s_d mn}{T}\left[DM\right] \\
&\quad + p\left(Pt_1 - DT\right) - \frac{nI_e s_d}{T}\left[D\left(\frac{T_3^2}{2}\right) + (N - T_3)DT_3\right] \\
TAC_{r1} &= \frac{mnA_r}{T} + \frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta} - T_4\right] + \frac{\tau_r mn}{T} + p\left(Pt_1 - DT\right) + \frac{mnI_p s_d}{T}\frac{DT_4^2}{2} \\
TAC_{r2} &= \frac{mnA_r}{T} + \frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta} - T_4\right] + \frac{\tau_r mn}{T} + p\left(Pt_1 - DT\right) \\
&\quad + \frac{mnI_p s_d}{T}\left[\frac{T_4^2}{2} - \frac{M^2}{2}\right] - \frac{mnI_e s_r}{T}\frac{DT_4^2}{2} \\
TAC_{r3} &= \frac{mnA_r}{T} + \frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta} - T_4\right] + \frac{\tau_r mn}{T} + p\left(Pt_1 - DT\right) - \frac{mnI_e s_r D}{T}\left[MT_4 - \frac{T_4^2}{2}\right]
\end{aligned}$$

$$\begin{aligned}
TCS_1 \;=\; & \frac{A_m}{T} + \frac{\tau_m n}{T} + \frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1 + \frac{e^{\theta T_1}}{\theta} - \frac{1}{\theta}\right] - \left[\frac{DT_2^2}{2}\right]\right] + \frac{I_m s_m n}{T}\left(DN\right) \\
& + p\left(PT_1 - DT\right) + \frac{nA_d}{T} + \frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta} - \frac{1}{\theta} - T_3\right] + \tau_{d1}\frac{n}{T} + \tau\frac{n}{T} + \frac{I_o s_d mn}{T}\left[DM\right] \\
& + p\left(Pt_1 - DT\right) + \frac{nI_p s_m D}{T\theta}\left[N - T_3 + \frac{e^{\theta(T_3-N)}}{\theta} - \frac{1}{\theta}\right] - \frac{nI_e s_d D}{T}\left[\frac{T_3^2}{2}\right] + \\
& \frac{mnA_r}{T} + \frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta} - T_4\right] + \frac{\tau_r mn}{T} + p\left(Pt_1 - DT\right) + \frac{mnI_p s_d}{T}\frac{DT_4^2}{2}
\end{aligned}$$

$$\begin{aligned}
TAS_2 \;=\; & \frac{A_m}{T} + \frac{\tau_m n}{T} + \frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1 + \frac{e^{\theta T_1}}{\theta} - \frac{1}{\theta}\right] - \left[\frac{DT_2^2}{2}\right]\right] + \frac{I_m s_m n}{T}\left(DN\right) \\
& + p\left(PT_1 - DT\right) + \frac{nA_d}{T} + \frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta} - \frac{1}{\theta} - T_3\right] + \tau_{d1}\frac{n}{T} + \tau\frac{n}{T} + \frac{I_o s_d mn}{T}\left[DM\right] \\
& + p\left(Pt_1 - DT\right) + \frac{nI_p s_m D}{T\theta}\left[N - T_3 + \frac{e^{\theta(T_3-N)}}{\theta} - \frac{1}{\theta}\right] - \frac{nI_e s_d D}{T}\left[\frac{T_3^2}{2}\right] + \\
& \frac{mnA_r}{T} + \frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta} - T_4\right] + \frac{\tau_r mn}{T} + p\left(Pt_1 - DT\right) + \frac{mnI_p s_d}{T}\left[\frac{T_4^2}{2} - \frac{M^2}{2}\right] - \frac{mnI_e s_r}{T}\frac{DT_4^2}{2}
\end{aligned}$$

$$\begin{aligned}
TAS_3 \;=\; & \frac{A_m}{T}+\frac{\tau_m n}{T}+\frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1+\frac{e^{\theta T_1}}{\theta}-\frac{1}{\theta}\right]-\left[\frac{DT_2^2}{2}\right]\right]+\frac{I_m s_m n}{T}\left(DN\right)\\
&+p\left(PT_1-DT\right)+\frac{nA_d}{T}+\frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta}-\frac{1}{\theta}-T_3\right]+\tau_{d1}\frac{n}{T}+\tau\frac{n}{T}+\frac{I_o s_d mn}{T}\left[DM\right]\\
&+p\left(Pt_1-DT\right)+\frac{nI_p s_m D}{T\theta}\left[N-T_3+\frac{e^{\theta(T_3-N)}}{\theta}-\frac{1}{\theta}\right]-\frac{nI_e s_d D}{T}\left[\frac{T_3^2}{2}\right]+\\
&\frac{mnA_r}{T}+\frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta}-T_4\right]+\frac{\tau_r mn}{T}+p\left(Pt_1-DT\right)-\frac{mnI_e s_r D}{T}\left[MT_4-\frac{T_4^2}{2}\right]
\end{aligned}$$

$$\begin{aligned}
TAS_4 \;=\; & \frac{A_m}{T}+\frac{\tau_m n}{T}+\frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1+\frac{e^{\theta T_1}}{\theta}-\frac{1}{\theta}\right]-\left[\frac{DT_2^2}{2}\right]\right]+\frac{I_m s_m n}{T}\left(DN\right)\\
&+p\left(PT_1-DT\right)+\frac{nA_d}{T}+\frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta}-\frac{1}{\theta}-T_3\right]+\tau_{d1}\frac{n}{T}+\tau\frac{n}{T}+\frac{I_o s_d mn}{T}\left[DM\right]\\
&+p\left(Pt_1-DT\right)-\frac{nI_e s_d}{T}\left[D\left(\frac{T_3^2}{2}\right)+(N-T_3)DT_3\right]+\\
&\frac{mnA_r}{T}+\frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta}-T_4\right]+\frac{\tau_r mn}{T}+p\left(Pt_1-DT\right)+\frac{mnI_p s_d}{T}\frac{DT_4^2}{2}
\end{aligned}$$

$$\begin{aligned}
TAS_5 \;=\; & \frac{A_m}{T}+\frac{\tau_m n}{T}+\frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1+\frac{e^{\theta T_1}}{\theta}-\frac{1}{\theta}\right]-\left[\frac{DT_2^2}{2}\right]\right]+\frac{I_m s_m n}{T}\left(DN\right)\\
&+p\left(PT_1-DT\right)+\frac{nA_d}{T}+\frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta}-\frac{1}{\theta}-T_3\right]+\tau_{d1}\frac{n}{T}+\tau\frac{n}{T}+\frac{I_o s_d mn}{T}\left[DM\right]\\
&+p\left(Pt_1-DT\right)-\frac{nI_e s_d}{T}\left[D\left(\frac{T_3^2}{2}\right)+(N-T_3)DT_3\right]+\\
&\frac{mnA_r}{T}+\frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta}-T_4\right]+\frac{\tau_r mn}{T}+p\left(Pt_1-DT\right)+\frac{mnI_p s_d}{T}\left[\frac{T_4^2}{2}-\frac{M^2}{2}\right]-\frac{mnI_e s_r}{T}\frac{DT_4^2}{2}
\end{aligned}$$

$$\begin{aligned}
TAS_6 \;=\; & \frac{A_m}{T}+\frac{\tau_m n}{T}+\frac{h_m}{T}\left[\left(\frac{P-D}{\theta}\right)\left[T_1+\frac{e^{\theta T_1}}{\theta}-\frac{1}{\theta}\right]-\left[\frac{DT_2^2}{2}\right]\right]+\frac{I_m s_m n}{T}\left(DN\right)\\
&+p\left(PT_1-DT\right)+\frac{nA_d}{T}+\frac{nh_d D}{T\theta}\left[\frac{e^{\theta T_3}}{\theta}-\frac{1}{\theta}-T_3\right]+\tau_{d1}\frac{n}{T}+\tau\frac{n}{T}+\frac{I_o s_d mn}{T}\left[DM\right]\\
&+p\left(Pt_1-DT\right)-\frac{nI_e s_d}{T}\left[D\left(\frac{T_3^2}{2}\right)+(N-T_3)DT_3\right]+\\
&\frac{mnA_r}{T}+\frac{mnh_r D}{T\theta}\left[-\frac{1}{\theta}-T_4\right]+\frac{\tau_r mn}{T}+p\left(Pt_1-DT\right)-\frac{mnI_e s_r D}{T}\left[MT_4-\frac{T_4^2}{2}\right]
\end{aligned}$$

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