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# Weak Forms Bitopological Semiopen Sets

Research Article

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**Abstract:** In this section, we introduce and study maximal $(i, j)$-semiopen and minimal $(i, j)$-semiclosed sets in bitopological space.

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**Keywords:** Bitopological space, $(i, j)$-semiopen sets.



## 1. Introduction

The concept of bitopological spaces was first introduced by Kelly [1]. After the introduction of the definition of a bitopological space by Kelly, a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. In this section, we introduce and study maximal $(i, j)$-semiopen and minimal $(i, j)$-semiclosed sets in bitopological space Throughout this paper, the triple $(X, \tau_1, \tau_2)$ where $X$ is a set and $\tau_1$ and $\tau_2$ are topologies on $X$, will always denote a bitopological space. For a subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$, the closure of $A$ and the interior of $A$ with respect to $\tau_i$ are denoted by $i\,\mathrm{Cl}(A)$ and $i\,\mathrm{Int}(A)$, respectively, for $i = 1, 2$.

## 2. Preliminaries

**Definition 2.1** ([2])**.** *A subset $S$ of a bitopological space $(X, \tau_1, \tau_2)$ is said to be $(i, j)$-semiopen if $S \subset j\,\mathrm{Cl}(i\,\mathrm{Int}(S))$. The complement of an $(i, j)$-semiopen set is called an $(i, j)$-semiclosed set.*

**Definition 2.2** ([2])**.** *The intersection of all $(i, j)$-semiclosed sets containing $S \subset X$ is called the $(i, j)$-semiclosure of $S$ and is denoted by $(i, j)$-$s\,\mathrm{Cl}(S)$. The family of all $(i, j)$-semiopen (resp. $(i, j)$-semiclosed) sets of $(X, \tau_1, \tau_2)$ is denoted by $(i, j)$-$SO(X)$ (resp. $(i, j)$-$SC(X)$). The family of all $(i, j)$-semiopen (resp. $(i, j)$-semiclosed) sets of $(X, \tau_1, \tau_2)$ containing a point $x \in X$ is denoted by $(i, j)$-$SO(X, x)$ (resp. $(i, j)$-$SC(X, x)$).*

## 3. Weak Forms Bitopological Semiopen Sets

In this section, we introduce and study maximal $(i, j)$-$b$-open and minimal $(i, j)$-$b$-closed sets in bitopological space.

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**Definition 3.1.** *A proper nonempty $(i,j)$-semiclosed subset $F$ of a bitopological space $(X,\tau_1,\tau_2)$ is said to be a minimal $(i,j)$-semiclosed set if any $(i,j)$-semiclosed set contained in $F$ is $\emptyset$ or $F$.*

**Definition 3.2.** *A proper nonempty $(i,j)$-semiopen $U$ of a bitopological space $(X,\tau_1,\tau_2)$ is said to be a maximal $(i,j)$-semiopen set if any $(i,j)$-semiopen set containing $U$ is either $X$ or $U$.*

The following theorem shows the relation between minimal $(i,j)$-semiclosed sets and maximal $(i,j)$-semiopen sets.

**Theorem 3.3.** *A proper nonempty subset $U$ of a bitopological space $(X,\tau_1,\tau_2)$ is maximal $(i,j)$-semiopen if, and only if $X\backslash U$ is minimal $(i,j)$-semiclosed.*

*Proof.* Let $U$ be a maximal $(i,j)$-semiopen set. Suppose $X\backslash U$ is not a minimal $(i,j)$-semiclosed set. Then there exists an $(i,j)$-semiclosed set $V \neq X\backslash U$ such that $\emptyset \neq V \subset X\backslash U$. That is $U \subset X\backslash V$ and $X\backslash V$ is an $(i,j)$-semiopen set, which is a contradiction for $U$ is a minimal $(i,j)$-semiclosed set. Conversely, let $X\backslash U$ be a minimal $(i,j)$-semiclosed set. Suppose $U$ is not a maximal $(i,j)$-semiopen set. Then there exists an $(i,j)$-semiopen set $E \neq U$ such that $U \subset E \neq X$. That is $\emptyset \neq X\backslash E \subset X\backslash U$ and $X\backslash E$ is an $(i,j)$-semiclosed set, which is a contradiction for $X\backslash U$ is a minimal $(i,j)$-semiclosed set. Therefore, $U$ is a maximal $(i,j)$-semiclosed set. □

**Lemma 3.4.** *For the subsets $U$ and $V$ of a bitopological space $(X,\tau_1,\tau_2)$, we have the following:*

*(1). If $U$ is minimal $(i,j)$-semiclosed and $V$ an $(i,j)$-semiclosed set, then $U \cap V = \emptyset$ or $U \subset V$.*

*(2). If $U$ and $V$ are minimal $(i,j)$-semiclosed sets, then $U \cap V = \emptyset$ or $U = V$.*

*Proof.*

(1). If $U \cap V = \emptyset$, then there is nothing to prove. If $U \cap V \neq \emptyset$, then $U \cap V \subset U$. Since $U$ is a minimal $(i,j)$-semiclosed set, $U \cap V = U$. Hence $U \subset V$.

(2). If $U \cap V \neq \emptyset$, then $U \subset V$ and $V \subset U$ by (1). Hence $U = V$.

□

**Theorem 3.5.** *Let $U$ be a minimal $(i,j)$-semiclosed subset of a bitopological space $(X,\tau_1,\tau_2)$. If $x \in U$, then $U \subset W$ for some $(i,j)$-semiclosed set $W$ containing $x$.*

*Proof.* Let $x \in U$ and $W$ be an $(i,j)$-semiclosed set such that $x \in G$. Then $U \cap W = \emptyset$. By Lemma 3.4 (1), $U \subset W$. □

**Theorem 3.6.** *If $U$ is a minimal $(i,j)$-semiclosed subset of a bitopological space $(X,\tau_1,\tau_2)$, then $U = \cap\{W : W \in (i,j)$-$SC(X,x)\}$.*

*Proof.* By Theorem 3.5 and $U$ is an $(i,j)$-semiclosed set containing $x$, we have $U \subset \cap\{W : W \in (i,j)$-$SC(X,x)\}$. Next let, $x \in \cap\{W : W \in (i,j)$-$SC(X,x)\}$. This implies that $x \in W$ for all $(i,j)$-semiclosed set $W$. As $U$ is $(i,j)$-semiclosed, $x \in U$; hence $\cap\{W : W \in (i,j)$-$SC(X,x)\} = U$. □

**Theorem 3.7.** *Let $U$ be a nonempty $(i,j)$-semiclosed subset of a bitopological space $(X,\tau_1,\tau_2)$. Then the following statements are equivalent:*

*(1). $U$ is a minimal $(i,j)$-semiclosed set.*

*(2). $U \subset (i,j)$-$s\,\mathrm{Cl}(S)$ for any nonempty subset $S$ of $U$.*

*(3). $(i,j)$-$s\,\mathrm{Cl}(U) = (i,j)$-$s\,\mathrm{Cl}(S)$ for any nonempty subset $S$ of $U$.*

*Proof.*
(1)⇒(2): Let $x \in U$; $U$ be a minimal $(i,j)$-semiclosed set and $S(\neq \emptyset) \subset U$. By Theorem 3.5, for any $(i,j)$-semiclosed set $W$ containing $x$, $S \subset U \subset W$ gives $S \subset W$. Now $S = S \cap U \subset S \cap W$. Since $S \neq \emptyset$, $S \cap W \neq \emptyset$. Since $W$ is any $(i,j)$-semiclosed set containing $x$, by Theorem 3.5, $x \in (i,j)$-$s\,\mathrm{Cl}(S)$. That is, $x \in U \Rightarrow x \in (i,j)$-$s\,\mathrm{Cl}(S)$. Hence $U \subset (i,j)$-$s\,\mathrm{Cl}(S)$ for any nonempty subset $S$ of $U$.

(2)⇒(3): Let $S$ be a nonempty subset of $U$. Then $(i,j)$-$s\,\mathrm{Cl}(S) \subset (i,j)$-$s\,\mathrm{Cl}(U)$. By (2), $(i,j)$-$s\,\mathrm{Cl}(U) \subset (i,j)$-$s\,\mathrm{Cl}((i,j)$-$s\,\mathrm{Cl}(S)) = (i,j)$-$s\,\mathrm{Cl}(S)$. That is, $(i,j)$-$s\,\mathrm{Cl}(U) \subset (i,j)$-$s\,\mathrm{Cl}(S)$. We have $(i,j)$-$s\,\mathrm{Cl}(U) = (i,j)$-$s\,\mathrm{Cl}(S)$ for any nonempty subset $S$ of $U$.

(3)⇒(1): Suppose $U$ is not a minimal $(i,j)$-semiclosed set. Then there exists a nonempty $(i,j)$-semiclosed set $V$ such that $V \subset U$ and $V \neq U$. Now, there exists an element $a$ in $U$ such that $a \notin V$. That is, $(i,j)$-$s\,\mathrm{Cl}(\{a\}) \subset (i,j)$-$s\,\mathrm{Cl}(X\backslash V) = X\backslash V$, as $X\backslash V$ is $(i,j)$-semiclosed in $X$. It follows that $(i,j)$-$s\,\mathrm{Cl}(\{a\}) \neq (i,j)$-$s\,\mathrm{Cl}(U)$. This is a contradiction for $(i,j)$-$s\,\mathrm{Cl}(\{a\}) = (i,j)$-$s\,\mathrm{Cl}(U)$ for any $\{a\}(\neq \emptyset) \subset U$. Therefore, $U$ is a minimal $(i,j)$-semiclosed set. □

**Theorem 3.8.** *If $V$ is a nonempty finite $(i,j)$-semiclosed subset of a bitopological space $(X,\tau_1,\tau_2)$, then there exists at least one (finite) minimal $(i,j)$-semiclosed set $U$ such that $U \subset V$.*

*Proof.* If $V$ is a minimal $(i,j)$-semiclosed set, we may set $U = V$. If $V$ is not a minimal $(i,j)$-semiclosed set, then there exists a (finite) $(i,j)$-semiclosed set $V_1$ such that $\emptyset \neq V_1 \subset V$. If $V_1$ is a minimal $(i,j)$-semiclosed set, we may set $U = V_1$. If $V_1$ is not a minimal $(i,j)$-semiclosed set, then there exists a (finite) $(i,j)$-semiclosed set $V_2$ such that $\emptyset \neq V_2 \subset V_1$. Continuing this process, we have a sequence of $(i,j)$-semiclosed sets $V \supset V_1 \supset V_2 \supset V_3 \supset ... \supset V_k \supset ....$ Since $V$ is a finite set, this process repeats only finitely many time and finally we get a minimal $(i,j)$-semiclosed set $U = V_n$ for some positive integer $n$. □

**Theorem 3.9.** *Let $U$ and $U_\alpha$ be minimal $(i,j)$-semiclosed subsets of a bitopological space $(X,\tau_1,\tau_2)$ for any element $\alpha \in \Delta$. If $U \subset \bigcup\limits_{\alpha\in\Delta} U_\alpha$, then there exists an element $\alpha \in \Delta$ such that $U = U_\alpha$.*

*Proof.* Let $U \subset \bigcup\limits_{\alpha\in\Delta} U_\alpha$. Then $U \cap (\bigcup\limits_{\alpha\in\Delta} U_\alpha) = U$. That is $\bigcup\limits_{\alpha\in\Delta}(U \cap U_\alpha) = U$. Also by Lemma 3.4 (2), $U \cap U_\alpha = \emptyset$ or $U = U_\alpha$ for any $\alpha \in \Delta$. It follows that there exists an element $\alpha \in \Delta$ such that $U = U_\alpha$. □

**Theorem 3.10.** *Let $U$ and $U_\alpha$ be minimal $(i,j)$-semiclosed subsets of a bitopological space $(X,\tau_1,\tau_2)$ for any $\alpha \in \Delta$. If $U \neq U_\alpha$ for any $\alpha \in \Delta$, then $(\bigcup\limits_{\alpha\in\Delta} U_\alpha) \cap U = \emptyset$.*

*Proof.* Suppose that $(\bigcup\limits_{\alpha\in\Delta} U_\alpha) \cap U \neq \emptyset$. Then there exists an element $\alpha \in \Delta$ such that $U \cap U_\alpha \neq \emptyset$. By Lemma 3.4 (2), we have $U = U_\alpha$, which contradicts the fact that $U \neq U_\alpha$ for any $\alpha \in \Delta$. Hence $(\bigcup\limits_{\alpha\in\Delta} U_\alpha) \cap U = \emptyset$. □

**Lemma 3.11.** *Let $A$ and $B$ be any two subsets of a bitopological space $(X,\tau_1,\tau_2)$. Then we have the following:*

*(1). If $A$ is maximal $(i,j)$-semiopen set and $B$ an $(i,j)$-semiopen set, then $A \cup B = X$ or $B \subset A$.*

*(2). Let $A$ and $B$ are maximal $(i,j)$-semiopen sets, then $A \cup B = X$ or $A = B$.*

*Proof.*

(1). If $A \cup B = X$, then there is nothing to prove. If $A \cup B \neq X$, then $A \cup B$ is an $(i, j)$-semiopen set such that $A \subset A \cup B$. Then $A \cup B = A$. Hence $B \subset A$.

(2). If $A \cup B \neq X$, then $A \cup B$ is an $(i, j)$-semiopen set such that $A$, $B \subset A \cup B$, that is, $A \cup B = A$ and $A \cup B = B$. Hence $A = B$.

□

**Theorem 3.12.** *Let $F$ be a maximal $(i, j)$-semiopen subset of a bitopological space $(X, \tau_1, \tau_2)$. If $x \in F$, then $S \subset F$ for some $(i, j)$-semiopen set $S$ containing $x$.*

*Proof.* Similar to the proof of Theorem 3.5. □

**Theorem 3.13.** *Let $A$, $B$ and $C$ be three maximal $(i, j)$-semiopen sets of a bitopological space $(X, \tau_1, \tau_2)$ such that $A \neq B$. If $A \cap B \subset C$, then either $A = C$ or $B = C$.*

*Proof.* If $A = C$, then there is nothing to prove. If $A \neq C$, then we have to prove $B = C$. Now $B \cap C = B \cap (C \cap X) = B \cap (C \cap (A \cup B)$ (by Theorem 3.11 (2)) $= B \cap ((C \cap A) \cup (C \cap B)) = (B \cap C \cap A) \cup (B \cap C) = (A \cap B) \cup (C \cap B) = (A \cup C) \cap B = X \cap B = B$ (Since $A$ and $C$ are maximal $(i, j)$-semiopen sets by Theorem 3.11 (2), $A \cup C = X$). That is, $B \cap C = B \Rightarrow B \subset C$. Since $B$ and $C$ are maximal $(i, j)$-semiopen sets, we have $B = C$. Hence $B = C$. □

**Theorem 3.14.** *Let $(X, \tau_1, \tau_2)$ be a bitopological space. If $A$, $B$ and $C$ are maximal $(i, j)$-semiopen sets which are different from each other, then $(A \cap B) \nsubseteq (A \cap C)$.*

*Proof.* Let $A \cap B \subset A \cap C$. Then $(A \cap B) \cup (C \cap B) \subset (A \cap C) \cup (C \cap B)$. That is, $(A \cup C) \cap B \subset C \cap (A \cup B)$. By Theorem 3.11 (2), $A \cup C = X = A \cup B$. Hence $X \cap B \subset C \cap X \Rightarrow B \subset C$. Thus from the definition of maximal $(i, j)$-semiopen set, we have $B = C$, which is a contradiction to the fact that $A$, $B$ and $C$ are different to each other. Therefore, $(A \cap B) \nsubseteq (A \cap C)$. □

**Theorem 3.15.** *Let $(X, \tau_1, \tau_2)$ be a bitopological space. If $F$ is a maximal $(i, j)$-semiopen set and $x$ be an element of $F$, then $F = \cup\{S : S$ is an $(i, j)$-semiopen set containing $x$ such that $F \cup S \neq X\}$.*

*Proof.* Similar to the proof of Theorem 3.6. □

We call a set cofinite if its complement is finite.

**Theorem 3.16.** *Let $(X, \tau_1, \tau_2)$ be a bitopological space. If $F$ is a proper nonempty cofinite $(i, j)$-semiopen set, then there exists (cofinite) maximal $(i, j)$-semiopen set $E$ such that $F \subset E$.*

*Proof.* If $F$ is a maximal $(i, j)$-semiopen set, we may set $E = F$. If $F$ is not a maximal $(i, j)$-semiopen set, then there exists a (cofinite) $(i, j)$-semiopen set $F_1$ such that $F \subset F_1 \neq X$. If $F_1$ is a maximal $(i, j)$-semiopen set, we may set $E = F_1$. If $F_1$ is not a maximal $(i, j)$-semiopen set, then there exists a (cofinite) $(i, j)$-semiopen set $F_2 \neq X$ such that $F \subset F_1 \subset F_2 (\neq X)$. Continuing this process, we have a sequence of $(i, j)$-semiopen sets such that $F \subset F_1 \subset F_2 \subset ... \subset F_k \subset ....$ Since $F$ is a cofinite set, this process repeats only finitely many times and finally we get a maximal $(i, j)$-semiopen set $E = F$. □

**Theorem 3.17.** *Let $(X, \tau_1, \tau_2)$ be a bitopological space. Then we have the following:*

*(1). Let $A$ be a maximal $(i, j)$-semiopen set and $x \in X \backslash A$. Then $X \backslash A \subset B$ for any $(i, j)$-semiopen set $B$ containing $x$.*

*(2). Let $A$ be a maximal $(i, j)$-semiopen set. Then either of the following (i) or (ii) holds:*

*(i) For each $x \in X \backslash A$ and each $(i, j)$-semiopen set $B$ containing $x$, $B = X$.*

*(ii) There exists an $(i, j)$-semiopen set $B$ such that $X \backslash A \subset B$.*

*(3). Let $A$ be a maximal $(i,j)$-semiopen set. Then either of the following (i) or (ii) holds:*

*(i) For each $x \in X \backslash A$ and each $(i,j)$-semiopen set $B$ containing $x$, $X \backslash A \subset B$.*

*(ii) There exists an $(i,j)$-semiopen set $B$ such that $X \backslash A = B$.*

*Proof.*

(1). Since $x \in X \backslash A$, we have $B \nsubseteq A$ for any $(i,j)$-semiopen set $B$ containing $x$. Then by Theorem 3.11 (1), $A \cup B = X \Rightarrow X \backslash A \subset B$.

(2). If (i) holds, we are done. Let (i) do not hold. Then there exists an element $x \in X \backslash A$ and an $(i,j)$-semiopen set $B$ containing $x$ such that $B \subset X$. Then by Theorem 3.11 (1), $A \cup B = X$ or $B \subset A$. But $B \nsubseteq A \Rightarrow A \cup B = X \Rightarrow X \backslash A \subset B$.

(3). If (ii) holds, we are done. Let (ii) do not hold. Then (by (i)) for each $x \in X \backslash A$ and each $(i,j)$-semiopen set $B$ containing $x$, $X \backslash A \subset B$. Hence by assumption $X \backslash A \subset B$.

□

**Theorem 3.18.** *Let $A$ be a maximal $(i,j)$-semiopen set in a bitopological space $(X, \tau_1, \tau_2)$. Then either $(i,j)$-$s\,\mathrm{Cl}(A)$ $=X$ or $(i,j)$-$s\,\mathrm{Cl}(A)= A$.*

*Proof.* Since $A$ is maximal $(i,j)$-semiopen set, only the following cases (i) and (ii) occur by Theorem 3.17 (3).

(i) For each $x \in X$ and $x \in X \backslash A$ and each $(i,j)$-semiopen set $B$ containing $x$, we have, $X \backslash A \subset B$: Let $x \in X \backslash A$ and $B$ be any $(i,j)$-semiopen set containing $x$. Since $X \backslash A \neq B$, we have $B \cap A \neq \emptyset$ and hence $X \backslash A \subset (i,j)$-$s\,\mathrm{Cl}(A)$. Since $X = A \cup (X \backslash A) \subset A \cup (i,j)$-$s\,\mathrm{Cl}(A) = (i,j)$-$s\,\mathrm{Cl}(A) \subset X$, $X = (i,j)$-$s\,\mathrm{Cl}(A)$.

(ii) There exists an $(i,j)$-semiopen set $B$ such that $X \backslash A = B (\neq X)$: Since $X \backslash A = B$, $A$ is an $(i,j)$-semiclosed set $\Rightarrow (i,j)$-$s\,\mathrm{Cl}(A) = A$.

□

**Theorem 3.19.** *Let $A$ be a maximal $(i,j)$-semiopen set in a bitopological space $(X, \tau_1, \tau_2)$. Then either $(i,j)$-$s\,\mathrm{Int}(X \backslash A)$ $=$ $X \backslash A$ or $(i,j)$-$s\,\mathrm{Int}(X \backslash A)$ $= \emptyset$.*

*Proof.* By Theorem 3.18, we have $(i,j)$-$s\,\mathrm{Cl}(A) = A$ or $(i,j)$-$s\,\mathrm{Cl}(A) = X$. That is, $(i,j)$-$s\,\mathrm{Int}(X \backslash A) = X \backslash A$ or $(i,j)$-$s\,\mathrm{Int}(X \backslash A) = \emptyset$. □

**Theorem 3.20.** *Let $A$ be a maximal $(i,j)$-semiopen set and $B$ be a nonempty subset of $X \backslash A$ in a bitopological space $(X, \tau_1, \tau_2)$. Then $(i,j)$-$s\,\mathrm{Cl}(B)$ $=X \backslash A$.*

*Proof.* Since $\emptyset \neq B \subset X \backslash A$, $W \cap B \neq \emptyset$ for any element $x \in X \backslash A$ and any $(i,j)$-semiopen set $W$ containing $x$, by Theorem 3.17 (1). Thus, $X \backslash A \subset (i,j)$-$s\,\mathrm{Cl}(B)$. Since $X \backslash A$ is $(i,j)$-semiclosed and $B \subset X \backslash A$, we have $(i,j)$-$s\,\mathrm{Cl}(B) \subset X \backslash A$. □

**Corollary 3.21.** *Let $A$ be a maximal $(i,j)$-semiopen set in a bitopological space $(X, \tau_1, \tau_2)$ and $A \subset B$. Then $(i,j)$-$s\,\mathrm{Cl}(B)$ $=X$.*

*Proof.* The proof follows from Theorem 3.18. □

**Theorem 3.22.** *Let $A$ be a maximal $(i,j)$-semiopen set in a bitopological space $(X, \tau_1, \tau_2)$ and let $X \backslash A$ have at least two elements. Then $(i,j)$-$s\,\mathrm{Cl}(X \backslash \{a\}) = X$ for any element $a$ of $X \backslash A$.*

*Proof.* As $A \subset X \backslash \{a\}$, we have, by Corollary 3.21, $(i,j)$-$s\,\mathrm{Cl}(X \backslash \{a\}) = X$. □

**Theorem 3.23.** *Let $A$ be a maximal $(i,j)$-semiopen set and $G$ be a proper subset of a bitopological space $(X, \tau_1, \tau_2)$ with $A \subset G$. Then $(i,j)$-$s\,\mathrm{Int}(G) = A$.*

*Proof.* If $G = A$, then $(i,j)$-$s\,\mathrm{Int}(G) = (i,j)$-$s\,\mathrm{Int}(A) = A$. If $G \neq A$, then we have $A \subset G$. Thus $A \subset (i,j)$-$s\,\mathrm{Int}(G)$. Since $A$ is maximal $(i,j)$-semiopen, we also have $(i,j)$-$s\,\mathrm{Int}(G) \subset A$. Hence $(i,j)$-$s\,\mathrm{Int}(G) = A$. □

**Theorem 3.24.** *Let $A$ be a maximal $(i,j)$-semiopen set and $F$ a nonempty subset of $X \backslash A$ in a bitopological space $(X, \tau_1, \tau_2)$. Then $X \backslash (i,j)$-$s\,\mathrm{Cl}(F) = A$.*

*Proof.* Since $A \subset X \backslash F \subset X$, by our assumption and by Theorem 3.20, $X \backslash (i,j)$-$s\,\mathrm{Cl}(F) = A$. □

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