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Minimum Dominating Partition Energies of Friendship Graph

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Abstract: In this paper we compute minimum dominating partition energies of friendship graphs.

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1. Introduction

The concept of energy of a graph was introduced by I. Gutman [4] in the year 1978. Let G be a graph with n vertices and m edges and let $A = (a_{ij})$ be the adjacency matrix of the graph. The eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of A, assumed in non increasing order, are the eigenvalues of the graph G. As A is real symmetric, the eigenvalues of G are real with sum equal to zero. The energy $E(G)$ of G is defined to be the sum of the absolute eigenvalues of G. i.e., $E(G) = \sum_{i=1}^{n} |\lambda_i|$. For details on the mathematical aspects of the theory of graph energy see the reviews[5], papers [1, 2, 6] and the references cited therein. The basic properties including various upper and lower bounds for energy of a graph have been established in [8, 9], and it has found remarkable chemical applications in the molecular orbital theory of conjugated molecules [3, 7].

1.1. Partition Energy

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E. Let $P_k = \{V_1, V_2, V_3, ..., V_k\}$ be a partition of a vertex set V. The partition matrix of G is the $n \times n$ matrix defined by $A(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 2 & \text{if } v_i \text{ and } v_j \text{ are adjacent where } v_i, v_j \in V_r \\ -1 & \text{if } v_i \text{ and } v_j \text{ are non adjacent where } v_i, v_j \in V_r \\ 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent between the sets } V_r \text{ and } V_s, \text{ for } r \neq s \text{ where } v_i \in V_r \text{ and } v_j \in V_s \\ 0 & \text{otherwise} \end{cases}$$

The eigenvalues of this matrix are called k - partition eigenvalues of G. The k - partition energy $P_kE(G)$ is defined as the sum of the absolute values of k - partition eigenvalues of G [10].

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1.2. Minimum Dominating Partition Energy

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E. Let $P_k = \{V_1, V_2, V_3, ..., V_k\}$ be a partition of a vertex set V. A subset D of V is called a dominating set of G if every vertex of $V - D$ is adjacent to some vertex in D. Any dominating set with minimum cardinality is called a minimum dominating set. Let D be a minimum dominating set of a graph G. The minimum dominating k - partition matrix of G is the $n \times n$ matrix defined by $P_k A_D(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 2 & \text{if } v_i \text{ and } v_j \text{ are adjacent where } v_i, v_j \in V_r \\ -1 & \text{if } v_i \text{ and } v_j \text{ are non adjacent where } v_i, v_j \in V_r \\ 1 & \text{if } i = j,\ v_i \in D \text{ or } v_i \text{ and } v_j \text{ are adjacent between the sets } V_r \text{ and } V_s, \text{ for } r \neq s \text{ where } v_i \in V_r \text{ and } v_j \in V_s \\ 0 & \text{otherwise} \end{cases}$$

The characteristic polynomial of $A_D(G)$ is denoted by $f_n(G, \lambda) = det(\lambda I - A_D(G))$. The minimum dominating k - partition eigenvalues of the graph G are the eigenvalues of $A_D(G)$. Since $A_D(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n$. The minimum dominating k - partition energy of G is defined as $P_k E_D(G) = \sum_{i=1}^{n} |\lambda_i|$. Note that the trace of $A_D(G) = |D|$.

2. Minimum Dominating Partition Energy of Friendship Graph

Definition 2.1. *The friendship graph is the graph obtained by taking m copies of the complete graph K_3 with a vertex in common. It is denoted by ${K_3}^m$.*

Theorem 2.2. *For $m \geq 2$, the minimum dominating 1-partition energy of friendship graph K_3^m is equal to $2(3m-2) + \sqrt{4m^2 + 20m + 9}$.*

Proof. Consider the friendship graph K_3^m with vertex set $V = \{u_0, u_1, u_2, \ldots, u_{2m}\}$. The minimum dominating set is $D = \{u_0\}$. Then the minimum dominating 1-partition matrix of friendship graph is

$$P_1 A_D((K_3)^m) = \left(\begin{array}{c|cccccccc} & u_0 & u_1 & u_2 & u_3 & \cdots & u_{2m-2} & u_{2m-1} & u_{2m} \\ \hline u_0 & 1 & 2 & 2 & 2 & \cdots & 2 & 2 & 2 \\ u_1 & 2 & 0 & 2 & -1 & \cdots & -1 & -1 & -1 \\ u_2 & 2 & 2 & 0 & -1 & \cdots & -1 & -1 & -1 \\ u_3 & 2 & -1 & -1 & 0 & \cdots & -1 & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ u_{2m-2} & 2 & -1 & -1 & -1 & \cdots & 0 & -1 & -1 \\ u_{2m-1} & 2 & -1 & -1 & -1 & \cdots & -1 & 0 & 2 \\ u_{2m} & 2 & -1 & -1 & -1 & \cdots & -1 & 2 & 0 \end{array} \right)_{n \times n}$$

Characteristic equation is $-(\lambda-4)^{m-1}(\lambda+2)^m(\lambda^2 + (2m-5)\lambda - (10m-4)) = 0$. The minimum dominating 1-partition spectrum of K_3^m is

$$\begin{pmatrix} 4 & -2 & \dfrac{-(2m-5)+\sqrt{4m^2+20m+9}}{2} & \dfrac{-(2m-5)-\sqrt{4m^2+20m+9}}{2} \\ (m-1) & m & 1 & 1 \end{pmatrix}$$

The minimum dominating 1-partition energy of K_3^m is

$$P_1E_D(K_3^m) = |4|(m-1) + |-2|(m) + \left|\frac{-(2m-5)+\sqrt{4m^2+20m+9}}{2}\right|(1) + \left|\frac{-(2m-5)-\sqrt{4m^2+20m+9}}{2}\right|(1)$$

$$P_1E_D(K_3^m) = 2(3m-2) + \sqrt{4m^2+20m+9}.$$

□

Theorem 2.3. *For $m \geq 2$, the minimum dominating 2-partition energy of friendship graph K_3^m is equal to $2(3m-2) + \sqrt{4m^2-4m+9}$.*

Proof. Consider the friendship graph K_3^m with vertex set $V = \{u_0, u_1, u_2, \ldots, u_{2m}\}$. Let $V_1 = \{u_0\}$ and $V_2 = \{u_1, u_2, \ldots, u_{2m}\}$ be the 2-partitions of the vertex set V. The minimum dominating set is $D = \{u_0\}$. Then the minimum dominating 2-partition matrix of friendship graph is

$$P_2A_D((K_3)^m) = \left(\begin{array}{c|cccccccc} & u_0 & u_1 & u_2 & u_3 & \ldots & u_{2m-2} & u_{2m-1} & u_{2m} \\ \hline u_0 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ u_1 & 1 & 0 & 2 & -1 & \cdots & -1 & -1 & -1 \\ u_2 & 1 & 2 & 0 & -1 & \cdots & -1 & -1 & -1 \\ u_3 & 1 & -1 & -1 & 0 & \cdots & -1 & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ u_{2m-2} & 1 & -1 & -1 & -1 & \cdots & 0 & -1 & -1 \\ u_{2m-1} & 1 & -1 & -1 & -1 & \cdots & -1 & 0 & 2 \\ u_{2m} & 1 & -1 & -1 & -1 & \cdots & -1 & 2 & 0 \end{array}\right)_{n\times n}$$

Characteristic equation is $-(\lambda-4)^{m-1}(\lambda+2)^m(\lambda^2+(2m-5)\lambda-(4m-4)) = 0$. The minimum dominating 2-partition spectrum of $(K_3)^m$ is

$$\begin{pmatrix} 4 & -2 & \frac{-(2m-5)+\sqrt{4m^2-4m+9}}{2} & \frac{-(2m-5)-\sqrt{4m^2-4m+9}}{2} \\ (m-1) & m & 1 & 1 \end{pmatrix}.$$

The minimum dominating 2-partition energy of $(K_3)^m$ is

$$P_1E_D(K_3)^m) = |4|(m-1) + |-2|(m) + \left|\frac{-(2m-5)+\sqrt{4m^2-4m+9}}{2}\right|(1) + \left|\frac{-(2m-5)-\sqrt{4m^2-4m+9}}{2}\right|(1)$$

$$P_1E_D(K_3^m) = 2(3m-2) + \sqrt{4m^2-4m+9}.$$

□

Theorem 2.4. *For $n \geq 2$, the minimum dominating $(m+1)$-partition energy of friendship graph K_3^m is equal to $2(2m-1) + \sqrt{8m+1}$.*

Proof. Consider the friendship graph K_3^m with vertex set $V = \{u_0, u_1, u_2, \ldots, u_{2m}\}$. Let $V_1 = \{u_0\}$ and $V_2 = \{u_1, u_2\}$, $V_3 = \{u_4, u_5\}$, ... , $V_{m+1} = \{u_{2m-1}, u_{2m}\}$ be $(m+1)$-partitions of the vertex set V. The minimum dominating set is $D = \{u_0\}$.

Then the minimum dominating $(m+1)$-partition matrix of friendship graph is

$$P_{m+1}A_D(K_3^m) = \left(\begin{array}{c|ccccccccc} & u_0 & u_1 & u_2 & u_3 & u_4 & \cdots & u_{2m-2} & u_{2m-1} & u_{2m} \\ \hline u_0 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ u_1 & 1 & 0 & 2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ u_2 & 1 & 2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ u_3 & 1 & 0 & 0 & 0 & 2 & \cdots & 0 & 0 & 0 \\ u_4 & 1 & 0 & 0 & 2 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ u_{2m-2} & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ u_{2m-1} & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 2 \\ u_{2m} & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 2 & 0 \end{array} \right)_{(2m+1)\times(2m+1)}$$

Characteristic equation is $-(\lambda-2)^{m-1}(\lambda+2)^m(\lambda^2-3\lambda-2(m-1)) = 0$. The minimum dominating $(m+1)$-partition spectrum of $(K_3)^m$ is

$$\begin{pmatrix} 2 & -2 & \dfrac{3+\sqrt{8m+1}}{2} & \dfrac{3-\sqrt{8m+1}}{2} \\ & & & \\ (m-1) & m & 1 & 1 \end{pmatrix}.$$

The minimum dominating $(m+1)$-partition energy of K_3^m is

$$P_{(m+1)}E_D(K_3^m) = |2|(m-1) + |-2|(m) + \left|\frac{3+\sqrt{8m+1}}{2}\right|(1) + \left|\frac{3-\sqrt{8m+1}}{2}\right|(1)$$

$$P_{(m+1)}E_D(K_3^m) = 2(2m-1) + \sqrt{8m+1}.$$

□

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