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Common Coupled Fixed Point Theorems in Dislocated Quasi Fuzzy B-Metric Spaces

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Abstract: In this paper we obtain some unique common coupled fixed point theorems in dislocated quasi fuzzy b-metric spaces. Also we give some examples which support our main results.

MSC: 54H25, 47H10.

Keywords: Fuzzy metric space , Dislocated Quasi fuzzy b- metric space, Coupled fixed point, w-compatible maps.

1. Introduction and Preliminaries

The concept of fuzzy sets was introduced initially by Zadeh [17] in 1965. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and application. George and Veeramani [7], Kramosil and Michalek [10] have introduced the concept of fuzzy topological spaces induced by fuzzy metric which have very important applications in quantum particle physics, particularly in connections with both string and E-infinity theory which were given and studied by El Naschie [3–6].

Definition 1.1. *Let Φ be the set all $\phi : [0,1] \to [0,1]$ satisfying*

(ϕ_1) *: ϕ is continuous,*

(ϕ_2) *: ϕ is monotonically non-decreasing and*

(ϕ_3) *: $\phi(t) > t$ for all $t \in (0,1)$.*

From (ϕ_1) and (ϕ_3) or (ϕ_2) and (ϕ_3) it clear that $\phi(1) = 1$.

Definition 1.2 ([8])**.** *A binary operation $* : [0.1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:*

(1). $*$ is associative and commutative,

(2). $*$ is continuous,

(3). $a * 1 = a$ for all $a \in [0,1]$,

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(4). $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$*, for each* $a, b, c, d \in [0, 1]$.

Two typical examples of a continuous t*-norm are* $a * b = a.b$ *and* $a * b = min\{a, b\}$.

Definition 1.3 ([8]). *A 3-tuple* $(X, M, *)$ *is called a fuzzy metric space if* X *is an arbitrary (non-empty) set,* $*$ *is a continuous* t*-norm and* M *is a fuzzy set on* $X^2 \times (0, \infty)$*, satisfying the following conditions for each* $x, y, z \in X$ *and* $t, s > 0$,

(1). $M(x, y, t) > 0$,

(2). $M(x, y, t) = 1$ *if and only if* $x = y$,

(3). $M(x, y, t) = M(y, x, t)$,

(4). $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(5). $M(x, y, .) : (0, \infty) \to [0, 1]$ *is continuous.*

The function M *is called a fuzzy metric.*

Definition 1.4 ([16]). *A 3-tuple* $(X, M, *)$ *is called a b-fuzzy metric space if* X *is an arbitrary (non-empty) set,* $*$ *is a continuous* t*-norm and* M *is a fuzzy set on* $X^2 \times (0, \infty)$*, satisfying the following conditions for each* $x, y, z \in X$ *and* $t, s > 0$ *and a given real number* $b \geq 1$,

(1). $M(x, y, t) > 0$,

(2). $M(x, y, t) = 1$ *if and only if* $x = y$,

(3). $M(x, y, t) = M(y, x, t)$,

(4). $M(x, y, \frac{t}{b}) * M(y, z, \frac{s}{b}) \leq M(x, z, t + s)$,

(5). $M(x, y, .) : (0, \infty) \to [0, 1]$ *is continuous.*

The function M *is called a b-fuzzy metric.*

Example 1.5. *Let* $M(x, y, t) = e^{\frac{-d(x,y)}{t}}$ *or* $M(x, y, t) = \frac{t}{t+d(x,y)}$*, where* d *is a* b *- metric on* X *and* $a * c = a.c$ *for all* $a, c \in [0, 1]$*. Then it is easy to show that* $(X, M, *)$ *is a* b *- fuzzy metric space.*

Definition 1.6 ([14]). *A 3-tuple* $(X, M, *)$ *is called a dislocated quasi fuzzy metric space if* X *is an arbitrary (non-empty) set,* $*$ *is a continuous* t*-norm and* M *is a fuzzy set on* $X^2 \times (0, \infty)$*, satisfying the following conditions:*

(1). $M(x, y, t) = M(y, x, t) = 1, \forall t > 0 \Rightarrow x = y$,

(2). $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$.

Combining these definitions, we introduce following definitions.

Definition 1.7. *A 3-tuple* $(X, M, *)$ *is called a dislocated quasi fuzzy b-metric space if* X *is an arbitrary (non-empty) set,* $*$ *is a continuous* t*-norm and* M *is a fuzzy set on* $X^2 \times (0, \infty)$*, satisfying the following conditions:*

(M1): $M(x, y, t) = M(y, x, t) = 1, \forall t > 0 \Rightarrow x = y$,

(M2): $M(x, y, \frac{t}{b}) * M(y, z, \frac{s}{b}) \leq M(x, z, t + s)$,

(M3): $M(x, y, .) : (0, \infty) \to [0, 1]$ *is continuous.*

Definition 1.8. *Let $(X, M, *)$ be a dislocated quasi fuzzy b- metric space.*

(1). A sequence $\{x_n\}$ in X is said to converge to $x \in X$ iff $M(x_n, x, t) \to 1$ and $M(x, x_n, t) \to 1$, $\forall t > 0$

(2). A sequence $\{x_n\}$ in X is said to be a Cauchy sequence iff $M(x_n, x_m, t) \to 1$ and $M(x_m, x_n, t) \to 1$, $\forall t > 0$.

One can easily prove the following

Proposition 1.9. *Let $(X, M, *)$ be a dislocated quasi fuzzy b- metric space and $\{x_n\}$ converge to x then we have*

$$M(x, y, \tfrac{t}{b}) \leq \lim_{n\to\infty} \sup M(x_n, y, t) \leq M(x, y, bt) \quad \text{and}$$
$$M(x, y, \tfrac{t}{b}) \leq \lim_{n\to\infty} \inf M(x_n, y, t) \leq M(x, y, bt).$$

Definition 1.10 ([2]). *An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.*

Definition 1.11 ([11]). *An element $(x, y) \in X \times X$ is called a coupled coincidence point of a mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x, y) = gx$ and $F(y, x) = gy$.*

Definition 1.12 ([11]). *An element $(x, y) \in X \times X$ is called a common coupled fixed point of mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x, y) = g(x) = x$ and $F(y, x) = g(y) = y$.*

Definition 1.13 ([1]). *The mappings $S : X \times X \to X$ and $f : X \to X$ are called w-compatible if $f(S(x, y)) = S(fx, fy)$ and $f(S(y, x)) = S(fy, fx)$ whenever $(x, y) \in X \times X$ such that $f(x) = S(x, y)$ and $f(y) = S(y, x)$.*

2. Main Section

Now we give our main results.

Theorem 2.1. *Let $(X, M, *)$ be a complete dislocated quasi fuzzy b-metric space, $F, G : X \times X \to X$ and $S, T : X \to X$ be mappings satisfying*

(2.1.1) $M(F(x, y), G(u, v), t) \geq \phi(\min\{M(Sx, Tu, t), M(Sy, Tv, t)\})$ *for all $x, y, u, v \in X$ and $\phi \in \Phi$*

(2.1.2) $M(G(x, y), F(u, v), t) \geq \phi(\min\{M(Tx, Su, t), M(Ty, Sv, t)\})$ *for all $x, y, u, v \in X$ and $\phi \in \Phi$*

(2.1.3) $F(X \times X) \subseteq T(X)$ *and* $G(X \times X) \subseteq S(X)$,

(2.1.4) $FS = SF$ *and* $GT = TG$,

(2.1.5) *F, G, S and T are continuous.*

Then F, G, S and T have a unique common coupled fixed point in $X \times X$.

Proof. Let $(x_0, y_0) \in X \times X$. From (2.1.3), we can construct sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ and $\{w_n\}$ in X such that $z_{2n} = F(x_{2n}, y_{2n}) = Tx_{2n+1}$, $w_{2n} = F(y_{2n}, x_{2n}) = Ty_{2n+1}$, $z_{2n+1} = G(x_{2n+1}, y_{2n+1}) = Sx_{2n+2}$ and $w_{2n+1} = G(y_{2n+1}, x_{2n+1}) = Sy_{2n+2}$.

Case (i): Suppose $\min \left\{ \begin{array}{l} M(z_{n-1}, z_n, t), M(z_n, z_{n-1}, t), \\ M(w_{n-1}, w_n, t), M(w_n, z_{n-1}, t) \end{array} \right\} = 1$ for some positive integer n and for some $t > 0$. Without loss of generality assume that $n = 2m$. Then $\min \left\{ \begin{array}{l} M(z_{2m-1}, z_{2m}, t), M(z_{2m}, z_{2m-1}, t), \\ M(w_{2m-1}, w_{2m}, t), M(w_{2m}, z_{2m-1}, t) \end{array} \right\} = 1$. Then from $(M1)$, we have $z_{2m-1} = z_{2m}$ and $w_{2m-1} = w_{2m}$. Consider

$$\begin{aligned} M(z_{2m}, z_{2m+1}, t) &= M(F(x_{2m}, y_{2m}), G(x_{2m+1}, y_{2m+1}), t) \\ &\geq \phi(\min\{M(z_{2m-1}, z_{2m}, t), M(w_{2m-1}, w_{2m}, t)\}), \\ M(z_{2m+1}, z_{2m}, t) &= M(G(x_{2m+1}, y_{2m+1}), F(x_{2m}, y_{2m}), t) \\ &\geq \phi(\min\{M(z_{2m}, z_{2m-1}, t), M(w_{2m}, w_{2m-1}, t)\}), \\ M(w_{2m}, w_{2m+1}, t) &= M(F(y_{2m}, x_{2m}), G(y_{2m+1}, x_{2m+1}), t) \\ &\geq \phi(\min\{M(w_{2m-1}, w_{2m}, t), M(z_{2m-1}, z_{2m}, t)\}) \\ M(w_{2m+1}, w_{2m}, t) &= M(G(y_{2m+1}, x_{2m+1}), F(y_{2m}, x_{2m}), t) \\ &\geq \phi(\min\{M(w_{2m}, w_{2m-1}, t), M(z_{2m}, z_{2m-1}, t)\}). \end{aligned}$$

Using (ϕ_2), we have

$$\begin{aligned} \min \left\{ \begin{array}{l} M(z_{2m}, z_{2m+1}, t), M(z_{2m+1}, z_{2m}, t), \\ M(w_{2m}, w_{2m+1}, t), M(w_{2m+1}, z_{2m}, t) \end{array} \right\} &\geq \phi \left(\min \left\{ \begin{array}{l} M(z_{2m}, z_{2m-1}, t), M(z_{2m-1}, z_{2m}, t), \\ M(w_{2m}, w_{2m-1}, t), M(w_{2m-1}, z_{2m}, t) \end{array} \right\} \right) \\ &= \phi(1) = 1 \end{aligned} \tag{1}$$

Hence by $(M1)$, we have $z_{2m} = z_{2m+1}$ and $w_{2m} = w_{2m+1}$. Continuing in this way, we can show that $z_{2m-1} = z_{2m} = z_{2m+1} = ...$ and $w_{2m-1} = w_{2m} = w_{2m+1} = ...$ Hence $\{z_n\}$, $\{w_n\}$ are Cauchy sequences in X.

Case (ii): Suppose $\min \left\{ \begin{array}{l} M(z_n, z_{n-1}, t), M(z_{n-1}, z_n, t), \\ M(w_n, z_{n-1}, t), M(w_{n-1}, w_n, t) \end{array} \right\} \neq 1$ for all n and for all $t > 0$. Let $a_n(t) = \min \left\{ \begin{array}{l} M(z_n, z_{n+1}, t), M(z_{n+1}, z_n, t), \\ M(w_n, w_{n+1}, t), M(w_{n+1}, w_n, t) \end{array} \right\}$ for every $t > 0$. As in (1), we have $a_{2n}(t) \geq \phi(a_{2n-1}(t))$ and $a_{2n+1}(t) \geq \phi(a_{2n}(t))$. Thus

$$\begin{aligned} a_n(t) &\geq \phi(a_{n-1}(t)) \\ &> a_{n-1}(t) \end{aligned} \tag{2}$$

Thus $\{a_n(t)\}$ is an non-decreasing sequence in [0, 1] for every $t > 0$. Hence $\{a_n(t)\}$ converges to some $a(t) \leq 1$ for every $t > 0$. If $a(t) < 1$, taking $n \to \infty$ in (2), we get $a(t) \geq \phi(a(t)) > a(t)$. Which is a contradiction. Hence $a(t) = 1$ for every $t > 0$. Thus for all $t > 0$,

$$\lim_{n\to\infty} M(z_n, z_{n+1}, t) = \lim_{n\to\infty} M(z_{n+1}, z_n, t) = 1. \tag{3}$$

and

$$\lim_{n\to\infty} M(w_n, w_{n+1}, t) = \lim_{n\to\infty} M(w_{n+1}, w_n, t) = 1. \tag{4}$$

Suppose $\{z_{2n}\}$ or $\{w_{2n}\}$ are not Cauchy sequences in X. Then there exists an $\epsilon \in (0, 1)$ such that for each integer k, there exist integers $m(k)$ and $n(k)$ with $m(k) > n(k) \geq k$ such that

$$\min \left\{ \begin{array}{l} M(z_{2n(k)}, z_{2m(k)}, t), M(z_{2m(k)}, z_{2n(k)}, t), \\ M(w_{2n(k)}, w_{2m(k)}, t), M(w_{2m(k)}, w_{2n(k)}, t) \end{array} \right\} \leq 1 - \epsilon. \tag{5}$$

for $k = 1, 2, 3, \cdots$. We may assume that

$$\min \left\{ \begin{array}{l} M(z_{2n(k)}, z_{2m(k)-2}, t), M(z_{2m(k)-2}, z_{2n(k)}, t), \\ M(w_{2n(k)}, w_{2m(k)-2}, t), M(w_{2m(k)-2}, w_{2n(k)}, t) \end{array} \right\} > 1 - \epsilon. \tag{6}$$

by choosing $m(k)$ be the smallest number exceeding $n(k)$ for which (5) holds. Let

$$d_k(t) = \min \left\{ \begin{array}{l} M(z_{2n(k)}, z_{2m(k)}, t), M(z_{2m(k)}, z_{2n(k)}, t), \\ M(w_{2n(k)}, w_{2m(k)}, t), M(w_{2m(k)}, w_{2n(k)}, t) \end{array} \right\}.$$

Using (5), we have

$$\begin{aligned} 1-\epsilon &\geq d_k(t) \\ &\geq \min \left\{ \begin{array}{l} M(z_{2n(k)}, z_{2m(k)-2}, \frac{t}{2b}) * M(z_{2m(k)-2}, z_{2m(k)}, \frac{t}{2b}), \\ M(z_{2m(k)}, z_{2m(k)-2}, \frac{t}{2b}) * M(z_{2m(k)-2}, z_{2n(k)}, \frac{t}{2b}), \\ M(w_{2n(k)}, w_{2m(k)-2}, \frac{t}{2b}) * M(w_{2m(k)-2}, w_{2m(k)}, \frac{t}{2b}), \\ M(w_{2m(k)}, w_{2m(k)-2}, \frac{t}{2b}) * M(w_{2m(k)-2}, w_{2n(k)}, \frac{t}{2b}) \end{array} \right\} \\ &\geq \min \left\{ \begin{array}{l} M\left(z_{2n(k)}, z_{2m(k)-2}, \frac{t}{2b}\right) * M\left(z_{2m(k)-2}, z_{2m(k)-1}, \frac{t}{4b}\right) * \\ M\left(z_{2m(k)-1}, z_{2m(k)}, \frac{t}{4b}\right), M\left(z_{2m(k)}, z_{2m(k)-2}, \frac{t}{2b}\right) * \\ M\left(z_{2m(k)-2}, z_{2m(k)-1}, \frac{t}{4b}\right) * M\left(z_{2m(k)-1}, z_{2n(k)}, \frac{t}{4b}\right), \\ M\left(w_{2n(k)}, w_{2m(k)-2}, \frac{t}{2b}\right) * M\left(w_{2m(k)-2}, w_{2m(k)-1}, \frac{t}{4b}\right) * \\ M\left(w_{2m(k)-1}, w_{2m(k)}, \frac{t}{4b}\right), M\left(w_{2m(k)}, w_{2m(k)-2}, \frac{t}{2b}\right) * \\ M\left(w_{2m(k)-2}, w_{2m(k)-1}, \frac{t}{4b}\right) * M\left(w_{2m(k)-1}, w_{2n(k)}, \frac{t}{4b}\right) \end{array} \right\} \\ &\geq \min \left\{ \begin{array}{l} (1-\varepsilon) * M\left(z_{2m(k)-2}, z_{2m(k)-1}, \frac{t}{4b}\right) * M\left(z_{2m(k)-1}, z_{2m(k)}, \frac{t}{4b}\right), \\ (1-\varepsilon) * M\left(z_{2m(k)-2}, z_{2m(k)-1}, \frac{t}{4b}\right) * M\left(z_{2m(k)-1}, z_{2n(k)}, \frac{t}{4b}\right), \\ (1-\varepsilon) * M\left(w_{2m(k)-2}, w_{2m(k)-1}, \frac{t}{4b}\right) * M\left(w_{2m(k)-1}, w_{2m(k)}, \frac{t}{4b}\right), \\ (1-\varepsilon) * M\left(w_{2m(k)-2}, w_{2m(k)-1}, \frac{t}{4b}\right) * M\left(w_{2m(k)-1}, w_{2n(k)}, \frac{t}{4b}\right) \end{array} \right\}, \text{ from (6)} \end{aligned}$$

Letting $k \to \infty$, we get $1-\epsilon \geq \lim_{k\to\infty} d_k(t) \geq 1-\epsilon$ by (3),(4). Thus

$$\lim_{k\to\infty} d_k(t) = 1-\epsilon \tag{7}$$

for all $t > 0$. On other hand, we have

$$\begin{aligned} d_k(t) &\geq \min \left\{ \begin{array}{l} M\left(z_{2n(k)}, z_{2n(k)+1}, \frac{t}{2b}\right) * M\left(z_{2n(k)+1}, z_{2m(k)}, \frac{t}{2b}\right), \\ M\left(z_{2m(k)}, z_{2m(k)+1}, \frac{t}{2b}\right) * M\left(z_{2m(k)+1}, z_{2n(k)}, \frac{t}{2b}\right), \\ M\left(w_{2n(k)}, w_{2n(k)+1}, \frac{t}{2b}\right) * M\left(w_{2n(k)+1}, w_{2m(k)}, \frac{t}{2b}\right), \\ M\left(w_{2m(k)}, w_{2m(k)+1}, \frac{t}{2b}\right) * M\left(w_{2m(k)+1}, w_{2n(k)}, \frac{t}{2b}\right) \end{array} \right\} \\ &= \min \left\{ \begin{array}{l} M\left(z_{2n(k)}, z_{2n(k)+1}, \frac{t}{2b}\right) * M\left(G(x_{2n(k)+1}, y_{2n(k)+1}), F(x_{2m(k)}, y_{2m(k)}), \frac{t}{2b}\right), \\ M\left(z_{2m(k)}, z_{2m(k)+1}, \frac{t}{2b}\right) * M\left(G(x_{2m(k)+1}, y_{2m(k)+1}), F(x_{2n(k)}, y_{2n(k)}), \frac{t}{2b}\right), \\ M\left(w_{2n(k)}, w_{2n(k)+1}, \frac{t}{2b}\right) * M\left(G(y_{2n(k)+1}, x_{2n(k)+1}), F(y_{2m(k)}, x_{2m(k)}), \frac{t}{2b}\right), \\ M\left(w_{2m(k)}, w_{2m(k)+1}, \frac{t}{2b}\right) * M\left(G(y_{2m(k)+1}, x_{2m(k)+1}), F(y_{2n(k)}, x_{2n(k)}), \frac{t}{2b}\right) \end{array} \right\} \end{aligned}$$

$$\geq \min \left\{ \begin{array}{l} M\left(z_{2n(k)}, z_{2n(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{\begin{array}{l} M\left(z_{2n(k)}, z_{2m(k)-1}, \frac{t}{2b}\right), \\ M\left(w_{2n(k)}, w_{2m(k)}, \frac{t}{2b}\right)\end{array}\right\}\right), \\ M\left(z_{2m(k)}, z_{2m(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{\begin{array}{l} M\left(z_{2m(k)}, z_{2n(k)-1}, \frac{t}{2b}\right), \\ M\left(w_{2m(k)}, w_{2n(k)}, \frac{t}{2b}\right)\end{array}\right\}\right), \\ M\left(w_{2n(k)}, w_{2n(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{\begin{array}{l} M\left(w_{2n(k)}, w_{2m(k)}, \frac{t}{2b}\right), \\ M\left(z_{2n(k)}, z_{2m(k)-1}, \frac{t}{2b}\right)\end{array}\right\}\right), \\ M\left(w_{2m(k)}, w_{2m(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{\begin{array}{l} M\left(w_{2m(k)}, w_{2n(k)}, \frac{t}{2b}\right), \\ M\left(z_{2m(k)}, z_{2n(k)-1}, \frac{t}{2b}\right)\end{array}\right\}\right) \end{array} \right\} \tag{8}$$

$M(z_{2n(k)}, z_{2m(k)-1, \frac{t}{2b}}) \geq M(z_{2n(k)}, z_{2m(k)-2, \frac{t}{4b^2}}) * M(z_{2m(k)-2}, z_{2m(k)-1, \frac{t}{4b^2}})$ and $M(z_{2m(k)}, z_{2n(k)-1, \frac{t}{2b}}) \geq M(z_{2m(k)}, z_{2n(k), \frac{t}{4b^2}}) * M(z_{2n(k)}, z_{2n(k)-1, \frac{t}{4b^2}})$, from (6). Write similarly for other values in (8)

$$d_k(t) \geq \min \left\{ \begin{array}{l} M\left(z_{2n(k)}, z_{2n(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{ 1-\varepsilon * M\left(z_{2m(k)-2}, z_{2m(k)-1}, \frac{t}{4b^2}\right), 1-\varepsilon \right\}\right), \\ M\left(z_{2m(k)}, z_{2m(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{ 1-\varepsilon * M\left(z_{2n(k)}, z_{2n(k)-1}, \frac{t}{4b^2}\right), 1-\varepsilon \right\}\right), \\ M\left(w_{2n(k)}, w_{2n(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{ 1-\varepsilon, 1-\varepsilon * M\left(z_{2m(k)-2}, z_{2m(k)-1}, \frac{t}{4b^2}\right) \right\}\right), \\ M\left(w_{2m(k)}, w_{2m(k)+1}, \frac{t}{2b}\right) * \phi\left(\min\left\{ 1-\varepsilon, 1-\varepsilon * M\left(z_{2n(k)}, z_{2n(k)-1}, \frac{t}{4b^2}\right) \right\}\right) \end{array} \right\}$$

Letting $k \to \infty$, using(7),(3),(4), (ϕ_1) and (ϕ_3), we get $1-\epsilon \geq \phi(1-\epsilon) > 1-\epsilon$, which is a contradiction. Thus $\{z_{2n}\}$ and $\{w_{2n}\}$ are Cauchy sequences. Hence $\lim\limits_{n,m\to\infty} M(z_{2n}, z_{2m}, t) = 1$ and $\lim\limits_{n,m\to\infty} M(w_{2n}, w_{2m}, t) = 1$, $\forall t > 0$. Now

$$\begin{aligned} M(z_{2n+1}, z_{2m+1}, t) &\geq M\left(z_{2n+1}, z_{2m}, \tfrac{2t}{3b}\right) * M\left(z_{2m}, z_{2m+1}, \tfrac{t}{3b}\right) \\ &\geq M\left(z_{2n+1}, z_{2n}, \tfrac{t}{3b^2}\right) * M\left(z_{2n}, z_{2m}, \tfrac{t}{3b^2}\right) * M\left(z_{2m}, z_{2m+1}, \tfrac{t}{3b}\right) \end{aligned}$$

Letting $n, m \to \infty$, we get $\lim\limits_{n,m\to\infty} M(z_{2n+1}, z_{2m+1}, t) \geq 1 * 1 * 1 = 1$. Thus $\lim\limits_{n,m\to\infty} M(z_{2n+1}, z_{2m+1}, t) = 1$. Similarly $\lim\limits_{n,m\to\infty} M(w_{2n+1}, w_{2m+1}, t) = 1$. Thus $\{z_{2n+1}\}$ and $\{w_{2n+1}\}$ are Cauchy. Hence $\{z_n\}$ and $\{w_n\}$ are Cauchy. Since X is complete there exist $z, w \in X$ such that $\{z_n\}$ converges to z and $\{w_n\}$ converges to w. Since S and F are continuous and $SF = FS$, we have

$$Sz = \lim_{n\to\infty} Sz_{2n} = \lim_{n\to\infty} S(F(x_{2n}, y_{2n})) = \lim_{n\to\infty} F(Sx_{2n}, Sy_{2n}) = \lim_{n\to\infty} F(z_{2n-1}, w_{2n-1}) = F(z, w). \tag{9}$$

Similarly we can show that

$$Sw = F(w, z). \tag{10}$$

Since G and T are continuous and $GT = TG$, we have

$$Tz = \lim_{n\to\infty} Tz_{2n+1} = \lim_{n\to\infty} T(G(x_{2n+1}, y_{2n+1})) = \lim_{n\to\infty} G(Tx_{2n+1}, Ty_{2n+1}) = \lim_{n\to\infty} G(z_{2n}, w_{2n}) = G(z, w). \tag{11}$$

Similarly we can show that

$$Tw = G(w, z). \tag{12}$$

Now

$$M(Sz, Tz, t) = M(F(z, w), G(z, w), t)$$

$$\geq \phi\left(\min\{M(Sz,Tz,t), M(Sw,Tw,t)\}\right)$$

Similarly we can show that

$$M(Tz,Sz,t) \geq \phi\left(\min\{M(Tz,Sz,t), M(Tw,Sw,t)\}\right),$$
$$M(Sw,Tw,t) \geq \phi\left(\min\{M(Sw,Tw,t), M(Sz,Tz,t)\}\right)$$
$$M(Tw,Sw,t) \geq \phi\left(\min\{M(Tw,Sw,t), M(Tz,Sz,t)\}\right).$$

Thus we have

$$\min\left\{\begin{array}{c} M(Sz,Tz,t), M(Tz,Sz,t), \\ M(Sw,Tw,t), M(Tw,Sw,t) \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{c} M(Sz,Tz,t), M(Tz,Sz,t) \\ M(Sw,Tw,t), M(Tw,Sw,t) \end{array}\right\}\right)$$

which in turn yields from (ϕ_3) and (M1) that $Sz = Tz$ and $Sw = Tw$. Let $\alpha = Sz = Tz$ and $\beta = Sw = Tw$. Then $S\alpha = S(Sz) = S(F(z,w)) = F(Sz,Sw) = F(\alpha,\beta)$. Similarly $S\beta = F(\beta,\alpha)$, $T\alpha = G(\alpha,\beta)$ and $T\beta = G(\beta,\alpha)$. Consider

$$\begin{aligned} M(S\alpha,\alpha,t) &= M(F(\alpha,\beta), G(z,w), t) \\ &\geq \phi\left(\min\{M(S\alpha,\alpha,t), M(S\beta,\beta,t)\}\right). \end{aligned}$$

Similarly

$$M(\alpha,S\alpha,t) \geq \phi\left(\min\{M(\alpha,S\alpha,t), M(\beta,S\beta,t)\}\right),$$
$$M(S\beta,\beta,t) \geq \phi\left(\min\{M(S\beta,\beta,t), M(S\alpha,\alpha,t)\}\right)$$
$$M(\beta,S\beta,t) \geq \phi\left(\min\{M(\beta,S\beta,t), M(\alpha,S\alpha,t)\}\right).$$

Hence

$$\min\left\{\begin{array}{c} M(S\alpha,\alpha,t), M(\alpha,S\alpha,t), \\ M(S\beta,\beta,t), M(\beta,S\beta,t) \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{c} M(S\alpha,\alpha,t), M(\alpha,S\alpha,t), \\ M(S\beta,\beta,t), M(\beta,S\beta,t) \end{array}\right\}\right)$$

which in turn yields from (ϕ_3) and (M1) that $S\alpha = \alpha$ and $S\beta = \beta$. Similarly we can show that $T\alpha = \alpha$ and $T\beta = \beta$. Thus $F(\alpha,\beta) = S\alpha = \alpha = T\alpha = G(\alpha,\beta)$ and $F(\beta,\alpha) = S\beta = \beta = T\beta = G(\beta,\alpha)$. Thus (α,β) is common coupled fixed point of F, G, S and T. Using (2.1.1), (2.1.2) we can show that (α,β) is unique common coupled fixed point of F, G, S and T. □

We give an example to illustrate Theorem 2.1.

Example 2.2. *Let $X = [0,1]$ and $a * c = ac$ for all $a, c \in [0,1]$ and M be a fuzzy set on $X \times X \times (0,\infty)$ defined by $M(x,y,t) = e^{-\frac{[|x-y|^2+|x|]}{t}}$. Let $F, G : X\times X \to X$ and $S, T : X \to X$ be defined by $F(x,y) = \frac{x+y}{64}$, $G(x,y) = \frac{x+y}{96}$, $Sx = \frac{x}{2}$ and $Tx = \frac{x}{3}$. Let $\phi : [0,1] \to [0,1]$ be defined by $\phi(t) = t^{\frac{1}{16}}$, for all $t \in [0,1]$. Now $M(x,y,t) = M(y,x,t) = 1 \Rightarrow e^{-\frac{[|x-y|^2+|x|]}{t}} = 1$ and $e^{-\frac{[|y-x|^2+|y|]}{t}} = 1 \Rightarrow x - y = 0, x = 0, y - x = 0, y = 0 \Rightarrow x = y = 0 \Rightarrow x = y$. Consider*

$$\begin{aligned} M(x,y,t+s) &= e^{\frac{-\left[|x-y|^2+|x|\right]}{t+s}} \\ &\geq e^{\frac{-2\left[\{|x-z|^2+|x|\}+\{|z-y|^2+|z|\}\right]}{t+s}} \\ &= e^{\frac{-2\left[|x-z|^2+|x|\right]}{t+s}}.e^{\frac{-2\left[|z-y|^2+|z|\right]}{t+s}} \\ &\geq e^{\frac{-2\left[|x-z|^2+|x|\right]}{t}}.e^{\frac{-2\left[|z-y|^2+|z|\right]}{s}} \\ &= e^{\frac{-\left[|x-z|^2+|x|\right]}{t/2}}.e^{\frac{-\left[|z-y|^2+|z|\right]}{s/2}} \\ &= M\left(x,z,\tfrac{t}{2}\right) * M\left(z,y,\tfrac{s}{2}\right). \end{aligned}$$

Thus M is a dislocated quasi fuzzy b-metric with $b = 2$. Now consider

$$\begin{aligned}\left|\frac{x+y}{64}-\frac{u+v}{96}\right|^2+\frac{x+y}{64} &= \left|\frac{(3x-2u)+(3y-2v)}{6\times 32}\right|^2+\frac{x+y}{64}\\ &= \left|\frac{\left(\frac{x}{2}-\frac{u}{3}\right)+\left(\frac{y}{2}-\frac{v}{3}\right)}{32}\right|^2+\frac{x+y}{64}\\ &\le \frac{2}{32\times 32}\left[\left(\frac{x}{2}-\frac{u}{3}\right)^2+\left(\frac{y}{2}-\frac{v}{3}\right)^2\right]+\frac{x+y}{64}\\ &\le \frac{1}{32}\left[\frac{1}{16}\left(\frac{x}{2}-\frac{u}{3}\right)^2+\frac{x}{2}+\frac{1}{16}\left(\frac{y}{2}-\frac{v}{3}\right)^2+\frac{y}{2}\right]\\ &\le \frac{1}{32}\left[\left\{\left(\frac{x}{2}-\frac{u}{3}\right)^2+\frac{x}{2}\right\}+\left\{\left(\frac{y}{2}-\frac{v}{3}\right)^2+\frac{y}{2}\right\}\right]\\ &\le \frac{2}{32}\left[\max\left\{\left(\frac{x}{2}-\frac{u}{3}\right)^2+\frac{x}{2},\left(\frac{y}{2}-\frac{v}{3}\right)^2+\frac{y}{2}\right\}\right].\end{aligned}$$

Hence

$$\begin{aligned}M\left(F(x,y),G(u,v),t\right) &= e^{\frac{-\left\{\left|\frac{x+y}{64}-\frac{u+v}{96}\right|^2+\frac{x+y}{64}\right\}}{t}}\\ &\ge e^{\frac{-\frac{1}{16}\max\left\{\left(\frac{x}{2}-\frac{u}{3}\right)^2+\frac{x}{2},\left(\frac{y}{2}-\frac{v}{3}\right)^2+\frac{y}{2}\right\}}{t}}\\ &= \left[e^{\frac{-\max\left\{\left(\frac{x}{2}-\frac{u}{3}\right)^2+\frac{x}{2},\left(\frac{y}{2}-\frac{v}{3}\right)^2+\frac{y}{2}\right\}}{t}}\right]^{\frac{1}{16}}\\ &= \left[\min\left\{e^{-\left\{\frac{\left(\frac{x}{2}-\frac{u}{3}\right)^2+\frac{x}{2}}{t}\right\}},e^{-\left\{\frac{\left(\frac{y}{2}-\frac{v}{3}\right)^2+\frac{y}{2}}{t}\right\}}\right\}\right]^{\frac{1}{16}}\\ &= \left[\min\left\{M(Sx,Tu,t),M(Sy,Tv,t)\right\}\right]^{\frac{1}{16}}\\ &= \phi\left(\min\left\{M(Sx,Tu,t),M(Sy,Tv,t)\right\}\right).\end{aligned}$$

Thus (2.1.1) is satisfied. Similarly we can easily verify (2.1.2), (2.1.3), (2.1.4) and (2.1.5). Clearly (0, 0) is the unique common coupled fixed point of F, G, S and T. Now replacing the completeness of X, continuities of F, G, S and T and commutativity of pairs (F, S) and (G, T) by w-compatible pairs (F, S) and (G, T) and completeness of $S(X)$ or $T(X)$, we prove a unique common coupled fixed point.Infact we prove the following theorem.

Theorem 2.3. *Let $(X, M, *)$ be a complete dislocated quasi fuzzy b-metric space, $F, G : X \times X \to X$ and $S, T : X \to X$ be mappings satisfying*

(2.3.1) $M(F(x, y), G(u, v), t) \ge \phi(\min\{M(Sx, Tu, 2b^2t), M(Sy, Tv, 2b^2t)\})$ for all $x, y, u, v \in X$ and $\phi \in \Phi$

(2.3.2) $M(G(x, y), F(u, v), t) \ge \phi(\min\{M(Tx, Su, 2b^2t), M(Ty, Sv, 2b^2t)\})$ for all $x, y, u, v \in X$ and $\phi \in \Phi$

(2.3.3) $F(X \times X) \subseteq T(X)$ and $G(X \times X) \subseteq S(X)$,

(2.3.4) one of $S(X)$ and $T(X)$ is a complete subspace of X,

(2.3.5) (F, S) and (G, T) are w-compatible.

Then F, G, S and T have a unique common coupled fixed point in $X \times X$.

Proof. As in proof of Theorem 2.1, the sequences $\{z_n\}$ and $\{w_n\}$ are Cauchy, where $z_{2n} = F(x_{2n}, y_{2n}) = Tx_{2n+1}$, $w_{2n} = F(y_{2n}, x_{2n}) = Ty_{2n+1}$, $z_{2n+1} = G(x_{2n+1}, y_{2n+1}) = Sx_{2n+2}$ and $w_{2n+1} = G(y_{2n+1}, x_{2n+1}) = Sy_{2n+2}$. Without loss of generality assume that $S(X)$ is a complete subspace of X. Since $z_{2n+1} = Sx_{2n+2} \in S(X)$ and $w_{2n+1} = Sy_{2n+2} \in S(X)$, there exist z, w, u and v in X such that $z_{2n+1} \to z = Su$, $w_{2n+1} \to w = Sv$. By Proposition (1.9), (2.3.1), (ϕ_1) and (ϕ_2), we have

$$\begin{aligned}M\left(F(u,v),z,bt\right) &\ge \lim_{n\to\infty}\sup\ M\left(F(u,v),G(x_{2n+1},y_{2n+1}),t\right)\\ &\ge \lim_{n\to\infty}\sup\ \phi\left(\min\left\{M\left(Su,z_{2n},2b^2t\right),M\left(Sv,w_{2n},2b^2t\right)\right\}\right)\\ &\ge \phi\left(\min\left\{M\left(z,z_{2n},2b^2t\right),M\left(w,w_{2n},2b^2t\right)\right\}\right)\\ &= \phi(1) = 1.\end{aligned}$$

$$
\begin{aligned}
M\left(z, F(u,v), bt\right) &\geq \lim_{n\to\infty} \sup\; M\left(G(x_{2n+1}, y_{2n+1}), F(u,v), t\right) \\
&\geq \lim_{n\to\infty} \sup\; \phi\left(\min\left\{M\left(z_{2n}, Su, 2b^2t\right), M\left(w_{2n}, Sv, 2b^2t\right)\right\}\right) \\
&\geq \phi\left(\min\left\{M\left(z_{2n}, z, 2b^2t\right), M\left(w_{2n}, w, 2b^2t\right)\right\}\right) \\
&= \phi(1) = 1.
\end{aligned}
$$

Thus $M(F(u,v), z, bt) = M(z, F(u,v), bt) = 1$ for all $t > 0$, $b \geq 1$. From (M1), we have $F(u,v) = z$ so that $Su = z = F(u,v)$. Similarly we can show that $Sv = w = F(v,u)$. Since the pair (F,S) is w-compatible, we have $Sz = S(Su) = S(F(u,v)) = F(Su, Sv) = F(z,w)$ and $Sw = S(Sv) = S(F(v,u)) = F(Sv, Su) = F(w,z)$. By Proposition 1.9, (2.3.1), (ϕ_1) and (ϕ_2), we have

$$
\begin{aligned}
M\left(Sz, z, bt\right) &= M\left(F(z,w), z, bt\right) \\
&\geq \lim_{n\to\infty} \sup\; M\left(F(z,w), G(x_{2n+1}, y_{2n+1}), t\right) \\
&\geq \lim_{n\to\infty} \sup\; \phi\left(\min\left\{M\left(Sz, z_{2n}, 2b^2t\right), M\left(Sw, w_{2n}, 2b^2t\right)\right\}\right) \\
&\geq \phi\left(\min\left\{M\left(Sz, z, bt\right), M\left(Sw, w, bt\right)\right\}\right).
\end{aligned}
$$

Similarly we can show that

$$
\begin{aligned}
M(z, Sz, bt) &\geq \phi\left(\min\left\{M\left(z, Sz, bt\right), M\left(w, Sw, bt\right)\right\}\right), \\
M(Sw, w, bt) &\geq \phi\left(\min\left\{M\left(Sw, w, bt\right), M\left(Sz, z, bt\right)\right\}\right) \\
M(w, Sw, bt) &\geq \phi\left(\min\left\{M\left(w, Sw, bt\right), M\left(z, Sz, bt\right)\right\}\right).
\end{aligned}
$$

From (ϕ_2), we have

$$
\min\left\{\begin{array}{l} M\left(Sz, z, bt\right), M\left(z, Sz, bt\right), \\ M\left(Sw, w, bt\right), M\left(w, Sw, bt\right) \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{l} M\left(Sz, z, bt\right), M\left(z, Sz, bt\right), \\ M\left(Sw, w, bt\right), M\left(w, Sw, bt\right) \end{array}\right\}\right)
$$

which in turn yields from (ϕ_3) and $(M1)$ that $z = Sz$ and $w = Sw$. Thus

$$
z = Sz = F(z,w) \tag{13}
$$

and

$$
w = Sw = F(w,z) \tag{14}
$$

Since $F(X \times X) = T(X)$, there exist α, β in X such that $T\alpha = F(z,w) = Sz = z$ and $T\beta = F(w,z) = Sw = w$. Now using (2.3.1) and (ϕ_2), we have

$$
\begin{aligned}
M\left(T\alpha, G(\alpha,\beta), t\right) &= M\left(F(z,w), G(\alpha,\beta), t\right) \\
&\geq \phi\left(\min\left\{M\left(Sz, T\alpha, 2b^2t\right), M\left(Sw, T\beta, 2b^2t\right)\right\}\right) \\
&\geq \phi\left(\min\left\{\begin{array}{l} M\left(T\alpha, G(\alpha,\beta), tb\right) * M\left(G(\alpha,\beta), T\alpha, tb\right), \\ M\left(T\beta, G(\beta,\alpha), tb\right) * M\left(G(\beta,\alpha), T\beta, tb\right) \end{array}\right\}\right) \\
&\geq \phi\left(\min\left\{\begin{array}{l} M\left(T\alpha, G(\alpha,\beta), t\right) * M\left(G(\alpha,\beta), T\alpha, t\right), \\ M\left(T\beta, G(\beta,\alpha), t\right) * M\left(G(\beta,\alpha), T\beta, t\right) \end{array}\right\}\right).
\end{aligned}
$$

Similarly we can show that

$$
M(G(\alpha,\beta), T\alpha, t) \geq \phi\left(\min\left\{\begin{array}{l} M\left(G(\alpha,\beta),, T\alpha, t\right) * M\left(T\alpha, G(\alpha,\beta), t\right), \\ M\left(G(\beta,\alpha), T\beta, t\right) * M\left(T\beta, G(\beta,\alpha), t\right) \end{array}\right\}\right),
$$

$$M(T\beta, G(\beta,\alpha), t) \geq \phi\left(\min\left\{\begin{array}{l} M\left(T\beta, G(\beta,\alpha), t\right) * M\left(G(\beta,\alpha), T\beta, t\right) \\ M\left(T\alpha, G(\alpha,\beta), t\right) * M\left(G(\alpha,\beta), T\alpha, t\right), \end{array}\right\}\right)$$

$$M(G(\beta,\alpha), T\beta, t) \geq \phi\left(\min\left\{\begin{array}{l} M\left(G(\beta,\alpha), T\beta, t\right) * M\left(T\beta, G(\beta,\alpha), t\right) \\ M\left(T\alpha, G(\alpha,\beta), t\right) * M\left(G(\alpha,\beta), T\alpha, t\right), \end{array}\right\}\right).$$

Hence

$$\min\left\{\begin{array}{l} M(T\alpha, G(\alpha,\beta), t), M(G(\alpha,\beta), T\alpha, t), \\ M(T\beta, G(\beta,\alpha), t), M(G(\beta,\alpha), T\beta, t) \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{l} M\left(T\alpha, G(\alpha,\beta), t\right) * M\left(G(\alpha,\beta), T\alpha, t\right), \\ M\left(T\beta, G(\beta,\alpha), t\right) * M\left(G(\beta,\alpha), T\beta, t\right) \end{array}\right\}\right)$$

which in turn yields from (ϕ_3) and $(M1)$ that $T\alpha = G(\alpha,\beta)$ and $T\beta = G(\beta,\alpha)$. Since the pair (G,T) is w-compatible, we have

$$Tz = T(T\alpha) = T(G(\alpha,\beta)) = G(T\alpha, T\beta) = G(z,w) \tag{15}$$

$$Tw = T(T\beta) = T(G(\beta,\alpha)) = G(T\beta, T\alpha) = G(w,z). \tag{16}$$

Now we have

$$\begin{aligned} M\left(z, G(z,w), t\right) &= M\left(F(z,w), G(z,w), t\right) \\ &\geq \phi\left(\min\left\{M\left(Sz, Tz, 2b^2t\right), M\left(Sw, Tw, 2b^2t\right)\right\}\right) \\ &= \phi\left(\min\left\{\begin{array}{l} M\left(z, G(z,w), 2b^2t\right), \\ M\left(w, G(w,z), 2b^2t\right) \end{array}\right\}\right), \text{ from } (13), (14), (15) \text{ and } (16) \\ &= \phi\left(\min\left\{ M\left(z, G(z,w), t\right), M\left(w, G(w,z), t\right) \right\}\right). \end{aligned}$$

Similarly

$$M(G(z,w), z, t) \geq \phi\left(\min\left\{ M\left(G(z,w), z, t\right), M\left(G(w,z), w, t\right) \right\}\right),$$

$$M(w, G(w,z), t) \geq \phi\left(\min\left\{ M\left(w, G(w,z), t\right), M\left(z, G(z,w), t\right) \right\}\right)$$

$$M(G(w,z), w, t) \geq \phi\left(\min\left\{ M\left(G(w,z), w, t\right), M\left(G(z,w), z, t\right) \right\}\right).$$

Thus from (ϕ_2), we have

$$\min\left\{\begin{array}{l} M\left(z, G(z,w), t\right), M\left(G(z,w), z, t\right), \\ M\left(w, G(w,z), t\right), M\left(G(w,z), w, t\right), \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{l} M\left(z, G(z,w), t\right), M\left(w, G(w,z), t\right), \\ M\left(G(w,z), w, t\right), M\left(G(z,w), z, t\right) \end{array}\right\}\right)$$

which in turn yields from (ϕ_3) and $(M1)$ that $z = G(z,w)$ and $w = G(w,z)$. Thus

$$z = G(z,w) = Tz \tag{17}$$

and

$$w = G(w,z) = Tw. \tag{18}$$

From (13), (14), (17) and (18), it follows that (z,w) is a common coupled fixed point of F, G, S and T. Uniqueness of common coupled fixed point of F, G, S and T follows easily from (2.3.1) and (2.3.2). $\square$

Now we give an example to support Theorem 2.3.

Example 2.4. *Let $X = [0,1]$ and $a*c = ac$ for all $a, c \in [0,1]$ and M be a fuzzy set on $X \times X \times (0,\infty)$ defined by $M(x,y,t) = e^{-\frac{[|x-y|^2+|x|]}{t}}$. Let $F, G : X \times X \to X$ and $S, T : X \to X$ be defined by $F(x,y) = \frac{x^2+y^2}{128}$, $G(x,y) = \frac{x^2+y^2}{256}$, $Sx = \frac{x^2}{2}$ and $Tx = \frac{x^2}{4}$ and Let $\phi : [0,1] \to [0,1]$ be defined by $\phi(t) = t^{\frac{1}{4}}$, for all $t \in [0,1]$. As in Example 2.2, M is a dislocated quasi fuzzy b-metric space with $b = 2$. Consider*

$$\begin{aligned}
\left|\frac{x^2+y^2}{128} - \frac{u^2+v^2}{256}\right|^2 + \frac{x^2+y^2}{128} &= \left|\frac{(2x^2-u^2)+(2y^2-v^2)}{256}\right|^2 + \frac{x^2+y^2}{128}\\
&= \left|\frac{\left(\frac{x^2}{2}-\frac{u^2}{4}\right)+\left(\frac{y^2}{2}-\frac{v^2}{4}\right)}{64}\right|^2 + \frac{x^2+y^2}{128}\\
&\leq \frac{2}{64\times 64}\left[\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2 + \left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2\right] + \frac{x^2+y^2}{128}\\
&= \frac{1}{64}\left[\frac{1}{32}\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2 + \frac{x^2}{2} + \frac{1}{32}\left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2 + \frac{y^2}{2}\right]\\
&\leq \frac{1}{64}\left[\left\{\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2 + \frac{x^2}{2}\right\} + \left\{\left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2 + \frac{y^2}{2}\right\}\right]\\
&\leq \frac{2}{64}\left[\max\left\{\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2 + \frac{x^2}{2}, \left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2 + \frac{y^2}{2}\right\}\right].
\end{aligned}$$

Hence

$$\begin{aligned}
M\left(F(x,y), G(u,v), t\right) &= e^{\frac{-\left\{\left|\frac{x^2+y^2}{128} - \frac{u^2+v^2}{256}\right|^2 + \frac{x^2+y^2}{128}\right\}}{t}}\\
&\geq e^{\frac{-\frac{1}{32}\max\left\{\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2+\frac{x^2}{2}, \left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2+\frac{y^2}{2}\right\}}{t}}\\
&= \left[e^{\frac{-\max\left\{\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2+\frac{x^2}{2}, \left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2+\frac{y^2}{2}\right\}}{8t}}\right]^{\frac{1}{4}}\\
&= \left[\min\left\{e^{-\left\{\frac{\left(\frac{x^2}{2}-\frac{u^2}{4}\right)^2+\frac{x^2}{2}}{8t}\right\}}, e^{-\left\{\frac{\left(\frac{y^2}{2}-\frac{v^2}{4}\right)^2+\frac{y^2}{2}}{8t}\right\}}\right\}\right]^{\frac{1}{4}}\\
&= \left[\min\left\{M(Sx,Tu,2b^2t), M(Sy,Tv,2b^2t)\right\}\right]^{\frac{1}{4}}\\
&= \phi\left(\min\left\{M(Sx,Tu,2b^2t), M(Sy,Tv,2b^2t)\right\}\right).
\end{aligned}$$

Since $\phi(t) = t^{\frac{1}{4}}$.

Thus (2.3.1) is satisfied. Similarly we can easily verify (2.3.2), (2.3.3), (2.3.4) and (2.3.5). Clearly (0, 0) is the unique common coupled fixed point of F, G, S and T. Wadhwa introduced the following definitions.

Definition 2.5 ([9]). *Let f and g be two self -maps of a fuzzy metric space $(X, M, *)$. We say that f and g satisfy E.A. Like property if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in f(X)$ or $z \in g(X)$, i.e., $z \in f(X) \cup g(X)$.*

Definition 2.6 ([9]). *Let A, B, S and T be self maps of a fuzzy metric space $(X, M, *)$, then the pairs (A, S) and (B, T) said to satisfy common E.A. Like property if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z$*
where $z \in S(X) \cup T(X)$ or $z \in A(X) \cup B(X)$.

Now we extend this definition to dislocated quasi fuzzy b-metric spaces as follows.

Definition 2.7. *Let $(X, M, *)$ be a dislocated quasi fuzzy b-metric space with $b \geq 1$ and $F, G : X \times X \to X$ and $S, T : X \to X$ be mappings. The pairs (F, S) and (G, T) are said to satisfy common E.A. like property if there exist sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ and $\{w_n\}$ in X such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} G(z_n, w_n) = \lim_{n\to\infty} Tz_n = \alpha$ and $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} G(w_n, z_n) = \lim_{n\to\infty} Tw_n = \alpha^1$ for some $\alpha, \alpha^1 \in S(X) \cap T(X)$ or $F(X \times X) \cap G(X \times X)$.*

Theorem 2.8. *Let $(X, M, *)$ be a dislocated quasi fuzzy b-metric space, $F, G : X \times X \to X$ and $S, T : X \to X$ be mappings satisfying*

(2.8.1) $M(F(x,y), G(u,v), t) \geq \phi(\min\{M(Sx, Tu, b^2t), M(Sy, Tv, b^2t)\})$ *for all* $x, y, u, v \in X$ *and* $\phi \in \Phi$

(2.8.2) $M(G(x,y), F(u,v), t) \geq \phi(\min\{M(Tx, Su, b^2t), M(Ty, Sv, b^2t)\})$ *for all* $x, y, u, v \in X$ *and* $\phi \in \Phi$

(2.8.3) *the pairs* (F, S) *and* (G, T) *satisfy common E.A. like property,*

(2.8.4) *the pairs* (F, S) *and* (G, T) *are* w*-compatible.*

Then F*,* G*,* S *and* T *have a unique common coupled fixed point in* $X \times X$*.*

Proof. Since (F, S) and (G, T) satisfy common E.A. like property, there exist sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ and $\{w_n\}$ in X such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} G(z_n, w_n) = \lim_{n\to\infty} Tz_n = \alpha$ and $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} G(w_n, z_n) = \lim_{n\to\infty} Tw_n = \alpha^1$ for some $\alpha, \alpha^1 \in S(X) \cap T(X)$ or $F(X \times X) \cap G(X \times X)$. Without loss of generality assume that $\alpha, \alpha^1 \in S(X) \cap T(X)$. Since $\lim_{n\to\infty} Sx_n = \alpha \in S(X)$ and $\lim_{n\to\infty} Sy_n = \alpha^1 \in S(X)$, there exist $u, v \in X$ such that $\alpha = Su$ and $\alpha^1 = Sv$. From Proposition 1.9, (2.8.1) and (ϕ_1), we have

$$\begin{aligned} M\left(F(u,v), \alpha, bt\right) &\geq \lim_{n\to\infty} \sup M\left(F(u,v), G(z_n, w_n), t\right) \\ &\geq \lim_{n\to\infty} \sup \phi\left(\min\left\{M(\alpha, Tz_n, b^2t), M(\alpha^1, Tw_n, b^2t)\right\}\right) \\ &= \phi(1) = 1. \end{aligned}$$

Hence $M(F(u,v), \alpha, bt) = 1$ for all $t > 0$ and $b \geq 1$. Similarly from (2.8.2), we can show that $M(\alpha, F(u,v), bt) = 1$. Hence from $(M1)$, $F(u,v) = \alpha$. Similarly we can prove that $F(v,u) = \alpha^1$. Thus $Su = \alpha = F(u,v)$ and $Sv = \alpha^1 = F(v,u)$. Since the pair (F, S) is w-compatible, we have $S\alpha = S(F(u,v)) = F(Su, Sv) = F(\alpha, \alpha^1)$ and $S\alpha^1 = S(F(v,u)) = F(Sv, Su) = F(\alpha^1, \alpha)$. From Proposition 1.9, (2.8.1) and (ϕ_1), we have

$$\begin{aligned} M\left(S\alpha, \alpha, bt\right) &= M\left(F(\alpha, \alpha^1), \alpha, bt\right) \\ &\geq \lim_{n\to\infty} \sup M\left(F(\alpha, \alpha^1), G(z_n, w_n), t\right) \\ &\geq \lim_{n\to\infty} \sup \phi\left(\min\left\{M(S\alpha, Tz_n, b^2t), M(S\alpha^1, Tw_n, b^2t)\right\}\right) \\ &\geq \phi\left(\min\left\{M(S\alpha, \alpha, b^2t), M(S\alpha^1, \alpha^1, b^2t)\right\}\right), \\ &\geq \phi\left(\min\left\{M(S\alpha, \alpha, bt), M(S\alpha^1, \alpha^1, bt)\right\}\right), \end{aligned}$$

$$\begin{aligned} M\left(\alpha, S\alpha, bt\right) &= M\left(\alpha, F(\alpha, \alpha^1), bt\right) \\ &\geq \lim_{n\to\infty} \sup M\left(G(z_n, w_n), F(\alpha, \alpha^1), t\right) \\ &\geq \lim_{n\to\infty} \sup \phi\left(\min\left\{M(Tz_n, S\alpha, b^2t), M(Tw_n, S\alpha^1, b^2t)\right\}\right) \\ &\geq \phi\left(\min\left\{M(\alpha, S\alpha, b^2t), M(\alpha^1, S\alpha^1, b^2t)\right\}\right), \\ &\geq \phi\left(\min\left\{M(\alpha, S\alpha, bt), M(\alpha^1, S\alpha^1, bt)\right\}\right), \end{aligned}$$

Similarly we can prove that $M(S\alpha^1, \alpha^1, bt) \geq \phi(\min\{M(S\alpha^1, \alpha^1, bt), M(S\alpha, \alpha, bt)\})$ and $M(\alpha^1, S\alpha^1, bt) \geq \phi(\min\{M(\alpha^1, S\alpha^1, bt), M(\alpha, S\alpha, bt)\})$. From (ϕ_2), we have

$$\min\left\{\begin{array}{l} M(S\alpha, \alpha, bt), M(\alpha, S\alpha, bt), \\ M(S\alpha^1, \alpha^1, bt), M(\alpha^1, S\alpha^1, bt) \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{l} M(S\alpha, \alpha, bt), M(\alpha, S\alpha, bt), \\ M(S\alpha^1, \alpha^1, bt), M(\alpha^1, S\alpha^1, bt) \end{array}\right\}\right)$$

which in turn yields from (ϕ_3) and $(M1)$ that $S\alpha = \alpha$ and $S\alpha^1 = \alpha^1$. Thus

$$S\alpha = \alpha = F(\alpha, \alpha^1) \text{ and } S\alpha^1 = \alpha^1 = F(\alpha^1, \alpha) \tag{19}$$

Since $\lim_{n\to\infty} Tz_n = \alpha \in T(X)$ and $\lim_{n\to\infty} Tw_n = \alpha^1 \in T(X)$, there exist β, β^1 such that $\alpha = T\beta$ and $\alpha^1 = T\beta^1$. Consider

$$\begin{aligned} M\left(\alpha, G(\beta,\beta^1), bt\right) &\geq \lim_{n\to\infty} \sup M\left(F(x_n, y_n), G(\beta, \beta^1), t\right) \\ &\geq \lim_{n\to\infty} \sup \phi\left(\min\left\{M(Sx_n, \alpha, b^2t), M(Sy_n, \alpha^1, b^2t)\right\}\right) \\ &= \phi(1) = 1. \end{aligned}$$

Hence $M(\alpha, G(\beta, \beta^1), bt) = 1$ for all $t > 0$. Similarly from (2.8.2) we can show that $M(G(\beta, \beta^1), \alpha, bt) = 1$. Hence from (M1), $G(\beta, \beta^1) = \alpha$. Similarly we can prove that $G(\beta^1, \beta) = \alpha^1$. Thus $\alpha = T\beta = G(\beta, \beta^1)$ and $\alpha^1 = T\beta^1 = G(\beta^1, \beta)$. Since the pair (G, T) is w-compatible, we have $T\alpha = T(G(\beta, \beta^1)) = G(T\beta, T\beta^1) = G(\alpha, \alpha^1)$ and $T\alpha^1 = T(G(\beta^1, \beta)) = G(T\beta^1, T\beta) = G(\alpha^1, \alpha)$. Now from Proposition 1.9, (2.8.1) and (ϕ_2), we have

$$\begin{aligned} M\left(\alpha, T\alpha, t\right) &= M\left(F(\alpha, \alpha^1), G(\alpha, \alpha^1), t\right) \\ &\geq \phi\left(\min\left\{M(S\alpha, T\alpha, b^2t), M(S\alpha^1, T\alpha^1, b^2t)\right\}\right) \\ &\geq \phi\left(\min\left\{M(\alpha, T\alpha, t), M(\alpha^1, T\alpha^1, t)\right\}\right), \end{aligned}$$

Similarly we can show that

$$M(T\alpha, \alpha, t) \geq \phi(\min\{M(T\alpha, \alpha, t), M(T\alpha^1, \alpha^1, t)\}),$$

$$M(\alpha^1, T\alpha^1, t) \geq \phi(\min\{M(\alpha^1, T\alpha^1, t), M(\alpha, T\alpha, t)\})$$

$$M(T\alpha^1, \alpha^1, t) \geq \phi(\min\{M(T\alpha^1, \alpha^1, t), M(T\alpha, \alpha, t)\}).$$

From (ϕ_2), we have

$$\min\left\{\begin{array}{l} M(\alpha, T\alpha, t), M(T\alpha, \alpha, t), \\ M(\alpha^1, T\alpha^1, t), M(T\alpha^1, \alpha^1, t) \end{array}\right\} \geq \phi\left(\min\left\{\begin{array}{l} M(\alpha, T\alpha, t), M(T\alpha, \alpha, t), \\ M(\alpha^1, T\alpha^1, t), M(T\alpha^1, \alpha^1, t) \end{array}\right\}\right)$$

which in turn yields from (ϕ_3) and $(M1)$ that $T\alpha = \alpha$ and $T\alpha^1 = \alpha^1$. Thus

$$\alpha = T\alpha = G(\alpha, \alpha^1) \text{ and } \alpha^1 = T\alpha^1 = G(\alpha^1, \alpha) \tag{20}$$

From (19) and (20), we have $F(\alpha, \alpha^1) = S\alpha = \alpha = T\alpha = G(\alpha, \alpha^1)$ and $F(\alpha^1, \alpha) = S\alpha^1 = \alpha^1 = T\alpha^1 = G(\alpha^1, \alpha)$. Thus (α, α^1) is a common coupled fixed point of F, G, S and T. □

Now we give an example to support Theorem 2.8.

Example 2.9. *Let $X = [0,1]$ and $a*c = ac$ for all $a, c \in [0,1]$ and M be a fuzzy set on $X \times X \times (0,\infty)$ defined by $M(x,y,t) = e^{-\frac{[|x-y|^2+|x|]}{t}}$. Let $F, G : X \times X \to X$ and $S, T : X \to X$ be defined by $F(x,y) = \frac{x^2+y^2}{32}$, $G(x,y) = \frac{x^2+y^2}{48}$, $Sx = \frac{x^2}{2}$ and $Tx = \frac{x^2}{3}$ and Let $\phi : [0,1] \to [0,1]$ be defined by $\phi(t) = t^{\frac{1}{2}}$, for all $t \in [0,1]$. As in Example 2.2, M is a dislocated quasi fuzzy b-metric space with $b = 2$. Consider*

$$\begin{aligned} \left|\frac{x^2+y^2}{32} - \frac{u^2+v^2}{48}\right|^2 + \frac{x^2+y^2}{32} &= \frac{1}{256}\left|\left(\frac{x^2}{2} - \frac{u^2}{3}\right) + \left(\frac{y^2}{2} - \frac{v^2}{3}\right)\right|^2 + \frac{x^2+y^2}{32} \\ &\leq \frac{1}{128}\left[\left(\frac{x^2}{2} - \frac{u^2}{3}\right)^2 + \left(\frac{y^2}{2} - \frac{v^2}{3}\right)^2\right] + \frac{x^2+y^2}{32} \\ &= \frac{1}{16}\left[\frac{1}{8}\left(\frac{x^2}{2} - \frac{u^2}{3}\right)^2 + \frac{1}{8}\left(\frac{y^2}{2} - \frac{v^2}{3}\right)^2 + \frac{x^2}{2} + \frac{y^2}{2}\right] \\ &\leq \frac{1}{16}\left[\left\{\left(\frac{x^2}{2} - \frac{u^2}{3}\right)^2 + \frac{x^2}{2}\right\} + \left\{\left(\frac{y^2}{2} - \frac{v^2}{3}\right)^2 + + \frac{y^2}{2}\right\}\right] \\ &\leq \frac{1}{8}\left[\max\left\{\left(\frac{x^2}{2} - \frac{u^2}{3}\right)^2 + \frac{x^2}{2}, \left(\frac{y^2}{2} - \frac{v^2}{3}\right)^2 + \frac{y^2}{2}\right\}\right]. \end{aligned}$$

Hence

$$\begin{aligned}
M\left(F(x,y),G(u,v),t\right) &= e^{\frac{-\left\{\left|\frac{x^2+y^2}{32}-\frac{u^2+v^2}{48}\right|^2+\frac{x^2+y^2}{32}\right\}}{t}} \\
&\geq e^{\frac{-\frac{1}{8}\max\left\{\left(\frac{x^2}{2}-\frac{u^2}{3}\right)^2+\frac{x^2}{2},\left(\frac{y^2}{2}-\frac{v^2}{3}\right)^2+\frac{y^2}{2}\right\}}{t}} \\
&= \left[e^{\frac{-\max\left\{\left(\frac{x^2}{2}-\frac{u^2}{3}\right)^2+\frac{x^2}{2},\left(\frac{y^2}{2}-\frac{v^2}{3}\right)^2+\frac{y^2}{2}\right\}}{4t}}\right]^{\frac{1}{2}} \\
&= \left[\min\left\{e^{-\left\{\frac{\left(\frac{x^2}{2}-\frac{u^2}{3}\right)^2+\frac{x^2}{2}}{4t}\right\}}, e^{-\left\{\frac{\left(\frac{y^2}{2}-\frac{v^2}{3}\right)^2+\frac{y^2}{2}}{4t}\right\}}\right\}\right]^{\frac{1}{2}} \\
&= \left[\min\left\{M(Sx,Tu,b^2t),M(Sy,Tv,b^2t)\right\}\right]^{\frac{1}{2}} \\
&= \phi\left(\min\left\{M(Sx,Tu,b^2t),M(Sy,Tv,b^2t)\right\}\right).
\end{aligned}$$

Since $\phi(t)=t^{\frac{1}{2}}$.

Thus (2.8.1) is satisfied. Similarly we can easily verify that (2.8.2).One can easily show that the pairs (F,S) and (G,T) satisfy common E.A. like property with $x_n=\frac{1}{n}$, $y_n=\frac{1}{n+1}$, $z_n=\frac{1}{2^n}$ and $w_n=\frac{1}{2^{n+1}}$ for $n=1,2,3,\ldots..$ Clearly the pairs (F,S) and (G,T) are w-compatible and (0, 0) is the unique common coupled fixed point of F, G, S and T.

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