Int. J. Math. And Appl., 6(1–C)(2018), 563–565
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

A Remark on Derivations and Bounded Nilpotence Index

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Abstract: Pace P. Nielsen and Michal Ziembowski has constructed a nil ring R which has bounded nilpotence index 2 and proved that this ring has a derivation δ for which the differential polynomial ring $R[x;\delta]$ is not even prime radical and thus obtained some interesting algebraically important results. The purpose of this note is to establish that the ring R constructed by these authors is nothing but an even square ring whose every element is a nil element.

MSC: 16D25, 16N40, 16N20.

Keywords: Even square rings, nil elements, nilpotent elements, nil ideals, rings of bounded nilpotence index.

1. Introduction

The notion of nil elements has been introduced recently [1–3]. Nil elements are special type of nilpotent elements. Recall that an element a of a ring R is called a nil element if $a^2 = 2a = 0$ and a ring R is called an even square ring if $a^2 \in 2R$, $\forall a \in R$ [1]. It has been noticed that a ring R whose every element is nil is a commutative ring [1] and a noncommutative ring of order n can contain maximum $n-1$ nil elements [3]. Thus there does not exist a noncommutative ring whose every element is nil element. In [4] authors have constructed an example of a ring R which is a commutative nil ring of bounded nilpotence index two. They have stated that existence of the example constructed by them is surprising because using that example they got surprising algebraically important results. In this note we establish that the ring R studied in [4] is an even square ring whose every element is nil element. In the light of this note one can safely state that all the results given in [4] hold for even square rings whose each element is nil element and there are several examples of such rings. In order to establish the said equivalence we note that if R is an even square ring whose every element is nil then it is a nil ring of bounded nilpotence index two. However if R be a nil ring of bounded nilpotence index two then each element of R is not necessarily a nil element.

2. Even Square Rings and the Ring Constructed in [4]

Proposition 2.1. *Each nilpotent element of index two in a commutative ring R is not necessarily a nil element.*

Proof. In order to prove this result we shall give the following example of a commutative ring. Consider the ring $R = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in Z_8 \right\}$. Then $a = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \in R$ is a nilpotent element of index two of the ring R. However it is not a nil element of R. □

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Proposition 2.2. *Let R be an even square ring whose every element is a nil element then R is a nil ring of bounded nilpotence index two.*

Proposition 2.3. *The converse of the Proposition 2.2 is not true.*

Proof. Let $R = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix} \right\}$. It is easy to see that R is a nil ring of bounded index two under addition and multiplication modulo four. One can see that each element of R is not a nil element. $\square$

Proposition 2.4. *The ring R constructed in [4] is an even square ring whose every element is nil element.*

Proof. The ring R constructed in [4] has the following property.

(1). $a^2 = 0, \ \forall \ a \in R$

(2). $ab + ba = 0, \ \forall \ a, b \in R$

(3). $2abc = 0, \ \forall \ a, b, c \in R$

(4). Characteristic of R is two.

(5). R is commutative.

In view of the above properties R is a commutative nil ring of bounded nilpotence index two. Now we shall see that if R be an even square ring whose every element is nil element then each of the above conditions hold. Let R be an even square ring whose every element is nil element then using the definition of even square rings and nil elements we have

(1). $a^2 \in 2R, \ \forall \ a \in R.$

(2). $a^2 = 2a = 0, \forall \ a \in R.$

(3). $(a + b) \in R, \forall \ a, b \in R \Rightarrow (a + b)^2 = 0 \Rightarrow ab + ba = 0, \forall a, b \in R.$

(4). $2a = 0, \ \forall \ a \in R \Rightarrow 2abc = 0, \ \forall \ a, b, c \in R$

(5). Since $2a = 0, \forall \ a \in R.$ Therefore Characteristic of $R$is two.

(6). $ab + ba = 0, \ \forall \ a, b \in R \Rightarrow ab + ba = 2ab \Rightarrow ab = ba, \ \forall \ a, b \in R.$ Therefore R is a commutative ring.

Hence the ring R constructed in [4] is an even square ring whose every element is nil element. $\square$

Proposition 2.5. *Let R be an even square ring whose every element is nil then R has a derivation δ such that each element of the differential polynomial ring $R[x; \delta]$ is not nil.*

3. Concluding Remarks

In this note we have established that the ring constructed in [4] is an even square ring whose every element is nil and such rings are a particular class of even square rings. Thus in the light of this note and [4] one can safely state that if R be an even square ring whose every element is nil then R has a derivation δ for which differential polynomial ring $R[x; \delta]$ fails to be prime radical. There are several examples of such rings.

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