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To Determine the Value of Game in Fuzzy Environment

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Abstract: In this research paper an optimization technique under uncertainty with payoffs as octagonal fuzzy numbers is developed. In present paper octagonal numbers in fuzzy environment are converted in crisp values to apply the classical methods to determine the game value and best strategies of players. Uncertainty and imperfect information arise only when players are unaware the observations of the actions of choices of each other.

MSC: 91A18, 91A80.

Keywords: Octagonal fuzzy numbers, crispness, fuzzy payoffs.

1. Introduction

Fuzzy linear programming problems were introduced and studied by [2, 9]. Tanaka et al. [10] studied a mathematical programming problem with fuzzy numbers, Zimmerman [7], illustrated a theory of fuzziness, Buckley and Feuring [6], developed an algorithm, Thakre et al. and Zhang et al. [8, 11] solved FLP problems using multi-objective linear programming technique in fuzzy environment. Pandian [3] has proposed a new approach. After the pioneering works on this area many authors have considered various kinds of the FLP problems and have proposed several approaches to solve these problems. In this paper we have developed an effective methodology to solve matrix games with payoffs as octagonal fuzzy numbers introduced in [1]. In this paper, the basic concepts and ranking order relations of octagonal fuzzy numbers are defined. The developed method in this paper is illustrated by a numerical example of the market share competition problem.

2. Octagonal Fuzzy Numbers

An octagonal fuzzy number denoted by $\tilde{A}_w$ is defined to be the ordered quadruple, $\tilde{A}_w = (l_1(r), s_1(t), s_2(t), l_2(r))$, where $r \in [0, k]$ and$t \in [k, w]$, where

(1). $l_1(r)$: a bounded left continuous non decreasing function over $[0, w_1]$, $[0 \leq w_1 \leq k]$.

(2). $s_1(t)$: a bounded left continuous non decreasing function over $[k, w_2]$, $[k \leq w_2 \leq w]$.

(3). $s_2(t)$: Is a bounded left continuous non increasing function over $[k, w_2]$, $[k \leq w_2 \leq w]$

(4). $l_2(r)$: Is a bounded left continuous non increasing function over $[0, w_1]$, $[0 \leq w_1 \leq k]$.

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According to above discussion, octagonal fuzzy number $\tilde{A}_w$ is the ordered quadruple $\tilde{A}_w = (l_1(r), s_1(t), s_2(t), l_2(r))$, where $r \in [0, k]$ and $t \in [k, w]$, where

$$l_1(r) = k\left(\frac{r - a_1}{a_2 - a_1}\right),\ s_1(t) = k + (1-k)\left(\frac{t - a_3}{a_4 - a_3}\right),\ s_2(t) = k + (1-k)\left(\frac{a_6 - t}{a_6 - a_5}\right)\ \textit{and}\ l_2(r) = k\left(\frac{a_8 - r}{a_8 - a_7}\right)$$

If $w = 1$, then the above defined number is called a normal octagonal fuzzy number. A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given as following,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a_1 \\ k\left(\frac{x-a_1}{a_2 - a_1}\right), a_1 \le x \le a_2 \\ k, a_2 \le x \le a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4 - a_3}\right), a_3 \le x \le a_4 \\ 1, a_4 \le x \le a_5 \\ k + (1-k)\left(\frac{a_6 - x}{a_6 - a_5}\right), a_5 \le x \le a_6 \\ k, a_6 \le x \le a_7 \\ k\left(\frac{a_8 - x}{a_8 - a_7}\right), a_7 \le x \le a_8 \\ 0, x \ge a_8 \end{cases}$$

Where $0<k<1$.

2.1. Operations in Octagonal Fuzzy Numbers

Definition 2.1 (α-cut of an octagonal fuzzy number)**.** *The α-cut of an normal octagonal fuzzy number $\tilde{A}$=$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ for $\alpha \in [0, 1]$ is given by*

$$[\tilde{A}]_\alpha = \begin{cases} [a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1), [a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)], \alpha \in [0, k] \\ [a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)], \alpha \in (k, 1] \end{cases}$$

Definition 2.2. *If $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$are two octagonal fuzzy numbers then their addition is*

$$[\tilde{A}]_\alpha + [\tilde{B}]_\alpha = \begin{cases} [a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1), a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)] + [b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1), b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)], \alpha \in [0, k] \\ [a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3), a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)] + [b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3), b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)], \alpha \in (k, 1] \end{cases}$$

$$[\tilde{A}]_\alpha + [\tilde{B}]_\alpha = \begin{cases} [a_1 + b_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1 + b_2 - b_1), a_8 + b_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7 + b_8 - b_7)], \alpha \in [0, k] \\ [a_3 + b_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3 + b_4 - b_3), a_6 + b_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5 + b_6 - b_5)]\ , \alpha \in (k, 1] \end{cases}$$

Definition 2.3. *If $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ are two octagonal fuzzy numbers then their subtraction is $[\tilde{A}]_\alpha - [\tilde{B}]_\alpha = [q_\alpha^L, q_\alpha^R]$, where*

$$q_\alpha^L = \min \begin{cases} \{(a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1)) - (b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1)) - (b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)), \\ (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)) - (b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)) - (b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7))\} \end{cases}$$

$$q_\alpha^R = \max \begin{cases} \{(a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1)) - (b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1)) - (b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)), \\ (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)) - (b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7)) - (b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7))\} \end{cases}$$

for $\alpha \in [0, k]$ *and*

$$q_\alpha^L = \min \begin{cases} \{(a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3)) - (b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3)) - (b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)), \\ (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)) - (b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)) - (b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5))\} \end{cases}$$

$$q_\alpha^R = \max \begin{cases} \{(a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3)) - (b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3)) - (b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)), \\ (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)) - (b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5)) - (b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5))\} \end{cases}$$

for $\alpha \in (k, 1]$.

Definition 2.4. *If* $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ *and* $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ *are two octagonal fuzzy numbers then their multiplication is* $[\tilde{A}]_\alpha * [\tilde{B}]_\alpha = [q_\alpha^L, q_\alpha^R]$*, where*

$$q_\alpha^L = \min \begin{cases} \{(a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1))(b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1))(b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)), \\ (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7))(b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7))(b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7))\} \end{cases}$$

$$q_\alpha^R = \max \begin{cases} \{(a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1))(b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_1 + \left(\frac{\alpha}{k}\right)(a_2 - a_1))(b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7)), \\ (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7))(b_1 + \left(\frac{\alpha}{k}\right)(b_2 - b_1)), (a_8 - \left(\frac{\alpha}{k}\right)(a_8 - a_7))(b_8 - \left(\frac{\alpha}{k}\right)(b_8 - b_7))\} \end{cases}$$

for $\alpha \in [0, k]$ *and*

$$q_\alpha^L = \min \begin{cases} \{(a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3))(b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3))(b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)), \\ (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5))(b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5))(b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5))\} \end{cases}$$

$$q_\alpha^R = \max \begin{cases} \{(a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3))(b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_3 + \left(\frac{\alpha-k}{1-k}\right)(a_4 - a_3))(b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5)), \\ (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5))(b_3 + \left(\frac{\alpha-k}{1-k}\right)(b_4 - b_3)), (a_6 - \left(\frac{\alpha-k}{1-k}\right)(a_6 - a_5))(b_6 - \left(\frac{\alpha-k}{1-k}\right)(b_6 - b_5))\} \end{cases}$$

for $\alpha \in (k, 1]$.

Definition 2.5. *A measure of a fuzzy number* $\tilde{A}_w$ *is a function* $M_\alpha : \mathrm{R}_w(I) \to \mathrm{R}^+$ *which assigns a non-negative real number* $M_\alpha(\tilde{A}_w)$ *that gives the measure of* $\tilde{A}_w$ *if* $w > \alpha$

$$\begin{aligned} M_0^{Oct}(\tilde{A}_w) &= \frac{1}{2}\int_\alpha^k (l_1(r) + l_2(r))dr + \frac{1}{2}\int_k^w (s_1(t) + s_2(t))dt, \text{ where } 0 \le \alpha < 1 \\ &= \frac{1}{2}\{[(a_1 + a_8) + \left(\frac{a_2 + a_7 - a_1 - a_8}{2kw}\right)(k + \alpha)](k - \alpha) + (a_3 + a_6) + \left(\frac{a_4 + a_5 - a_3 - a_6}{2w(1-k)}\right)(w + k - 2kw)](w - k)\} \end{aligned}$$

And when $w \le \alpha$ *then the above quantities will be zero. If* $w = 1$ *then it becomes a normal octagonal number and then*

$$\begin{aligned} M_0^{Oct}(\tilde{A}_w) &= \frac{1}{2}\int_\alpha^k (l_1(r) + l_2(r))dr + \frac{1}{2}\int_k^1 (s_1(t) + s_2(t))dt, \text{ where } 0 \le \alpha < 1 \\ &= \frac{1}{2}\{[(a_1 + a_8) + \left(\frac{a_2 + a_7 - a_1 - a_8}{2k}\right)(k + \alpha)](k - \alpha) + (a_3 + a_6) + \left(\frac{a_4 + a_5 - a_3 - a_6}{2(1-k)}\right)(1 + k)](1 - k)\} \end{aligned}$$

3. Numerical Example

$$
\begin{array}{cc}
 & \textit{Player } B \\
\textit{Player } A & \begin{pmatrix}
(0,1,2,3,4,5,6,7) & (8,9,10,11,12,13,14,15) & (4,5,6,7,8,9,10,11) & (-1,0,1,2,3,4,5,6) \\
(2,3,4,5,6,7,8,9) & (11,12,14,15,16,17,18,21) & (5,6,8,9,10,11,12,15) & (1,2,3,5,6,7,8,10) \\
(3,6,7,8,9,10,12,13) & (5,6,8,10,12,13,15,17) & (1,3,5,6,7,8,10,12) & (-2,-1,0,1,2,3,4,5) \\
(7,8,9,10,11,12,13,14) & (3,4,5,6,7,8,9,10) & (1,2,3,4,5,6,7,8) & (6,7,8,9,10,11,12,13)
\end{pmatrix}
\end{array}
$$

Convert the octagonal fuzzy number problem into crisp value problem using the measures given by definitions and the problem is done by taking k as 0.4

$$
\begin{aligned}
M_0^{Oct}(\tilde{A}) &= \frac{1}{2}\int_0^k (l_1(r)+l_2(r))dr + \frac{1}{2}\int_k^1 (s_1(t)+s_2(t))dt \; where \; 0 \le k \le 1 \\
&= \frac{1}{4}[(a_1+a_2+a_7+a_8)k + (a_3+a_4+a_5+a_6)] \;\; where \; 0 \le k \le 1
\end{aligned}
$$

$\tilde{a}_{11} = (0,1,2,3,4,5,6,7)$	$M_0^{Oct}(\tilde{a}_{11}) = \frac{1}{4}[\frac{2}{5}(0+1+6+7) + \frac{3}{5}(2+3+4+5)] = 3.5$
$\tilde{a}_{12} = (8,9,10,11,12,13,14,15)$	$M_0^{Oct}(\tilde{a}_{12}) = \frac{1}{4}[\frac{2}{5}(8+9+14+15) + \frac{3}{5}(10+11+12+13)] = 11.5$
$\tilde{a}_{13} = (4,5,6,7,8,9,10,11)$	$M_0^{Oct}(\tilde{a}_{13}) = \frac{1}{4}[\frac{2}{5}(4+5+10+11) + \frac{3}{5}(6+7+8+9)] = 7.5$
$\tilde{a}_{14} = (-1,0,1,2,3,4,5,6)$	$M_0^{Oct}(\tilde{a}_{14}) = \frac{1}{4}[\frac{2}{5}(-1+0+5+6) + \frac{3}{5}(1+2+3+4)] = 2.5$
$\tilde{a}_{21} = (2,3,4,5,6,7,8,9)$	$M_0^{Oct}(\tilde{a}_{21}) = \frac{1}{4}[\frac{2}{5}(2+3+8+9) + \frac{3}{5}(4+5+6+7)] = 5.5$
$\tilde{a}_{22} = (11,12,14,15,16,17,18,21)$	$M_0^{Oct}(\tilde{a}_{22}) = \frac{1}{4}[\frac{2}{5}(11+12+18+21) + \frac{3}{5}(14+15+16+17)] = 15.5$
$\tilde{a}_{23} = (5,6,8,9,10,11,12,15)$	$M_0^{Oct}(\tilde{a}_{23}) = \frac{1}{4}[\frac{2}{5}(5+6+12+15) + \frac{3}{5}(8+9+10+11)] = 9.5$
$\tilde{a}_{24} = (2,3,4,5,6,7,8,9)$	$M_0^{Oct}(\tilde{a}_{24}) = \frac{1}{4}[\frac{2}{5}(2+3+8+9) + \frac{3}{5}(4+5+6+7)] = 5.5$
$\tilde{a}_{31} = (3,6,7,8,9,10,12,13)$	$M_0^{Oct}(\tilde{a}_{31}) = \frac{1}{4}[\frac{2}{5}(3+6+12+13) + \frac{3}{5}(7+8+9+10)] = 8.5$
$\tilde{a}_{32} = (7,8,9,10,11,12,13,14)$	$M_0^{Oct}(\tilde{a}_{32}) = \frac{1}{4}[\frac{2}{5}(7+8+13+14) + \frac{3}{5}(9+10+11+12)] = 10.5$
$\tilde{a}_{33} = (1,3,5,6,7,8,10,12)$	$M_0^{Oct}(\tilde{a}_{33}) = \frac{1}{4}[\frac{2}{5}(1+3+10+12) + \frac{3}{5}(5+6+7+8)] = 6.5$
$\tilde{a}_{34} = (-2,-1,0,1,2,3,4,5)$	$M_0^{Oct}(\tilde{a}_{34}) = \frac{1}{4}[\frac{2}{5}(-2-1+4+5) + \frac{3}{5}(0+1+2+3)] = 1.5$
$\tilde{a}_{41} = (7,8,9,10,11,12,13,14)$	$M_0^{Oct}(\tilde{a}_{41}) = \frac{1}{4}[\frac{2}{5}(7+8+13+14) + \frac{3}{5}(9+10+11+12)] = 10.5$
$\tilde{a}_{42} = (3,4,5,6,7,8,9,10)$	$M_0^{Oct}(\tilde{a}_{42}) = \frac{1}{4}[\frac{2}{5}(3+4+9+10) + \frac{3}{5}(5+6+7+8)] = 6.5$
$\tilde{a}_{43} = (1,2,3,4,5,6,7,8)$	$M_0^{Oct}(\tilde{a}_{43}) = \frac{1}{4}[\frac{2}{5}(1+2+7+8) + \frac{3}{5}(3+4+5+6)] = 4.5$
$\tilde{a}_{44} = (6,7,8,9,10,11,12,13)$	$M_0^{Oct}(\tilde{a}_{44}) = \frac{1}{4}[\frac{2}{5}(6+7+12+13) + \frac{3}{5}(8+9+10+11)] = 9.5$

So the problem reduces to

$$
\begin{array}{cc}
 & \textit{player } B \\
\textit{player } A & \begin{pmatrix}
3.5 & 11.5 & 7.5 & 2.5 \\
5.5 & 15.5 & 9.5 & 5.5 \\
8.5 & 10.5 & 6.5 & 1.5 \\
10.5 & 6.5 & 4.5 & 9.5
\end{pmatrix}
\end{array}
$$

Solution by Simplex Method: $Max\ Z = X1 + X2 + X3 + X4$

Subject To

$$3.5X_1 + 11.5X_2 + 7.5X_3 + 2.5X_4 \le 1$$

$$5.5X_1 + 15.5X_2 + 9.5X_3 + 5.5X_4 \le 1$$

$$8.5X_1 + 10.5X_2 + 6.5X_3 + 1.5X_4 \le 1$$

$$10.5X_1 + 6.5X_2 + 4.5X_3 + 9.5X_4 \le 1$$

Objective Function Value = 0.1374046.

Variable	Value	Reduced Cost
X_1	0.000000	0.061069
X_2	0.000000	0.580153
X_3	0.061069	0.000000
X_4	0.076336	0.000000

Slack Or Surplus	Dual Prices
0.351145	0.000000
0.000000	0.076336
0.488550	0.000000
0.000000	0.061069

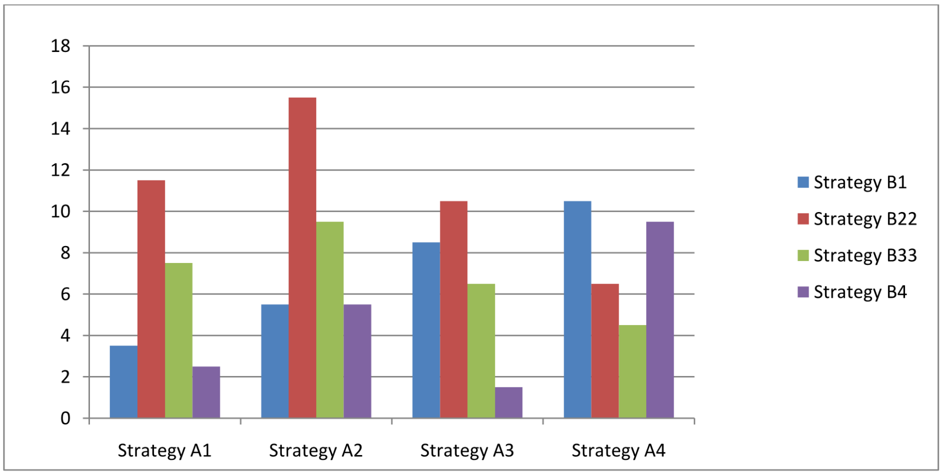
Number Iterations is 2. Ranges in which the Basis Is Unchanged: Objective Coefficient Ranges

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X_1	1.000000	0.061069	Infinity
X_2	1.000000	0.580153	Infinity
X_3	1.000000	0.727273	0.340807
X_4	1.000000	1.111111	0.053333

Righthand Side Ranges

RHS	Increase	Decrease
1.000000	Infinity	0.351145
1.000000	0.383333	0.421053
1.000000	Infinity	0.488550
1.000000	0.727273	0.526316

Row (Basis)	X_1	X_2	X_3	X_4	Slack 2	Slack 3
1	-1.000	-1.000	-1.000	-1.000	0.000	0.000
2	3.500	11.500	7.500	2.500	1.000	0.000
3	5.500	15.500	9.500	5.500	0.000	1.000
4	8.500	10.500	6.500	1.500	0.000	0.000
5	10.500	6.500	4.500	9.500	0.000	0.000



4. Conclusion

Value game, we obtained is $v = 7.2777767265$. In the above discussion a new method for solving the games having payoffs in form of octagonal fuzzy numbers is suggested. A numerical example is given to clarify the proposed method. Many problems in the present scenarios having competitive situations that require best strategy for the players. These situations may be seen in business between firms, military battles between countries, sports between teams, elections forecasting, advertising, publicity and marketing in different cases of conflict.

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