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Pair Sum Labeling of Splitting, Shadow and Middle Graphs of Path Graphs

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Abstract

Let G be a finite, undirected and simple graph with p vertices and q edges. A *pair sum labeling* of G is an injective map $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ such that the induced edge function, $f_e : E(G) \to \mathbb{Z} - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-to-one and $f_e(E(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q}{2}}\right\}$ or $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q-1}{2}}\right\} \cup \left\{k_{\frac{q+1}{2}}\right\}$ according as q is even or odd number. A graph with pair sum labeling will be referred to as a pair sum graph. Here, we show that the splitting graph, shadow graph, and middle graph of a path graph are pair sum graphs.

Keywords: pair sum graph; sum labeling; splitting graph; shadow graph; middle graph; path graph.
2020 Mathematics Subject Classification: 05C78.

1. Introduction

In graph theory, graph labeling remains to be an interest of study as it finds applications to the encryption and decryption process in cryptography (see [3, 10] and references therein). A graph labeling is an assignment of labels, usually represented by integers, to the vertices or edges (or both) of a graph, subject to certain conditions. In this work, we consider pair sum labeling of (p, q) graph G where G is finite, simple and undirected. We denote the vertex set and edge set of G as $V(G)$ and $E(G)$ respectively. In a (p, q) graph G, the number of elements of $V(G)$ and $E(G)$ are p and q, respectively.
A *pair sum labeling* of G is an injective map $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ such that the induced edge function, $f_e : E(G) \to \mathbb{Z} - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-to-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q}{2}}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q-1}{2}}\} \cup \{k_{\frac{q+1}{2}}\}$ according as q is even or odd number. A graph with a pair sum labeling is called *pair sum graph*.
Pair sum labeling had been introduced by Ponraj and Parthipan in [4]. They have also several works with Kala on this topic. In their series of work, they showed that the following are pair sum graphs:

(1). On 2010, in [4,6]: path P_n, cycle C_n, complete graph K_n for $n \leq 4$, star $K_{1,n}$, complete bipartite $K_{2,n}$, bistar $B_{m,n}$, subdivision graph $S(K_{1,2})$ of $K_{1,2}$, and the combs $P_n \odot K_1$ and $P_n \odot (K_1 \cup K_1)$, the union graphs $K_{1,m} \cup K_{1,n}$, $P_m \cup K_{1,n}$, $C_m \cup C_m$, the graph mK_n obtained from the union of m copies of K_n

*Corresponding author (eduardtaganap@clsu.edu.ph) for $n \leq 4$, ladder graph L_n, quadrilateral snake Q_n for odd integer n, triangular snake T_n, and the crown $C_n \odot K_1$.

(2). On 2011, in [5,7]: some trees derive from stars and bistars, all the trees of order 9, the graph $G \cup mK_1$ where G is a pair sum graph, all pair sum graphs of order at most 5.

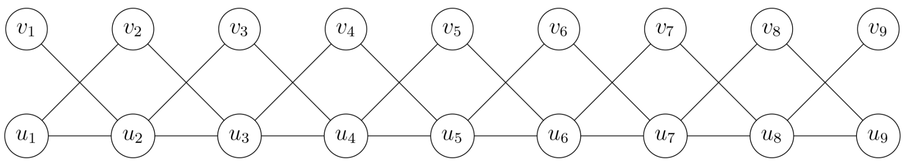
(3). On 2012, in [8,9]: the graph $P_n \times P_n$ for even n, the graph $C_n \times P_2$ for even n, the graph $L_n \odot K_1$, the graph $[C_m, P_n]$ obtained from two copies of C_m connected by the path P_n, the graph G where $V(G) = V(C_n) \cup \{v\}$ and $E(G) = E(C_n) \cup \{u_1v, u_3v\}$ ($V(C_n) = \{u_1, u_2, \ldots, u_n\}$), the graph G_n with vertex set $V(G_n) = V(C_n) \cup \{v_i : 1 \leq i \leq n$ and $E(G_n) = E(C_n) \cup \{u_iv_i, u_{i(i+1) \bmod n} : 1 \leq i \leq n\}$, and the graph G_n^* with vertex set $V(G_n^*) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n$ and $E(G_n^*) = E(C_n) \cup \{u_iv_i, u_iw_iv_iw_i : 1 \leq i \leq n\}$, the dragon graph $C_n@P_m$ for even integer n, the graph $K_n^c + 2K_2$, the subdivision graph $S(L_n)$ of ladder graph L_n of order n, and the subdivision graphs $S(C_nK_1)$, $S(P_nK_1)$, $S(T_n)$, $S(Q_n)$.

These results were also documented in the survey on graph labeling in [1]. For standard terminology and notations we follow [2].

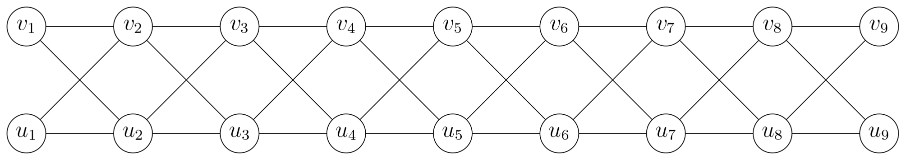
2. Preliminaries

Given a path graph P_n with vertices $u_1, u_2, \ldots, u_n$, many graphs can be constructed from it by applying certain graph operations. We focus our attention to the splitting graph, shadow graph, and middle graph of P_n.

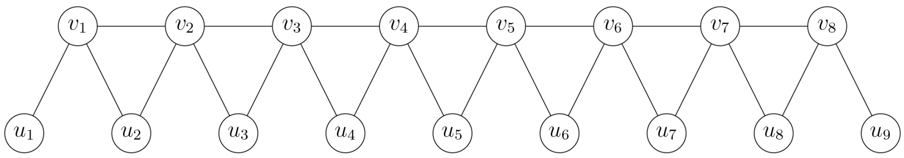
Let u be a vertex in P_n. The *open neighbourhood set* $N(u)$ is the set of all vertices adjacent to u in P_n. The *splitting graph* $S'(P_n)$ of graph P_n is obtained by adding a new vertex v corresponding to each vertex u of P_n such that $N(u) = N(v)$. Shown in Figure 1a is $S'(P_9)$, constructed from path graph P_9 with vertices $u_1, u_2, \ldots, u_9$. In the construction of $S'(P_9)$, the new vertex corresponding to u_5 is v_5. Since u_5 is adjacent to u_4 and u_6 in P_9, v_5 is also adjacent to u_4 and u_6 in $S'(P_9)$.



(a)



(b)



(c)

Figure 1: The (a) splitting graph $S'(P_9)$, (b) shadow graph $D_2(P_9)$, and (c) middle graph $M(P_9)$

The *shadow graph* $D_2(P_n)$ of P_n is obtained by taking two copies of P_n, namely P_n and P'_n, and joining each vertex u_i in P_n to the neighbours of corresponding vertex v_i in P'_n. The shadow graph $D_2(P_9)$ is shown in Figure 1b where the vertices of P'_9 are named $v_1, v_2, \ldots, v_9$.

The *middle graph* $M(P_n)$ of P_n is the graph whose vertex set is $V(P_n) \cup E(P_n)$ and in which two vertices are adjacent whenever either they are adjacent edges of P_n, or one is a vertex of P_n and the other is an edge incident with it. The middle graph $M(P_9)$ in Figure 1c is constructed from P_9 with vertices $u_1, u_2, \ldots, u_9$.

3. Main Results

In this section, we provide pair sum labelings of splitting, shadow and middle graphs of a path graph. Examples of these labelings are shown in Figures 2, 3 and 4.

Theorem 3.1. *The splitting graph $S'(P_n)$ of path graph P_n is a pair sum graph.*

Proof. Let P_n be a path with $V(P_n) = \{u_1, u_2, \ldots, u_n\}$. Consider the splitting graph $S'(P_n)$ of P_n. Suppose $V(S'(P_n)) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ and $E(S'(P_n)) = \{u_a u_{a+1} : 1 \le a \le n-1\} \cup \{u_b v_{b+1}, v_{b+1} u_{b+2} : 1 \le b \le n-2\} \cup \{u_{n-1} v_n, v_1 u_2\}$ (See Figure 1a).

When $n = 1$, $S'(P_1) \cong \overline{K}_2$ where $\overline{K}_2$ is the empty graph on 2 vertices. When $n = 2$, $S'(P_2) \cong P_4$, path of order 4. Thus, for $n \in \{1, 2\}$, $S'(P_n)$ is a pair sum graph.

Let $n \ge 3$. We have two cases.

Case 1: Suppose $n \equiv 1 \pmod 2$

Define the map $f : V(S'(P_n)) \to \{\pm 1, \pm 2, \ldots, \pm 2n\}$ as follows:

$$f(u_i) = \begin{cases} 1 - 2i & \text{if } i \equiv 1 \pmod 2, 1 \le i \le n-2 \\ 2n + 1 - 2i & \text{if } i \equiv 0 \pmod 2, 2 \le i \le n-1 \\ 1 & \text{if } i = n \end{cases}$$

and

$$f(v_i) = \begin{cases} -2n + 2 & \text{if } i = 1 \\ 1 - 2i & \text{if } i \equiv 0 \pmod 2, 2 \le i \le n-1 \\ 2n + 1 - 2i & \text{if } i \equiv 1 \pmod 2, 3 \le i \le n-2 \\ -2 & \text{if } i = n \end{cases}$$

Now,

$$\begin{aligned} f_e[E(S'(P_n))] &= \{2n-4, 2n-8, \ldots, 2\} \cup \{-2, -6, \ldots, 8-2n\} \cup \{4\} \\ &\quad \cup \{-4, -12, \ldots, 8-4n\} \cup \{4n-8, 4n-16, \ldots, 12\} \cup \{1\} \\ &\quad \cup \{-1\} \cup \{4n-12, 4n-20, \ldots, 16, 8\} \cup \{-8, -16, -24, \ldots, -4n+12\} \cup \{4-2n\} \\ f_e[E(S'(P_n))] &= \{\pm 2, \pm 6, \ldots, \pm(2n-4)\} \cup \{\pm 8, \pm 16, \pm 24, \ldots, \pm(4n-12)\} \\ &\quad \cup \{\pm 4, \pm 12, \ldots, \pm(4n-8)\} \cup \{\pm 1\} \end{aligned}$$

Thus f is a pair sum labeling of $S'(P_n)$.

Case 2: Suppose $n \equiv 0 \pmod 2$

Define the map $f : V(S'(P_n)) \to \{\pm 1, \pm 2, \ldots, \pm 2n\}$ as follows:

$$f(u_i) = \begin{cases} -1-2i & \text{if } i \equiv 1 \pmod 2, 1 \leq i \leq n-1 \\ 2n+1-2i & \text{if } i \equiv 0 \pmod 2, 2 \leq i \leq n-2 \\ n-2 & \text{if } i = n \end{cases}$$

and

$$f(v_i) = \begin{cases} 2n+1-2i & \text{if } i \equiv 1 \pmod 2, 1 \leq i \leq n-1 \\ -1-2i & \text{if } i \equiv 0 \pmod 2, 2 \leq i \leq n-2 \\ 2n & \text{if } i = n \end{cases}$$

Now,

$$\begin{aligned} f_e[E(S'(P_n))] &= \{2n-6, 2n-10, \ldots, 2\} \cup \{-2, -6, \ldots, 6-2n\} \cup \{-n-1\} \\ &\cup \{-8, -16, -24, \ldots, -4n+8\} \cup \{1\} \cup \{4n-8, 4n-16, \ldots, 16, 8\} \\ &\cup \{4n-4, 4n-12, \ldots, 12\} \cup \{n+1\} \cup \{-12, -20, \ldots, -4n+4\} \\ f_e[E(S'(P_n))] &= \{\pm 2, \pm 6, \ldots, \pm(2n-6)\} \cup \{\pm(n+1)\} \cup \{\pm 8, \pm 16, \pm 24, \ldots, \pm(4n-8)\} \\ &\cup \{\pm 12, \pm 20, \ldots, \pm(4n-4)\} \cup \{1\} \end{aligned}$$

Thus f is a pair sum labeling of $S'(P_n)$. □

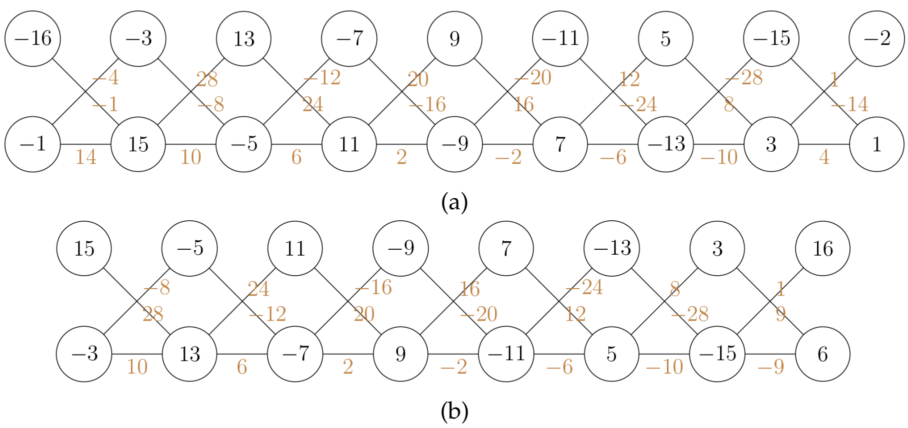


Figure 2: (a) Pair sum labeling of $S'(P_9)$, and (b) pair sum labeling of $S'(P_8)$

Theorem 3.2. *The shadow graph $D_2(P_n)$ of path graph P_n is a pair sum graph.*

Proof. Let P_n be a path with $V(P_n) = \{u_1, u_2, \ldots, u_n\}$. Consider the shadow graph $D_2(P_n)$ of P_n. Suppose $V(D_2(P_n)) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ and $E(D_2(P_n)) = \{u_a u_{a+1}, v_a v_{a+1} : 1 \leq a \leq n-1\} \cup \{u_b v_{b+1}, v_{b+1} u_{b+2} : 1 \leq b \leq n-2\} \cup \{u_{n-1} v_n, v_1 u_2\}$ (See Figure 1b).

For $n \in \{1, 2\}$, $D_2(P_1) \cong \overline{K}_2$ and $D_2(P_2) \cong C_4$, cycle of order 4. Thus, for $n \in \{1, 2\}$, $D_2(P_n)$ is a pair sum graph. Let $n \geq 3$. We have two cases.

Case 1: Suppose $n \equiv 1 \pmod 2$

Define the map $f : V(D_2(P_n)) \to \{\pm 1, \pm 2, \ldots, \pm 2n\}$ as follows:

$$f(u_i) = \begin{cases} \frac{3n-1}{2} - \frac{i-1}{2} & \text{if } i \equiv 1 \pmod 2, 1 \leq i \leq n \\ -\frac{3n+3}{2} - \frac{i-2}{2} & \text{if } i \equiv 0 \pmod 2, 2 \leq i \leq n-1 \end{cases}$$

and

$$f(v_i) = \begin{cases} -n - \frac{i-1}{2} & \text{if } i \equiv 1 \pmod 2, 1 \leq i \leq n \\ 2n - \frac{i-2}{2} & \text{if } i \equiv 0 \pmod 2, 2 \leq i \leq n-1 \end{cases}$$

Now,

$$\begin{aligned} f_e[E(D_2(P_n))] &= \{-2, -3, -4 \ldots, -n\} \cup \{n, n-1, -n-2, \ldots, 2\} \\ &\cup \left\{\frac{7n-1}{2}, \frac{7n-5}{2}, \ldots, \frac{5n+9}{2}, \frac{5n+5}{2}\right\} \\ &\cup \left\{\frac{-5n-5}{2}, \frac{-5n-9}{2}, \ldots, \frac{5-7n}{2}, \frac{1-7n}{2}\right\} \\ &\cup \left\{\frac{-5n-3}{2}, \frac{-5n-7}{2}, \frac{-5n-11}{2}, \ldots, \frac{-7n+3}{2}\right\} \\ &\cup \left\{\frac{7n-3}{2}, \frac{7n-7}{2}, \ldots, \frac{5n+7}{2}, \frac{5n+3}{2}\right\} \end{aligned}$$

Thus, f is a pair sum labeling of $D_2(P_n)$.

Case 2: Suppose $n \equiv 0 \pmod 2$

Define the map $f : V(D_2(P_n)) \to \{\pm 1, \pm 2, \ldots, \pm 2n\}$ as follows:

$$f(u_i) = \begin{cases} n - \frac{i-1}{2} & \text{if } i \equiv 1 \pmod 2, 1 \leq i \leq n-1 \\ -2n + 3(\frac{i-2}{2}) & \text{if } i \equiv 0 \pmod 2, 2 \leq i \leq n \end{cases}$$

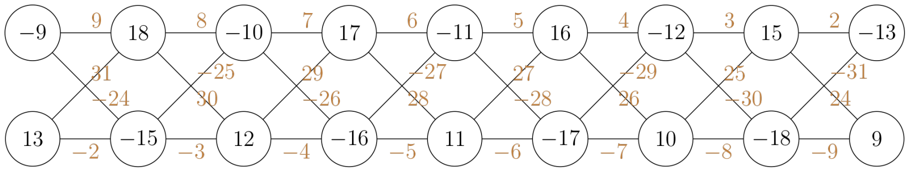
and

$$f(v_i) = \begin{cases} -n + \frac{i-1}{2} & \text{if } i \equiv 1 \pmod 2, 1 \leq i \leq n-1 \\ 2n - 3(\frac{i-2}{2}) & \text{if } i \equiv 0 \pmod 2, 2 \leq i \leq n \end{cases}$$

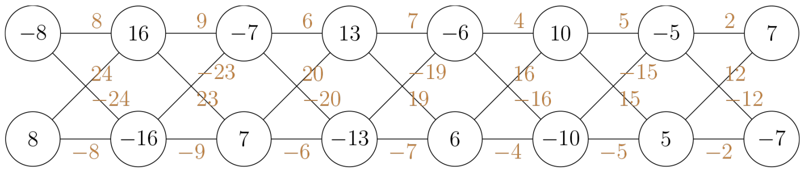
We have

$$\begin{aligned} f_e[E(D_2(P_n))] &= \{-n, -n+2, -n+4 \ldots, -2\} \cup \{-n-1, -n+1, -n+3 \ldots, -5\} \\ &\cup \{n, n-2, n-4 \ldots, 2\} \cup \{n+1, n-1, n-3 \ldots, 5\} \\ &\cup \{3n, 3n-4, 3n-8 \ldots, n+4\} \cup \{-3n+1, -3n+5, \ldots, -n-7\} \\ &\cup \{-3n, -3n+4, -3n+8 \ldots, -n-4\} \cup \{3n-1, 3n-5, \ldots, n+7\} \end{aligned}$$

Thus f is a pair sum labeling of $D_2(P_n)$. $\square$



(a)



(b)

Figure 3: (a) Pair sum labeling of $D_2(P_9)$, and (b) pair sum labeling of $D_2(P_8)$

Theorem 3.3. *The middle graph $M(P_n)$ of path graph P_n is a pair sum graph.*

Proof. Let P_n be a path with $V(P_n) = \{u_1, u_2, \dots, u_n\}$. Consider the middle graph $D_2(P_n)$ of P_n. Suppose $V(D_2(P_n)) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}\}$ and $E(D_2(P_n)) = \{v_a v_{a+1} : 1 \leq a \leq n-2\} \cup \{u_b v_b, v_b u_{b+1} : 1 \leq b \leq n-1\}$ (See Figure 1c).

When $n = 1$, $M(P_1) \cong \overline{K}_1$, the empty graph on 1 vertex. When $n = 2$, $M(P_2) \cong P_3$, path of order 3. Thus, for $n \in \{1, 2\}$, $M(P_n)$ is a pair sum graph. When $n = 3$, $M(P_2)$ is a simple graph with 5 vertices. In [7], Theorem 3.18, that $M(P_2)$ is a pair sum graph. Let $n \geq 4$, we have the following cases.

Case 1: Suppose $n \equiv 0 \pmod 2$

Define the map $f : V(M(P_n)) \to \{\pm 1, \pm 2, \dots, \pm(2n-1)\}$ as follows:

$$f(u_i) = \begin{cases} \frac{-3n}{2} & \text{if } i = 1 \\ -5 - n + 2i & \text{if } 2 \leq i \leq \frac{n-2}{2} \\ 3 & \text{if } i = \frac{n}{2} \\ -5 & \text{if } i = \frac{n+2}{2} \\ 3 - n + 2i & \text{if } \frac{n+4}{2} \leq i \leq n-1 \\ \frac{3n}{2} & \text{if } i = n \end{cases}$$

and

$$f(v_i) = \begin{cases} 1 & \text{if } i = \frac{n}{2} \\ 2i - n & \text{if } 1 \leq i \leq n-1, i \neq \frac{n}{2} \end{cases}$$

We have

$$\begin{aligned} f_e[E(M(P_n))] &= \{6-2n, 10-2n, 14-2n, \dots, -6\} \cup \{-1, 3\} \cup \{6, 10, 14, \dots, 2n-6\} \\ &\cup \left\{2 - \frac{5n}{2}\right\} \cup \{3-2n, 7-2n, 11-2n, \dots, -9\} \cup \{4, -3\} \cup \{11, 15, 19 \dots, 2n-1\} \\ &\cup \{1-2n, 5-2n, 9-2n \dots, -11\} \cup \{1, -4\} \cup \{9, 13, 17, \dots, 2n-3\} \cup \left\{\frac{5n}{2} - 2\right\} \\ f_e[E(M(P_n))] &= \{\pm 1, \pm 3, \pm 4\} \cup \{\pm 6, \pm 10, \pm 14, \dots, \pm(2n-6)\} \cup \{\pm 9, \pm 13, \pm 17, \dots, \pm(2n-3)\} \end{aligned}$$

$$\cup \{\pm 11, \pm 15, \pm 19 \ldots, \pm(2n-1)\} \cup \left\{ \pm \left(\frac{5n}{2} - 2 \right) \right\}$$

Thus f is a pair sum labeling of $M(P_n)$.

Case 2: Suppose $n \equiv 1 \pmod 4$

Define the map $f : V(M(P_n)) \to \{\pm 1, \pm 2, \ldots, \pm(2n-1)\}$ as follows:

$$f(u_i) = \begin{cases} 2n-5 & \text{if } i = 1 \\ 2i-2 & \text{if } 2 \le i \le \frac{n-1}{2} \\ -2n+5 & \text{if } i = \frac{n+1}{2} \\ n+1-2i & \text{if } \frac{n+3}{2} \le i \le n-1 \\ -n & \text{if } i = n \end{cases}$$

and

$$f(v_i) = \begin{cases} 2i-1 & \text{if } 1 \le i \le \frac{n-1}{2} \\ n-2i & \text{if } \frac{n+1}{2} \le i \le n-1 \end{cases}$$

We have

$$\begin{aligned} f_e[E(M(P_n))] &= \{4, 8, 12, \ldots, 2n-6\} \cup \{n-3\} \cup \{-4, -8, -12 \ldots, -2n+6\} \\ &\cup \{2n-4\} \cup \{5, 9, 13 \ldots, 2n-5\} \cup \{-2n+4\} \cup \{-5, -9, -13 \ldots, -2n+5\} \cup \{-2n+2\} \\ &\cup \{3, 7, 11 \ldots, 2n-7\} \cup \{-n+3\} \cup \{-3, -7, -11 \ldots, -2n+7\} \\ f_e[E(M(P_n))] &= \{\pm 3, \pm 7, \pm 11 \ldots, \pm(2n-7)\} \cup \{\pm 4, \pm 8, \pm 12, \ldots, \pm(2n-6)\} \\ &\cup \{\pm 5, \pm 9, \pm 13 \ldots, \pm(2n-5)\} \cup \{\pm(n-3)\} \cup \{\pm(2n-4)\} \cup \{-2n+2\} \end{aligned}$$

Thus f is a pair sum labeling of $M(P_n)$.

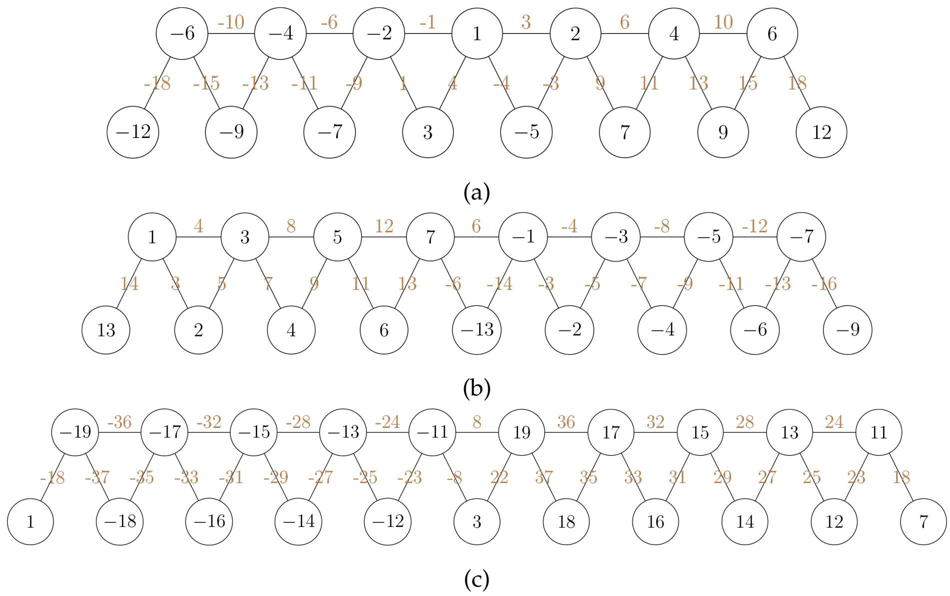


Figure 4: (a) Pair sum labeling of $M(P_8)$, (b) pair sum labeling of $M(P_9)$, and (c) pair sum labeling of $M(P_{11})$

Case 3: Suppose $n \equiv 3 \pmod 4$

Define the map $f : V(M(P_n)) \to \{\pm 1, \pm 2, \ldots, \pm(2n-1)\}$ as follows:

$$f(u_i) = \begin{cases} 1 & \text{if } i = 1 \\ -2n + 2i & \text{if } 2 \leq i \leq \frac{n-1}{2} \\ 3 & \text{if } i = \frac{n+1}{2} \\ 3n - 1 - 2i & \text{if } \frac{n+3}{2} \leq i \leq n-1 \\ n - 4 & \text{if } i = n \end{cases}$$

and

$$f(v_i) = \begin{cases} -2n + 1 + 2i & \text{if } 1 \leq i \leq \frac{n-1}{2} \\ 3n - 2 - 2i & \text{if } \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

We have

$$\begin{aligned} f_e[E(M(P_n))] &= \{8 - 4n, 12 - 4n, 16 - 4n, \ldots, -2n - 2\} \cup \{n - 3\} \\ &\cup \{4n - 8, 4n - 12, 4n - 16, \ldots, 2n + 2\} \\ &\cup \{4 - 2n\} \cup \{9 - 4n, 13 - 4n, 17 - 4n, \ldots, -2n - 1\} \cup \{2n\} \\ &\cup \{4n - 9, 4n - 13, 4n - 17 \ldots, 2n + 1\} \cup \{7 - 4n, 11 - 4n, 15 - 4n \ldots, -2n - 3\} \\ &\cup \{3 - n\} \cup \{4n - 7, 4n - 11, 4n - 15, \ldots, 2n + 3\} \cup \{2n - 4\} \\ f_e[E(M(P_n))] &= \{\pm(n - 3), \pm(2n - 4)\} \cup \{\pm(4n - 8), \pm(4n - 12), \pm(4n - 16), \ldots, \pm(2n + 2)\} \\ &\cup \{\pm(4n - 9), \pm(4n - 13), \pm(4n - 17) \ldots, \pm(2n + 1)\} \\ &\cup \{\pm(4n - 7), \pm(4n - 11), \pm(4n - 15), \ldots, \pm(2n + 3)\} \cup \{2n\} \end{aligned}$$

Thus, f is a pair sum labeling of $M(P_n)$. $\square$

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