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Availability Analysis of a Repairable Redundant System Under Preemptive-Resume Repair Discipline

Shikha Bansal[1,*]

1 Department of Mathematics, SRM University, NCR Campus, Modinagar, Ghaziabad, Uttar Pradesh, India.

Abstract: This paper addresses the availability evaluation of a complex system comprising of two independent repairable subsystems in 1-out-of 2: F, under preemptive resume repair discipline. The failure rate follow exponential time distribution and repair time follow and general time distribution. The problem is formulated using the supplementary variable technique and various state probabilities have been obtained. A numerical example along with graphical representation has been appended to highlight the important conclusions.

Keywords: Redundancy, Supplementary variables technique, Laplace transform, Preemptive-Resume repair.

1. Introduction

Redundancy is a very important technique of availability improvement used in industries. In a cold standby redundant system whenever the operating unit fails, the standby unit takes its existence and the failed unit goes under repair. During last few decades, a lot of research papers are written by researchers and scientists. Aggarwal and Gupta [2] performed a study on the minimizing the cost of reliable systems. Mokaddis [7] examined Cost Analysis of a two dissimilar-unit cold standby redundant system subject to inspection and two types of repair. Cau [6] performed a new stochastic model for system under general repair. Parashar [11] analyzed the Reliability and profit evaluation of a PLC hot standby system based on a master-stove concept and two types of repair facilities. Agarwal & Bansal [1] examined the Reliability Characteristic of Cold-Standby Redundant System. Many researchers have evaluated the operational availability of various complex systems but not much work has been reported so far incorporating the concept of environmental effects. In this paper the model consists of two independent repairable subsystems A and B. Subsystem A has two identical units one unit is in operative mode and other in cold standby. The cold standby unit becomes operative after failure of the operative unit. Both units of subsystem A have the same failure rate at the time of installation but due to adverse catastrophic effects, the failure rate of the standby unit increases by the time. Subsystem B is a simple system having minor and major failure. Minor, reduces the efficiency of the system causing degraded state while major results into a non-operative state of the system. This system can also fail due to environmental failure like, Temperature, humidity etc. Laplace Transform of various state probabilities have been obtained by employing the Supplementary variable technique under preemptive resume repair discipline. The Ergodic behaviour and particular cases have also been evaluated and effect of minor failure rate on the operational availability of the system in the steady state has been computed both numerically and graphically.

* *E-mail: srbansal2008@gmail.com*

1.1. Assumptions

The mathematical model of the system under preemptive resume repair discipline policy is formulated under the following assumptions:

(a). Initially the system is in operable state.

(b). Only one change in state of the system can take place at an instant.

(c). Perfect switching over device is used.

(d). When the system starts functioning, both the A-units have the same failure rates say λ_1. But as the time passes, due to adverse catastrophic failure rate of the standby unit increases to λ_2 by the time it is needed to operate.

(e). During the degraded state of the system due to minor failure in subsystem B, major failure may also occur.

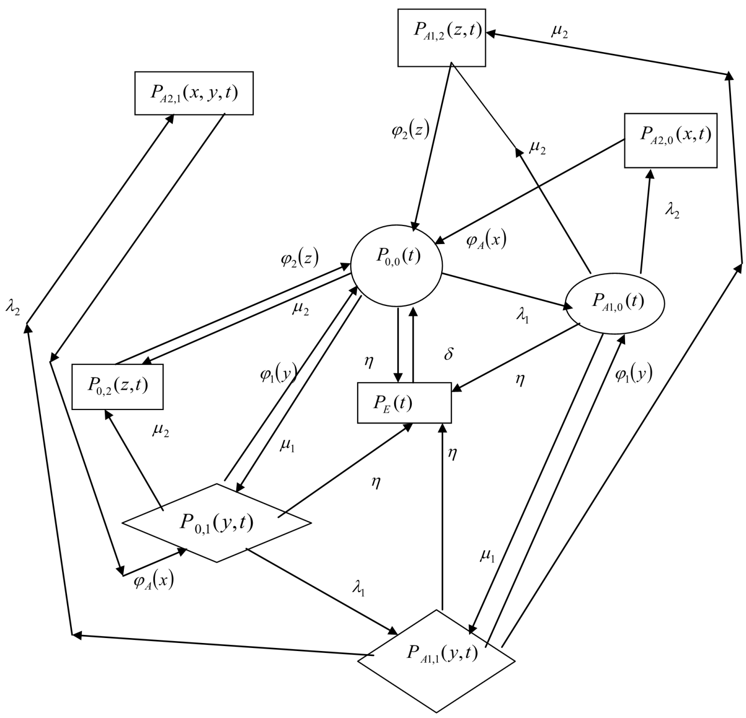

Figure 1. **State transition diagram**

1.2. Notations

$\lambda_1, \lambda_2 = (\lambda_1 \angle \lambda_2)$	: Failure rates of the principal and standby units respectively of Subsystem A.
μ_1, μ_2	: Minor and major failure rates respectively of B.
x, y, z	: Elapsed repair times for both A-units and Minor and major failure of B-unit respectively.
$\varphi_A(x), \varphi_1(y), \varphi_2(z)$, $S_A(x), S_1(y), S_2(z)$	: Transition rates and probability, density functions respectively repairs of A class and Minor and major failure of B-unit and completed in time x, y, z respectively.
M_A, M_1, M_2	: Mean time to repair of A units and Minor and major failure of B-unit respectively.
η	: Failure rate of Subsystem A and Subsystem B due to environmental Failure.
δ	: Constant repair rate due to environmental Failure.

1.3. Additional Notations

$$J_E(s,\alpha) = \left[1 - \overline{S}_E(s+\alpha)\right](s+\alpha)^{-1} \quad \text{where } E = \ \text{A},\ 1 \text{ or } 2$$

$$E_1(s,\alpha,\beta) = \left[J_1(s,\alpha) - J_1(s,\beta)\right](\beta-\alpha)^{-1}$$

$$D_1(s,\alpha,\beta) = \left[\overline{S}_1(s+\alpha) - \overline{S}_1(s+\beta)\right](\beta-\alpha)^{-1}$$

$$K(s) = \bar{S}_A(s)E_1(s,\lambda_2+\mu_2+\eta,\lambda_1+\mu_2+\eta)$$

$$L(s) = J_1(s,\lambda_1+\mu_2+\eta)\left[1-\lambda_1\lambda_2K(s)\right]^{-1}$$

$$N(s) = K(s)\left[1-\lambda_1\lambda_2K(s)\right]^{-1}$$

$$H(s) = s+\lambda_2+\mu_1+\mu_2+\eta-\mu_1\left[1+\lambda_1\lambda_2N(s)\right]\overline{S}_1(s+\lambda_2+\mu_2+\eta)$$

$$Q(s) = \lambda_1\left[1+\mu_1L(s)\bar{S}_1(s+\lambda_2+\mu_2+\eta)\right]$$

$$\begin{aligned} A(s) = {} & s+\lambda_1+\mu_1+\mu_2+\eta-\mu_1\lambda_1\lambda_2L(s)\bar{S}_A(s)D_1\left(s,\lambda_2+\mu_2+\eta,\lambda_1+\mu_2+\eta\right) \\ & -\mu_1\bar{S}_1(s+\lambda_1+\mu_2+\eta)-\mu_2\bar{S}_2(s)\left\{1+\mu_1L(s)\right\}-\lambda_1\mu_1\mu_2L(s)\bar{S}_2(s)J_1\left(s,\lambda_2+\mu_2+\eta\right) \\ & -\frac{\eta\delta}{(s+\delta)}\left\{1+\mu_1L(s)+\lambda_1\mu_1L(s)J_1\left(s,\lambda_2+\mu_2+\eta\right)\right\} \end{aligned}$$

$$\begin{aligned} B(s) = {} & \lambda_2\bar{S}_A(s)+\lambda_2\mu_1\overline{S_A}(s)D_1(s,\lambda_2+\mu_2+\eta,\lambda_1+\mu_2+\eta)\left[1+\lambda_1\lambda_2N(s))\right] \\ & +\lambda_2\mu_1\mu_2N(s)\bar{S}_2(s)+\mu_2\bar{S}_2(s)\left[1+\mu_1J_1(s,\lambda_2+\mu_2+\eta)\left\{1+\lambda_1\lambda_2N(s)\right\}\right] \\ & +\frac{\eta\delta}{(s+\delta)}\left[1+\mu_1J_1(s,\lambda_2+\mu_2+\eta)\left\{1+\lambda_1\lambda_2N(s)\right\}+\lambda_2\mu_1N(s)\right] \end{aligned}$$

$$T(s) = H(s).A(s)-Q(s).B(s)$$

2. Formulation of the Mathematical Model

By elementary probability and continuity arguments, difference differential equations governing the stochastic behaviour of the complex system are.

$$\left[\frac{\partial}{\partial t}+\lambda_1+\mu_1+\mu_2+\eta\right]P_{0,0}(t) = \int_0^\infty P_{A2,0}(x,t)\varphi_A(x)dx\int_0^\infty P_{0,1}(y,t)\varphi_1(y)dy\int_0^\infty\left[P_{0,2}(z,t)+P_{A1,2}(z,t)\right]\varphi_2(z)dz+\delta P_E(t) \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\lambda_2+\mu_2+\mu_1+\eta\right]P_{A1,0}(t) = \lambda_1P_{0,0}(t)+\int_0^\infty P_{A1,1}(y,t)\varphi_1(y)dy \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi_A(x)\right]P_{A2,0}(x,t) = 0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_1+\mu_2+\eta+\varphi_1(y)\right]P_{0,1}(y,t) = \int_0^\infty P_{A2,1}\left(x,y,t\right)\varphi_A\left(x\right)dx \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_2+\mu_2+\eta+\varphi_1(y)\right]P_{A1,1}(y,t) = 0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi_A(x)\right]P_{A2,1}(x,y,t) = 0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\varphi_2(z)\right]P_{a,2}(z,t) = 0 \quad (a = 0, A_1) \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\delta\right]P_E(t) = \eta\left[P_{A1,0}(t)+P_{0,0}(t)+\int_0^\infty P_{A1,1}(y,t)dy+\int_0^\infty P_{0,1}(y,t)dy\right] \tag{8}$$

2.1. Boundary Conditions

$$P_{A2,0}(0,t) = \lambda_2P_{A1,0}(t) \tag{9}$$

$$P_{0,1}(0,t) = \mu_1 P_{0,0}(t) \tag{10}$$

$$P_{A1,1}(0,t) = \mu_1 P_{A1,0}(t) + \lambda_1 P_{0,1}(t) \tag{11}$$

$$P_{A2,1}(0,y,t) = \lambda_2 P_{A1,1}(y,t) \tag{12}$$

$$P_{0,2}(0,t) = \mu_2 \left[P_{0,0}(t) + \int P_{0,1}(y,t)dy \right] \tag{13}$$

$$P_{A1,2}(0,t) = \mu_2 \left[P_{A1,0}(t) + \int P_{A1,1}(y,t)dy \right] \tag{14}$$

2.2. Initial Conditions

$$P_{0,0}(t) = 1 \text{ and other state probabilities are zero at } t = 0 \tag{15}$$

3. Solution of the Model

Taking Laplace Transform of (1) to (15) and on further simplification, one may obtain the following L.T. of the probabilities that the system is in up (operable) state and the down (failed) state

$$\begin{aligned}\overline{P}_{up}(s) &= [1 + \mu_1 L(s) + \lambda_1 \mu_1 L(s) J_1(s, \lambda_2 + \mu_2 + \eta)][H(s)/T(s)] \\ &\quad + [1 + \lambda_2 \mu_1 N(s) + \mu_1 \{1 + \lambda_1 \lambda_2 N(s)\} J_1(s, \lambda_2 + \mu_2 + \eta)][Q(s)/T(s)]\end{aligned} \tag{16}$$

$$\begin{aligned}\overline{P}_{down}(s) &= \begin{bmatrix} \lambda_1 \lambda_2 \mu_1 L(s) J_A(s,0) J_1(s, \lambda_2 + \mu_2 + \eta) + \mu_2 J_2(s,0)\{1 + \mu_1 L(s)\} \\ + \mu_2 \lambda_1 \mu_1 L(s) J_2(s,0) J_1(s, \lambda_2 + \mu_2 + \eta) \\ + \frac{\eta}{(s+\delta)} \{1 + \mu_1 \lambda_1 L(s) J_1(s, \lambda_2 + \mu_2 + \eta) + \mu_1 L(s)\} \end{bmatrix} [H(s)/T(s)] \\ &\quad + \begin{bmatrix} \lambda_2 J_A(s,0) + \lambda_2 \mu_1 \{1 + \lambda_1 \lambda_2 N(s)\} J_A(s,0) J_1(s, \lambda_2 + \mu_2 + \eta) \\ + \lambda_2 \mu_1 \mu_2 N(s) J_2(s,0) + \mu_2 J_2(s,0)[1 + \mu_1 J_1(s, \lambda_2 + \mu_2 + \eta)(1 + \lambda_1 \lambda_2 N(s))] \\ + \frac{\eta}{(s+\delta)} \{1 + \mu_1 (1 + \lambda_1 \lambda_2 N(s)) J_1(s, \lambda_2 + \mu_2 + \eta) + \lambda_2 \mu_1 N(s)\} \end{bmatrix} [Q(s)/T(s)]\end{aligned} \tag{17}$$

4. Ergodic Behaviour

Using Abel's Lemma viz;

$$\lim_{s \to 0} \left[s\overline{\mathrm{F}}(s) \right] = \lim_{t \to \infty} F(t) = F_1 \ \ (say)$$

Provided that the limit on the right hand side exists, the following time independent up and down state probabilities are:

$$\begin{aligned}P_{up} &= [1 + \mu_1 L(0) + \lambda_1 \mu_1 L(0) J_1(0, \lambda_2 + \mu_2 + \eta)] \left[H(0)/T'(0) \right] \\ &\quad + \left[1 + \lambda_2 \mu_1 N(0) + \mu_1 J_1(0, \lambda_2 + \mu_2 + \eta)\{1 + \lambda_1 \lambda_2 N(0)\} \right][Q(0)/T'(0)]\end{aligned} \tag{18}$$

$$\begin{aligned}P_{down} &= \begin{bmatrix} \lambda_1 \lambda_2 \mu_1 L(0) M_A J_1(0, \lambda_2 + \mu_2 + \eta) + \mu_2 M_2 \{1 + \mu_1 L(0)\} \\ + \mu_2 \lambda_1 \mu_1 L(0) M_2 J_1(0, \lambda_2 + \mu_2 + \eta) \\ + \frac{\eta}{\delta} \{1 + \mu_1 \lambda_1 L(0) J_1(0, \lambda_2 + \mu_2 + \eta) + \mu_1 L(0)\} \end{bmatrix} [H_1(s)/T_1(s)] \\ &\quad + \begin{bmatrix} \lambda_2 M_A + \lambda_2 \mu_1 \{1 + \lambda_1 \lambda_2 N(0)\} M_A J_1(0, \lambda_2 + \mu_2 + \eta) \\ + \lambda_2 \mu_1 \mu_2 N(0) M_2 + \mu_2 M_2 [1 + \mu_1 J_1(0, \lambda_2 + \mu_2 + \eta)(1 + \lambda_1 \lambda_2 N(0))] \\ + \frac{\eta}{\delta} \{1 + \mu_1 (1 + \lambda_1 \lambda_2 N(0)) J_1(0, \lambda_2 + \mu_2 + \eta) + \lambda_2 \mu_1 N(0)\} \end{bmatrix} [Q(s)/T(s)]\end{aligned} \tag{19}$$

5. Particular Cases

Repair follow exponential time distributions: Setting $\overline{S}_A(s) = \frac{\phi_A}{s+\phi_A}$, $\overline{S}_1(s) = \frac{\phi_1}{s+\phi_1}$, $\overline{S}_2(s) = \frac{\phi_2}{s+\phi_2}$. Laplace Transform of operational availability and non-availability of the system are:-

$$\begin{aligned}\overline{P}_{up}(s) &= \left[1 + \frac{\mu_1\left(s+\varphi_A\right)\left(s+\lambda_1+\lambda_2+\mu_2+\eta+\varphi_1\right)}{\left(s+\varphi_A\right)\left(s+\lambda_1+\mu_2+\eta+\varphi_1\right)\left(s+\lambda_2+\mu_2+\eta+\varphi_1\right)-\lambda_1\lambda_2\varphi_A}\right][H_1(s)/T_1(s)] \\ &+ \left[1 + \frac{\lambda_2\mu_1\varphi_A + \mu_1\left(s+\varphi_A\right)\left(s+\lambda_1+\mu_2+\eta+\varphi_1\right)}{\left(s+\varphi_A\right)\left(s+\lambda_1+\mu_2+\eta+\varphi_1\right)\left(s+\lambda_2+\mu_2+\eta+\varphi_1\right)-\lambda_1\lambda_2\varphi_A}\right][Q_1(s)/T_1(s)] \end{aligned} \tag{20}$$

$$\begin{aligned}\overline{P}_{down}(s) &= \left[\begin{array}{l} \frac{\mu_1\lambda_1(\lambda_2+\mu_2)}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A} \\ +\frac{\mu_2}{(s+\phi_2)}\left\{1+\frac{\mu_1(s+\phi_A)(s+\lambda_2+\mu_2+\eta+\phi_1)}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A}\right\} \\ +\frac{\eta}{(s+\delta)}\left\{1+\frac{\mu_1(s+\phi_A)(s+\lambda+_1\lambda_2+\mu_2+\eta+\phi_1)}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A}\right\} \end{array}\right][H_1(s)/T_1(s)] \\ &+ \left[\begin{array}{l} \frac{\lambda_2}{(s+\phi_A)}+\frac{\lambda_2\mu_1(s+\lambda_1+\mu_2+\eta+\phi_1)}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A} \\ +\frac{\lambda_2\mu_1\mu_2\phi_A}{(s+\phi_2)\{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A\}} \\ +\frac{\mu_2}{(s+\phi_2)}\left\{1+\frac{\mu_1(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A}\right\} \\ +\frac{\eta}{(s+\delta)}\left\{\begin{array}{l} 1+\frac{\lambda_2\mu_1\phi_A}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A} \\ +\frac{\mu_1(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)}{(s+\phi_A)(s+\lambda_1+\mu_2+\eta+\phi_1)(s+\lambda_2+\mu_2+\eta+\phi_1)-\lambda_1\lambda_2\phi_A} \end{array}\right\} \end{array}\right][Q_1(s)/T_1(s)] \end{aligned} \tag{21}$$

6. Effect of Minor Failure on the Long Run Availability of the System

Setting $\lambda_1 = .01$, $\lambda_2 = .02$, $\mu_2 = .1$, $\eta = .03$, $\phi_A = 0.40$, $\phi_1 = 0.5$, $\phi_2 = 1$ and $\delta = 0.7$. We obtain the following table and three graphs The behaviour of all have been also shown graphically.

μ_1	Pr. Of normal availability $P_{0,0} + P_{A1,0}$
0.01	0.337268
0.02	0.2090292
0.03	0.1514454
0.04	0.1187358
0.05	0.097646
0.06	0.0829181
0.07	0.0720507
0.08	0.0637018
0.09	0.0570869
0.1	0.0517165

Table 1.

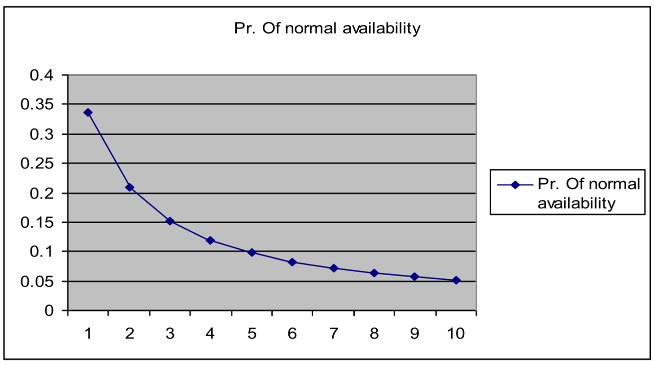

Figure 2. **Normal availability v/s minor failure**

μ_1	Operation availability Pup
0.01	0.872614
0.02	0.872614
0.03	0.872614
0.04	0.872614
0.05	0.872614
0.06	0.872614
0.07	0.872614
0.08	0.872614
0.09	0.872614
0.1	0.872614

Table 2.

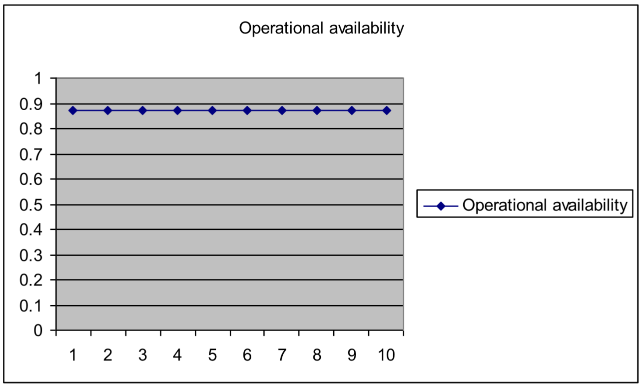

Figure 3. **Operational availability v/s minor failure**

7. Interpretation of the Results

The inspection of Table 1 and Figure 2 "Normal Availability V/S Minor Failure" results that, as the minor failure increases, the availability of the system with normal efficiency decreases rapidly. From Table 2 & Figure 3 "Operational Availability V/S Minor Failure "effect of minor failure for a complex system under given parameters is such that the availability is throughout constant subject to different variations in μ_1.

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