Int. J. Math. And Appl., 6(1–D)(2018), 687–697
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Square Divisor Cordial Labeling in the Context of Vertex Switching

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Abstract: A square divisor cordial labeling of a graph G with vertex set $V(G)$ is a bijection f from $V(G)$ to $\{1, 2, \ldots, |V(G)|\}$ such that an edge $e = uv$ is assigned the label 1 if $[f(u)]^2|f(v)$ or $[f(v)]^2|f(u)$ and the label 0 otherwise, then $|e_f(0) - e_f(1)| \leq 1$. A graph which admits square divisor cordial labeling is called a square divisor cordial graph. In this research article we prove that the graphs obtained by switching of a vertex in bistar, comb graph, crown and armed crown are square divisor cordial. In addition to this we also prove that the graphs obtained by switching of a vertex except apex vertex in helm and gear graph are square divisor cordial.

MSC: 05C78

Keywords: Square Divisor Cordial Labeling; Vertex Switching.

1. Introduction

Throughout this work, by a graph we mean finite, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$. For any undefined notations and terminology we follow Gross and Yellen [4] while for number theory we follow Burton [1].

1.1. Preliminaries

Definition 1.1. *If the vertices or edges or both are assigned numbers subject to certain condition(s) then it is known as graph labeling.*

A dynamic survey on graph labeling is regularly updated by Gallian [3].

Definition 1.2. *A mapping $f : V(G) \longrightarrow \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

Notation 1.3. *If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \longrightarrow \{0, 1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Then*

$v_f(i)$=number of vertices of G having label i under f,

$e_f(i)$=number of edges of G having label i under f^*

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Definition 1.4. *A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$. A graph which admits cordial labeling is called a cordial graph.*

Definition 1.5. *Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \longrightarrow \{1, 2, \ldots, |V(G)|\}$ be a bijection. For each edge $e = uv$, assign the label 1 if $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$.*
A graph which admits divisor cordial labeling is called a divisor cordial graph.

The divisor cordial labeling was introduced by Varatharajan et al. [7]. Vaidya and Shah [6] proved that

- H_n is a divisor cordial graph for every n.
- G_n is a divisor cordial graph for every n.
- Switching of a vertex in cycle C_n admits divisor cordial labeling.
- Switching of a rim vertex in a wheel W_n admits divisor cordial labeling.
- Switching of the apex vertex in helm H_n admits divisor cordial labeling.

Definition 1.6. *Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \longrightarrow \{1, 2, \ldots, |V(G)|\}$ be a bijection. For each edge $e = uv$, assign the label 1 if $[f(u)]^2|f(v)$ or $[f(v)]^2|f(u)$ and the label 0 otherwise. The function f is called a square divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits square divisor cordial labeling is called a square divisor cordial graph.*

The square divisor cordial labeling was introduced by Murugesan et al. [5] and they proved the following results:

- The complete bipartite graph $K_{2,n}$ is square divisor cordial.
- The complete bipartite graph $K_{3,n}$ is square divisor cordial if and only if $n = 1, 2, 3, 5, 6, 7$ or 9.
- The complete graph Kn is square divisor cordial if and only if $n = 1, 2, 3$ or 5.

Definition 1.7. *A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.*

Definition 1.8. *Bistar $B_{m,n}$ is the graph obtained by joining the center(apex) vertices of $K_{1,m}$ and $K_{1,n}$ by an edge.*

Definition 1.9. *Comb graph $P_n \odot K_1$ is the graph obtained by joining a pendant edge to each vertex of path P_n.*

Definition 1.10. *Crown $C_n \odot K_1$ is the graph obtained by joining a pendant edge to each vertex of cycle C_n.*

Definition 1.11. *Armed crown is the graph obtained by attaching a path P_2 at each vertex of cycle C_n. It is denoted by AC_n, where n is the number of vertices in cycle C_n.*

Definition 1.12. Helm *H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.*

Definition 1.13. *Let $G = (V(G), E(G))$ be a graph. Let $e = uv$ be an edge of G and w is not a vertex of G. The edge e is subdivided when it is replaced by the edges $e^{'} = uw$ and $e^{''} = wv$.*

Definition 1.14. *Gear graph G_n is the graph obtained from a wheel W_n by subdividing each of its rim edge.*

2. Main Results

Theorem 2.1. *The graph G_v obtained by switching of a vertex in the bistar $B_{m,n}$ is square divisor cordial.*

Proof. Let $G = B_{m,n}$ be the bistar with the vertex set $\{u_0, v_0, u_j, v_i : 1 \leq j \leq m, 1 \leq i \leq n\}$, where u_0, v_0 are the apex vertices and u_j, v_i are pendant vertices for all $j = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$. Without loss of generality we are assuming that $m \leq n$ because $B_{m,n}$ and $B_{n,m}$ are isomorphic graphs. Let G_v be the graph obtained by switching of a vertex v in G. The proof is divided into following three cases:

Case 1: Switching of the pendant vertex.

Without loss of generality we are assuming that the switched pendant vertex is v_1. We note that $|V(G_v)| = m+n+2$ and $|E(G_v)| = 2m+2n$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, m+n+2\}$ as follows:

Let p be the largest prime number such that $p_1 \leq m+n+2$ and p_2 be the second largest prime number such that $p_2 < p_1 \leq m+n+2$. $f(v_1) = 1$, $f(v_0) = p_1$, $f(u_0) = p_2$. Now, label the vertices $v_2, v_3, \ldots, v_n, u_1, u_2, \ldots, u_m$ from the set $\{2, 3, 4, \ldots, m+n+2\} - \{p_1, p_2\}$. In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = m+n$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 2: Switching of the vertex of degree m.

Here, switched vertex is u_0. We note that $|V(G_v)| = m+n+2$ and $|E(G_v)| = 2n$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, m+n+2\}$ as follows:

Let p be the largest prime number such that $p \leq m+n+2$. $f(v_0) = 1$, $f(u_0) = p$. Now, label the vertices $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m$ from the set $\{2, 3, 4, \ldots, m+n+2\} - \{p\}$. In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 3: Switching of the vertex of degree n.

Here, switched vertex is v_0. We note that $|V(G_v)| = m+n+2$ and $|E(G_v)| = 2m$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, m+n+2\}$ as follows:

Let p be the largest prime number such that $p \leq m+n+2$. $f(v_0) = 1$, $f(u_0) = p$. Now, label the vertices $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m$ from the set $\{2, 3, 4, \ldots, m+n+2\} - \{p\}$. In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = m$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph G_v obtained by switching of a vertex in the bistar $B_{m,n}$ is square divisor cordial. □

Illustration 2.2. *The bistar $G = B_{4,5}$ and the graph G_v obtained by switching of vertex v_1 with its square divisor cordial labeling are shown in the Figure 1.*

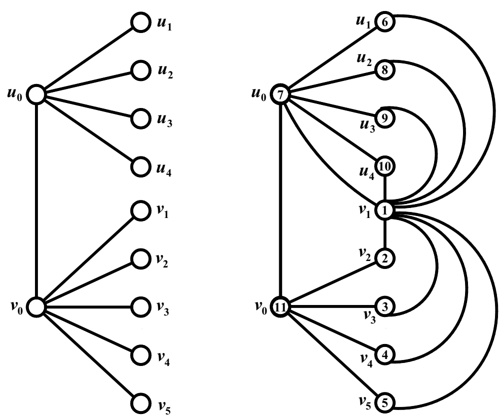

Figure 1. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.3. *The bistar $G = B_{4,5}$ and the graph G_v obtained by switching of vertex u_0 with its square divisor cordial labeling are shown in the Figure 2.*

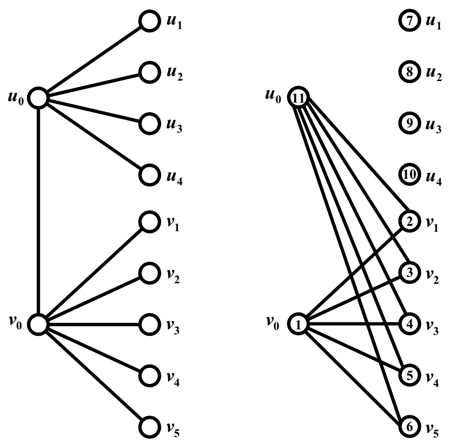

Figure 2. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.4. *The bistar $G = B_{4,5}$ and the graph G_v obtained by switching of vertex v_0 with its square divisor cordial labeling are shown in the Figure 3.*

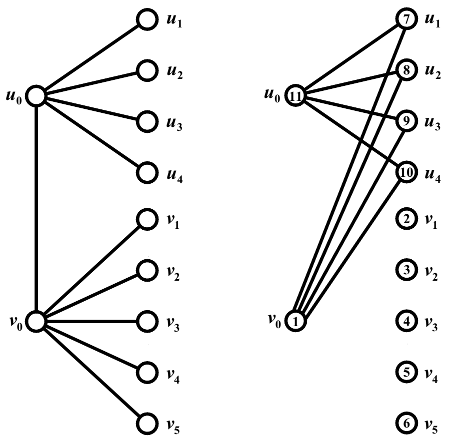

Figure 3. **The graph G and square divisor cordial labeling of graph G_v.**

Theorem 2.5. *The graph G_v obtained by switching of a vertex in the comb graph $P_n \odot K_1$ is square divisor cordial.*

Proof. Let $G = P_n \odot K_1$ be the comb graph with the vertex set $V(G) = \{v_i, u_i : 1 \leq i \leq n\}$, where v_i and u_i are pendant and path vertices respectively for all $i = 1, 2, \ldots, n$. Let G_v be the graph obtained by switching of a vertex v in G. The proof is divided into following three cases:

Case 1: Switching of the pendant vertex.

Without loss of generality we are assuming that the switched pendant vertex is v_1. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n - 4$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n\}$ as follows:

$$f(v_1) = 1,\ f(u_1) = 2n.$$

$$f(u_i) = 2(n - i) + 3;\ 2 \leq i \leq n.$$

$$f(v_i) = f(u_i) - 1;\ 2 \leq i \leq n.$$

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = 2n - 2$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 2: Switching of the vertex of degree two.

Without loss of generality we are assuming that the switched vertex is u_1. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n-6$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n\}$ as follows:

$$f(u_1) = 1, \;\; f(v_1) = 2n.$$

$$f(u_i) = 2(n-i) + 3; \;\; 2 \leq i \leq n.$$

$$f(v_i) = f(u_i) - 1; \;\; 2 \leq i \leq n.$$

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = 2n - 3$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 3: Switching of the vertex of degree three.

Without loss of generality we are assuming that the switched vertex is u_2. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n-8$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n\}$ as follows:

$$f(u_2) = 1, \;\; f(u_1) = 2n - 1.$$

$$f(u_i) = 2(n-i) + 3; \;\; 3 \leq i \leq n.$$

$$f(v_i) = f(u_i) + 1; \;\; 1 \leq i \leq n.$$

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = 2n - 4$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph G_v obtained by switching of a vertex in the comb graph $P_n \odot K_1$ is square divisor cordial. $\square$

Illustration 2.6. *The comb graph $G = P_5 \odot K_1$ and the graph G_v obtained by switching of vertex v_1 with its square divisor cordial labeling are shown in the Figure 4.*

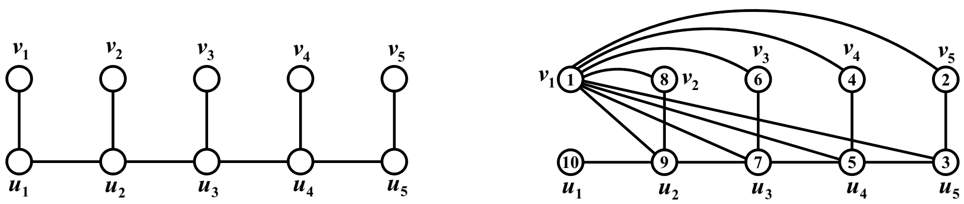

Figure 4. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.7. *The comb graph $G = P_5 \odot K_1$ and the graph G_v obtained by switching of vertex u_1 with its square divisor cordial labeling are shown in the Figure 5.*

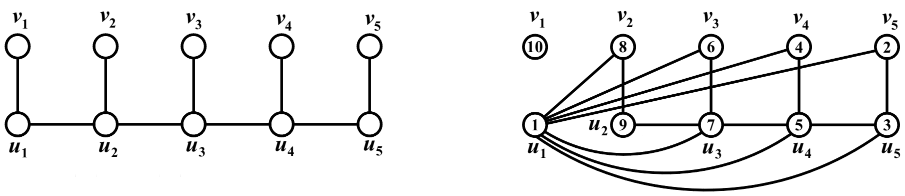

Figure 5. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.8. *The comb graph $G = P_5 \odot K_1$ and the graph G_v obtained by switching of vertex u_2 with its square divisor cordial labeling are shown in the Figure 6.*

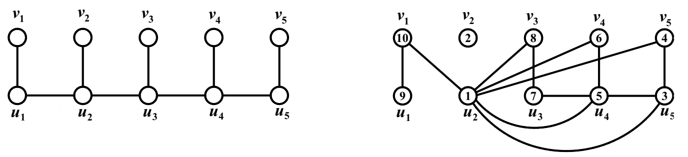

Figure 6. **The graph G and square divisor cordial labeling of graph G_v.**

Theorem 2.9. *The graph G_v obtained by switching of a vertex in the crown $C_n \odot K_1$ is square divisor cordial.*

Proof. Let $G = C_n \odot K_1$ be the crown with the vertex set $V(G) = \{v_i, u_i : 1 \leq i \leq n\}$, where v_i and u_i are vertices of degree one and three for all $i = 1, 2, \ldots, n$. Let G_v be the graph obtained by switching of a vertex v in G. The proof is divided into following two cases:

Case 1: Switching of the vertex of degree one.

Without loss of generality we are assuming that the switched pendant vertex is v_1. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n - 3$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n\}$ as follows:

$$f(v_1) = 1$$

$$f(u_i) = \begin{cases} 2i; & 1 \leq i \leq n-1 \\ 2n-1; & i = n \end{cases}$$

$$f(v_i) = \begin{cases} 2i-1; & 2 \leq i \leq n-1 \\ 2n; & i = n \end{cases}$$

In view of the above defined labeling pattern we have $e_f(0) = 2n - 2$, $e_f(1) = 2n - 1$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 2: Switching of the vertex of degree three.

Without loss of generality we are assuming that the switched vertex is u_1. We note that $|V(G_v)| = 2n$ and $|E(G_v)| = 4n - 7$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n\}$ as follows:

$$f(u_1) = 1, \ f(u_i) = 2i; \ 2 \leq i \leq n.$$

$$f(v_1) = 2, \ f(v_i) = 2i - 1; \ 2 \leq i \leq n.$$

In view of the above defined labeling pattern we have $e_f(0) = 2n - 3$ and $e_f(1) = 2n - 4$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph G_v obtained by switching of a vertex in the crown $C_n \odot K_1$ is square divisor cordial. □

Illustration 2.10. *The crown $G = C_7 \odot K_1$ and the graph G_v obtained by switching of vertex v_1 with its square divisor cordial labeling are shown in the Figure 7.*

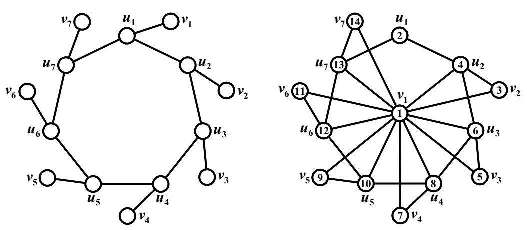

Figure 7. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.11. *The crown $G = C_7 \odot K_1$ and the graph G_v obtained by switching of vertex u_1 with its square divisor cordial labeling are shown in the Figure 8.*

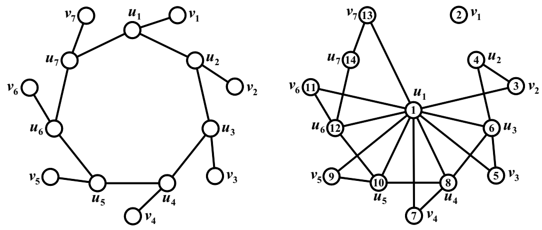

Figure 8. **The graph G and square divisor cordial labeling of graph G_v.**

Theorem 2.12. *The graph G_v obtained by switching of a vertex in the armed crown AC_n is square divisor cordial.*

Proof. Let $G = AC_n$ be the armed crown with the vertex set $V(G) = \{v_i, w_i, u_i : 1 \leq i \leq n\}$, where v_i, w_i and u_i are vertices of degree one, two and three respectively for all $i = 1, 2, \dots, n$. Let G_v be the graph obtained by switching of a vertex v in G. The proof is divided into following three cases.

Case 1: Switching of the vertex of degree one.

Without loss of generality we are assuming that the switched pendant vertex is v_1. We note that $|V(G_v)| = 3n$ and $|E(G_v)| = 6n - 3$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \dots, 3n\}$ as follows:

$$f(v_1) = 1.$$

$$f(u_i) = 3i;\ 1 \leq i \leq n$$

$$f(v_i) = f(u_i) - 2;\ 2 \leq i \leq n$$

$$f(w_i) = f(u_i) - 1;\ 1 \leq i \leq n$$

In view of the above defined labeling pattern we have $e_f(0) = 3n - 2$, $e_f(1) = 3n - 1$ if $3n \equiv 0(mod\ 9)$ and $e_f(0) = 3n - 1$, $e_f(1) = 3n - 2$ if $3n \not\equiv 0(mod\ 9)$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 2: Switching of the vertex of degree two.

Without loss of generality we are assuming that the switched vertex is w_1. We note that $|V(G_v)| = 3n$ and $|E(G_v)| = 6n-5$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \dots, 3n\}$ as follows:

$$f(w_1) = 1,\ f(u_i) = 3i;\ 1 \leq i \leq n$$

$$f(v_1) = 2,\ f(v_i) = f(u_i) - 2;\ 2 \leq i \leq n$$

$$f(w_i) = f(u_i) - 1;\ 2 \leq i \leq n$$

In view of the above defined labeling pattern we have $e_f(0) = 3n - 3$, $e_f(1) = 3n - 2$ if $3n \equiv 0(mod\ 9)$ and $e_f(0) = 3n - 2$, $e_f(1) = 3n - 3$ if $3n \not\equiv 0(mod\ 9)$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 3: Switching of the vertex of degree three.

Without loss of generality we are assuming that the switched vertex is u_1. We note that $|V(G_v)| = 3n$ and $|E(G_v)| = 6n-7$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \dots, 3n\}$ as follows:

$$f(u_1) = 1,\ f(w_1) = 2,\ f(v_1) = 3$$

$$f(u_i) = 3i;\ 2 \le i \le n$$

$$f(v_i) = f(u_i) - 2;\ 2 \le i \le n$$

$$f(w_i) = f(u_i) - 1;\ 2 \le i \le n$$

In view of the above defined labeling pattern we have $e_f(0) = 3n - 3$ and $e_f(1) = 3n - 4$. Thus, $|e_f(0) - e_f(1)| \le 1$. Hence, the graph G_v obtained by switching of a vertex in the armed crown AC_n is square divisor cordial. □

Illustration 2.13. *The armed crown $G = AC_5$ and the graph G_v obtained by switching of vertex v_1 with its square divisor cordial labeling are shown in the Figure 9.*

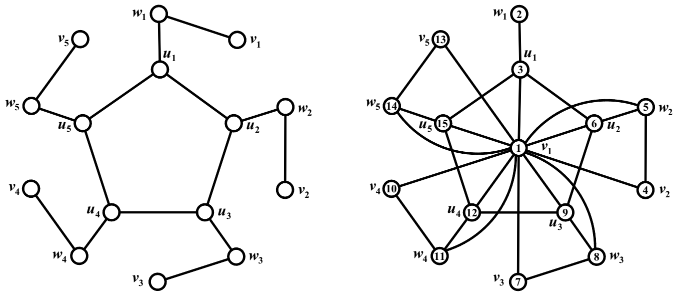

Figure 9. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.14. *The armed crown $G = AC_5$ and the graph G_v obtained by switching of vertex w_1 with its square divisor cordial labeling are shown in the Figure 10.*

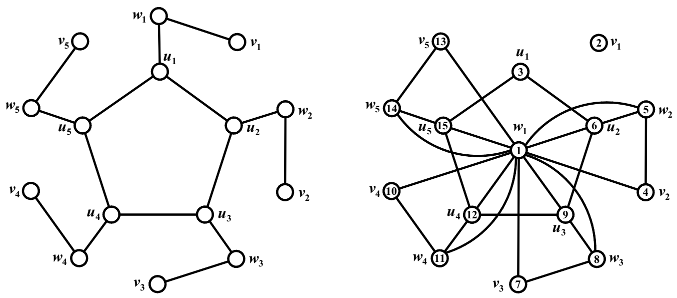

Figure 10. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.15. *The armed crown $G = AC_5$ and the graph G_v obtained by switching of vertex u_1 with its square divisor cordial labeling are shown in the Figure 11.*

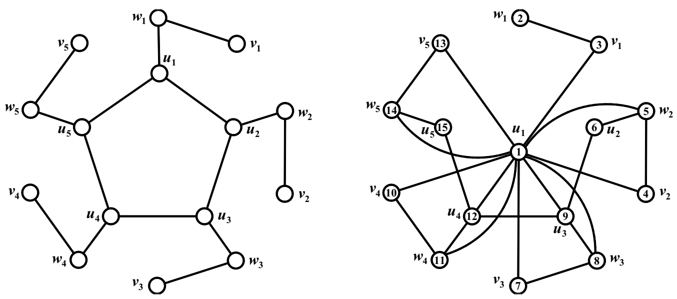

Figure 11. **The graph G and square divisor cordial labeling of graph G_v.**

Theorem 2.16. *The graph G_v obtained by switching of a vertex except apex vertex in the helm H_n is square divisor cordial.*

Proof. Let $G = H_n$ be the helm with the vertex set $\{u_0, u_i, v_i : 1 \le i \le n\}$, where u_0 is the apex vertex, u_i is pendant vertex and v_i is vertex of degree four for all $i = 1, 2, \dots, n$. Let G_v be the graph obtained by switching of a vertex v in G. The proof is divided into following two cases.

Case 1: Switching of the pendant vertex.

Without loss of generality we are assuming that the switched pendant vertex is v_1. We note that $|V(G_v)| = 2n+1$ and $|E(G_v)| = 5n-2$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \dots, 2n+1\}$ as follows:

$$f(v_1) = 1,\ f(u_1) = 2n+1$$

$$f(u_i) = 2i;\ 2 \le i \le n$$

$$f(v_i) = f(u_i) - 1;\ 2 \le i \le n$$

In view of the above defined labeling pattern we have $e_f(0) = \left\lceil \frac{5n-2}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{5n-2}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \le 1$.

Case 2: Switching of the vertex of the degree four.

Without loss of generality we are assuming that the switched pendant vertex is u_1. We note that $|V(G_v)| = 2n+1$ and $|E(G_v)| = 5n-8$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \dots, 2n+1\}$ as follows:

$$f(u_1) = 1,\ f(v_1) = 2n+1$$

$$f(u_i) = 2i;\ 2 \le i \le n$$

$$f(v_i) = f(u_i) - 1;\ 2 \le i \le n$$

In view of the above defined labeling pattern we have $e_f(0) = \left\lceil \frac{5n-8}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{5n-8}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \le 1$.

Hence, the graph G_v obtained by switching of a vertex except apex vertex in the helm H_n is square divisor cordial. $\square$

Illustration 2.17. *The helm $G = H_6$ and the graph G_v obtained by switching of vertex v_1 with its square divisor cordial labeling are shown in the Figure 12.*

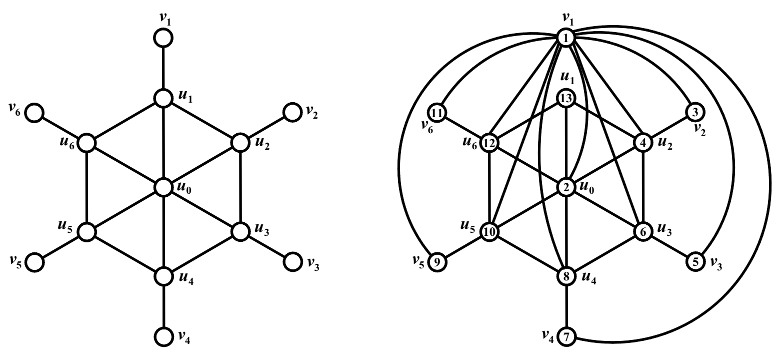

Figure 12. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.18. *The helm $G = H_6$ and the graph G_v obtained by switching of vertex u_1 with its square divisor cordial labeling are shown in the Figure 13.*

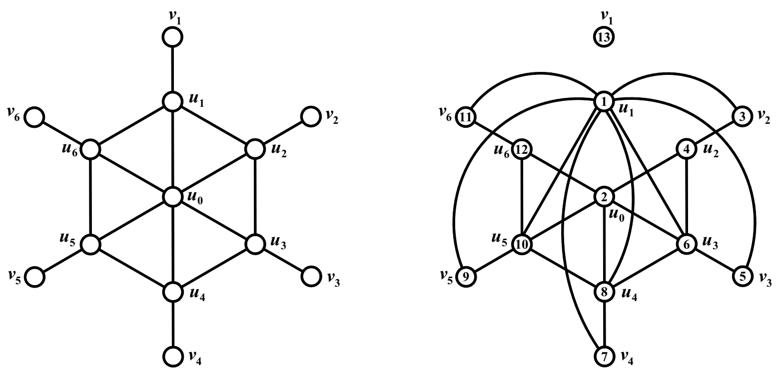

Figure 13. **The graph G and square divisor cordial labeling of graph G_v.**

Theorem 2.19. *The graph G_v obtained by switching of a vertex except apex vertex in the gear graph G_n is square divisor cordial.*

Proof. Let $G = G_n$ be the gear graph with the vertex set $V(G) = \{u_0, u_i, v_i : 1 \leq i \leq n\}$, where u_0 is the apex vertex, v_i and u_i are vertices of degree two and three respectively for all $i = 1, 2, \ldots, n$. Let G_v be the graph obtained by switching of a vertex v in G. The proof is divided into following two cases.

Case 1: Switching of the vertex of degree two.

Without loss of generality we are assuming that the switched vertex is v_1. We note that $|V(G_v)| = 2n+1$ and $|E(G_v)| = 5n-4$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n+1\}$ as follows:

$$f(v_1) = 1$$

$$f(u_0) = 2,\ f(u_1) = 2n+1$$

$$f(u_i) = 2i;\ 2 \leq i \leq n$$

$$f(v_i) = 2i-1;\ 2 \leq i \leq n$$

In view of the above defined labeling pattern we have $e_f(0) = \left\lceil \frac{5n-4}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{5n-4}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Case 2: Switching of the vertex of degree three.

Without loss of generality we are assuming that the switched vertex is u_1. We note that $|V(G_v)| = 2n+1$ and $|E(G_v)| = 5n-6$. Define vertex labeling $f : V(G_v) \longrightarrow \{1, 2, \ldots, 2n+1\}$ as follows:

$$f(u_1) = 1, f(u_0) = 2$$

$$f(u_i) = 2i;\ 2 \leq i \leq n$$

$$f(v_i) = 2i+1;\ 1 \leq i \leq n$$

In view of the above defined labeling pattern we have $e_f(0) = \left\lceil \frac{5n-6}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{5n-6}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph G_v obtained by switching of a vertex except apex vertex in the gear graph G_n is square divisor cordial. □

Illustration 2.20. *The gear graph $G = G_6$ and the graph G_v obtained by switching of vertex v_1 with its square divisor cordial labeling are shown in the Figure 14.*

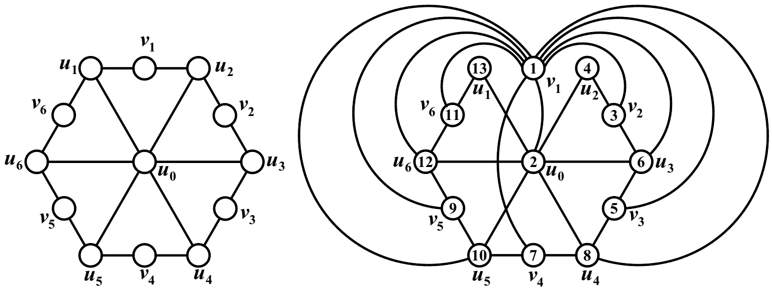

Figure 14. **The graph G and square divisor cordial labeling of graph G_v.**

Illustration 2.21. *The gear graph $G = G_6$ and the graph G_v obtained by switching of vertex u_1 with its square divisor cordial labeling are shown in the Figure 15.*

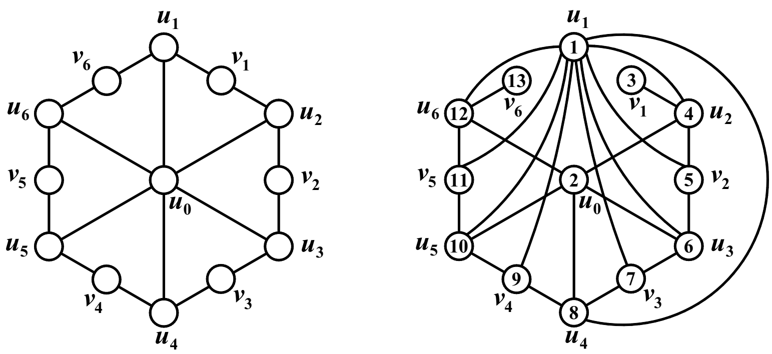

Figure 15. **The graph G and square divisor cordial labeling of graph G_v.**

3. Concluding Remark

Here, we have derived six new results related to the graph operation vertex switching for the square divisor cordial labeling technique. To explore some new square divisor cordial graphs with respect to other graph operations is an open problem.

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