International Journal of
Mathematics And its Applications
ISSN: 2347-1557

Int. J. Math. And Appl., **11(3)**(2023), 11–20
Available Online: http://ijmaa.in

Multiplicative Domination Nirmala Indices of Graphs

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Abstract

In this study, we introduce the multiplicative domination Nirmala index, multiplicative modified domination Nirmala index of a graph. Furthermore, we compute these multiplicative domination Nirmala indices for some standard graphs, French windmill graphs, friendship graphs and book graphs.

Keywords: multiplicative domination Nirmala index; multiplicative modified domination Nirmala index; graph.
2020 Mathematics Subject Classification: 05C10, 05C69.

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For undefined terms and notations, we refer the books [1, 2]. Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [3]. The domination degree $d_d(u)$ [4] of a vertex u in a graph G is defined as the number of minimal dominating sets of G which contains u. Recently, some domination indices were studied in [5, 6, 7, 8]. In [9], Kulli introduced the domination Nirmala index of a graph G and it is defined as

$$DN(G) = \sum_{uv \in E(G)} \sqrt{d_d(u) + d_d(v)}$$

where $d_d(u)$ is the domination degree of a vertex u in G.
The modified domination Nirmala index [9] of a graph G is defined as

$${}^{m}DN(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_d(u) + d_d(v)}}.$$

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We define the multiplicative domination Nirmala index of a graph G as

$$DNII(G) = \prod_{uv \in E(G)} \sqrt{d_d(u) + d_d(v)}.$$

We also define the multiplicative modified domination Nirmala index of a graph G as

$${}^{m}DNII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_d(u) + d_d(v)}}.$$

Recently, some Nirmala indices were studied in [10-33].

In this paper, the multiplicative domination Nirmala index, multiplicative modified domination Nirmala index of some standard graphs, French windmill graphs, book graphs are computed.

2. Multiplicative Domination Nirmala Index

2.1 Results for Some Standard Graphs

Proposition 2.1. *If K_n is a complete graph with n vertices, then*

$$DNII(K_n) = \left(\sqrt{2}\right)^{\frac{n(n-1)}{2}}.$$

Proof. If K_n is a complete graph, then $d_d(u) = 1$. From definition, we have

$$\begin{aligned} DNII(K_n) &= \prod_{uv \in E(K_n)} \sqrt{d_d(u) + d_d(v)} \\ &= \left(\sqrt{1+1}\right)^{\frac{n(n-1)}{2}} \\ &= \left(\sqrt{2}\right)^{\frac{n(n-1)}{2}}. \end{aligned}$$

□

Proposition 2.2. *If S_{n+1} is a star graph with $d_d(u) = 1$, then*

$$DNII(S_{n+1}) = \left(\sqrt{2}\right)^{n}.$$

Proposition 2.3. *If $S_{p+1,q+1}$, is a double star graph with $d_d(u) = 2$, then*

$$DNII\left(S_{p+1,q+1}\right) = (2)^{p+q+1}.$$

Proposition 2.4. *Let $K_{m,n}$ be a complete bipartite graph with $2 \le m \le n$. Then*

$$DNII(K_{m,n}) = \left(\sqrt{m+n+2}\right)^{mn}.$$

Proof. Let $G = K_{m,n}$, $m, n \geq 2$ with

$$d_d(u) = \begin{cases} m+1 \\ n+1 & \text{for all } u \in V(G) \end{cases}$$

From definition, we have

$$\begin{aligned} DNII(K_{m,n}) &= \prod_{uv \in E(K_{m,n})} \sqrt{d_d(u) + d_d(v)} \\ &= \left(\sqrt{(m+1)+(n+1)}\right)^{mn} \\ &= \left(\sqrt{m+n+2}\right)^{mn}. \end{aligned}$$

□

2.2 Results for French Windmill Graphs

The French windmill graph F_n^m is the graph obtained by taking $m \geq 3$ copies of K_n, $n \geq 3$ with a vertex in common. The graph F_n^m is presented in Figure 1. The French windmill graph F_3^m is called a friendship graph.

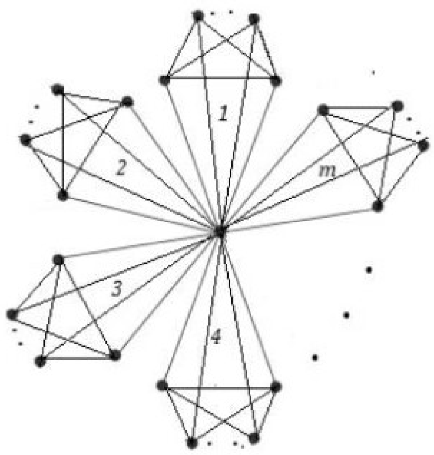

Figure 1: French windmill graph F_n^m

Let F be a French windmill graph F_n^m. Then

$$d_d(u) = \begin{cases} 1, & \text{if } u \text{ is the center vertex;} \\ (n-1)^{m-1}, & \text{otherwise.} \end{cases}$$

Theorem 2.5. *Let F be a French windmill graph F_n^m. Then*

$$DNII(F) = \left(\sqrt{1+(n-1)^{(m-1)}}\right)^{m(n-1)} \times \left(\sqrt{2(n-1)^{(m-1)}}\right)^{[(mn(n-1)/2)-m(n-1)]}.$$

Proof. In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$\begin{aligned}
DNII(F) &= \prod_{uv\in E(F)} \sqrt{d_d(u)+d_d(v)} \\
&= \prod_{uv\in E_1(F)} \sqrt{d_d(u)+d_d(v)} \times \prod_{uv\in E_2(F)} \sqrt{d_d(u)+d_d(v)} \\
&= \left(\sqrt{1+(n-1)^{(m-1)}}\right)^{m(n-1)} \times \left(\sqrt{(n-1)^{(m-1)}+(n-1)^{(m-1)}}\right)^{[(mn(n-1)/2)-m(n-1)]} \\
&= \left(\sqrt{1+(n-1)^{(m-1)}}\right)^{m(n-1)} \times \left(\sqrt{2(n-1)^{(m-1)}}\right)^{[(mn(n-1)/2)-m(n-1)]}.
\end{aligned}$$

□

Corollary 2.6. *Let F_3^m be a friendship graph. Then*

$$DNII(F_3^m) = \left(\sqrt{1+2^{(m-1)}}\right)^{2m} \times \left(\sqrt{2^m}\right)^{m}.$$

2.3 Results for GoK_p

Theorem 2.7. *Let $H = GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then*

$$DNII(H) = \left(\sqrt{2(p+1)^{n-1}}\right)^{\frac{1}{2}(2m+np^2+np)}.$$

Proof. If $H = GoK_p$, then $d_d(u) = (p+1)^{n-1}$. In K_p, there are $\frac{p(p-1)}{2}$ edges. Thus H has $\frac{1}{2}(2m+np^2+np)$ edges. Thus

$$\begin{aligned}
DNII(H) &= \prod_{uv\in E(H)} \sqrt{d_d(u)+d_d(v)} \\
&= \left(\sqrt{(p+1)^{n-1}+(p+1)^{n-1}}\right)^{\frac{1}{2}(2m+np^2+np)} \\
&= \left(\sqrt{2(p+1)^{n-1}}\right)^{\frac{1}{2}(2m+np^2+np)}
\end{aligned}$$

□

2.4 Results for B_n

The book graph B_n, $n \geq 3$, is a cartesian product of star S_{n+1} and path P_2. For B_n, $n \geq 3$, we have

$$d_d(u) = \begin{cases} 3, & \text{if } u \text{ is the center vertex;} \\ 2^{n-1}+1, & \text{otherwise.} \end{cases}$$

Theorem 2.8. *If B_n, $n \geq 3$, is a book graph, then*

$$DNII\left(B_n\right) = \left(\sqrt{6}\right)^1 \times \left(\sqrt{4+2^{n-1}}\right)^{2n} \times \left(\sqrt{2\left(2^{n-1}+1\right)}\right)^n.$$

Proof. In B_n, there are three types of edges as follow:

$$E_1 = \{uv \in E(B_n) | d_d(u) = d_d(v) = 3\}, \qquad |E_1| = 1$$
$$E_2 = \{uv \in E(B_n) | d_d(u) = 3, d_d(v) = 2^{n-1}+1\}, \qquad |E_2| = 2r$$
$$E_3 = \{uv \in E(B_n) | d_d(u) = d_d(v) = 2^{n-1}+1\}, \qquad |E_3| = r$$

By definition, we have

$$\begin{aligned} DNII\left(B_n\right) &= \prod_{uv \in E(B_n)} \sqrt{d_d\left(u\right) + d_d\left(v\right)} \\ &= \left(\sqrt{3+3}\right)^1 \times \left(\sqrt{3+\left(2^{n-1}+1\right)}\right)^{2n} \times \left(\sqrt{\left(2^{n-1}+1\right)+\left(2^{n-1}+1\right)}\right)^n \\ &= \left(\sqrt{6}\right)^1 \times \left(\sqrt{4+2^{n-1}}\right)^{2n} \times \left(\sqrt{2\left(2^{n-1}+1\right)}\right)^n \end{aligned}$$

□

3. Multiplicative Modified Domination Nirmala Index

3.1 Results for Some Standard Graphs

Proposition 3.1. *If K_n is a complete graph with n vertices, then*

$$^{m}DNII\left(K_n\right) = \left(\frac{1}{\sqrt{2}}\right)^{\frac{n(n-1)}{2}}.$$

Proof. If K_n is a complete graph, then $d_d(u) = 1$. From definition, we have

$$\begin{aligned} ^{m}DNII\left(K_n\right) &= \prod_{uv \in E(K_n)} \frac{1}{\sqrt{d_d\left(u\right) + d_d\left(v\right)}} \\ &= \left(\frac{1}{\sqrt{1+1}}\right)^{\frac{n(n-1)}{2}} \\ &= \left(\frac{1}{\sqrt{2}}\right)^{\frac{n(n-1)}{2}} \end{aligned}$$

□

Proposition 3.2. *If S_{n+1} is a star graph with $d_d(u) = 1$, then*

$$^{m}DNII\left(S_{n+1}\right) = \left(\frac{1}{\sqrt{2}}\right)^{n}.$$

Proposition 3.3. *If $S_{p+1,q+1}$ is a double star graph with $d_d(u) = 2$, then*

$$^{m}DNII\left(S_{p+1,q+1}\right) = \left(\frac{1}{2}\right)^{p+q+1}.$$

Proposition 3.4. *Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$. Then*

$$^{m}DNII\left(K_{m,n}\right) = \left(\frac{1}{\sqrt{m+n+2}}\right)^{mn}.$$

3.2 Results for French Windmill Graphs

Theorem 3.5. *Let F be a French windmill graph F_n^m. Then*

$$^{m}DNII\left(F\right) = \left(\frac{1}{\sqrt{1+(n-1)^{(m-1)}}}\right)^{m(n-1)} \times \left(\frac{1}{\sqrt{2\left(n-1\right)^{(m-1)}}}\right)^{[(mn(n-1)/2)-m(n-1)]}.$$

Proof. In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$\begin{aligned}
^{m}DNII\left(F\right) &= \prod_{uv \in E(F)} \frac{1}{\sqrt{d_d\left(u\right)+d_d\left(v\right)}} \\
&= \prod_{uv \in E_1(F)} \frac{1}{\sqrt{d_d\left(u\right)+d_d\left(v\right)}} \times \prod_{uv \in E_2(F)} \frac{1}{\sqrt{d_d\left(u\right)+d_d\left(v\right)}} \\
&= \left(\frac{1}{\sqrt{1+(n-1)^{(m-1)}}}\right)^{m(n-1)} \times \left(\frac{1}{\sqrt{(n-1)^{(m-1)}+(n-1)^{(m-1)}}}\right)^{[(mn(n-1)/2)-m(n-1)]} \\
&= \left(\frac{1}{\sqrt{1+(n-1)^{(m-1)}}}\right)^{m(n-1)} \times \left(\frac{1}{\sqrt{(n-1)^{(m-1)}+(n-1)^{(m-1)}}}\right)^{[(mn(n-1)/2)-m(n-1)]} \\
&= \left(\frac{1}{\sqrt{1+(n-1)^{(m-1)}}}\right)^{m(n-1)} \times \left(\frac{1}{\sqrt{2\left(n-1\right)^{(m-1)}}}\right)^{[(mn(n-1)/2)-m(n-1)]}
\end{aligned}$$

□

Corollary 3.6. *Let F_3^m be a friendship graph. Then*

$$^{m}DSNII\left(F_3^m\right) = \left(\frac{1}{\sqrt{1+2^{(m-1)}}}\right)^{2m} \times \left(\frac{1}{\sqrt{2^m}}\right)^{m}.$$

3.3 Results for GoK_p

Theorem 3.7. *Let $H = GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then*

$$^{m}DNII\,(H) = \left(\frac{1}{\sqrt{2\,(p+1)^{n-1}}} \right)^{\frac{(2m+np^2+np)}{2}}.$$

Proof. If $H = GoK_p$, then $d_d\,(u) = (p+1)^{n-1}$. In K_p, there are $\frac{p(p-1)}{2}$ edges. Thus H has $\frac{1}{2}(2m + np^2 + np)$ edges. Hence

$$\begin{aligned} ^{m}DNII\,(F) &= \prod_{uv \in E(F)} \frac{1}{\sqrt{d_d\,(u) + d_d\,(v)}} \\ &= \left(\frac{1}{\sqrt{(p+1)^{n-1} + (p+1)^{n-1}}} \right)^{\frac{(2m+np^2+np)}{2}} \\ &= \left(\frac{1}{\sqrt{2\,(p+1)^{n-1}}} \right)^{\frac{(2m+np^2+np)}{2}} \end{aligned}$$

□

3.4 Results for B_n

The book graph B_n, $n \geq 3$, is a cartesian product of star S_{n+1} and path P_2. For B_n, $n \geq 3$, we have

$$d_d\,(u) = \begin{cases} 3, & \text{if } u \text{ is center vertex,} \\ 2^{n-1}+1, & \text{otherwise.} \end{cases}$$

Theorem 3.8. *If B_n, $n \geq 3$, is a book graph, then*

$$^{m}DNII\,(B_n) = \frac{1}{\sqrt{6}} \times \left(\frac{1}{\sqrt{4+2^{n-1}}} \right)^{2n} \times \left(\frac{1}{\sqrt{2\,(2^{n-1}+1)}} \right)^{n}.$$

Proof. In B_n, there are three types of edges as follow:

$$\begin{aligned} E_1 &= \{uv \in E(B_n) | d_d(u) = d_d(v) = 3\}, & |E_1| &= 1 \\ E_2 &= \{uv \in E(B_n) | d_d(u) = 3, d_d(v) = 2^{n-1}+1\}, & |E_2| &= 2r \\ E_3 &= \{uv \in E(B_n) | d_d(u) = d_d(v) = 2^{n-1}+1\}, & |E_3| &= r \end{aligned}$$

By definition, we have

$$^{m}DNII\,(B_n) = \prod_{uv \in E(B_n)} \frac{1}{\sqrt{d_d\,(u) + d_d\,(v)}}$$

$$= \left(\frac{1}{\sqrt{3+3}}\right)^1 \times \left(\frac{1}{\sqrt{3+(2^{n-1}+1)}}\right)^{2n} \times \left(\frac{1}{\sqrt{(2^{n-1}+1)+(2^{n-1}+1)}}\right)^n$$
$$= \frac{1}{\sqrt{6}} \times \left(\frac{1}{\sqrt{4+2^{n-1}}}\right)^{2n} \times \left(\frac{1}{\sqrt{2\,(2^{n-1}+1)}}\right)^n$$

□

4. Conclusion

In this study, the multiplicative domination Nirmala index, multiplicative modified domination Nirmala index for some standard graphs, French windmill graphs, book graphs are determined.

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