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Graph Theory in Different Networks

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Abstract: This paper gives a brief description of graph theory and networks .Various papers based on graph theory related to electrical network, communication network , biological network, transportation network, computer science applications have been studied and an overview has been presented here. More over author tries to find out the scope of graph theoretical concepts in further research.

Keywords: Bipartite graph, Tree, Cutset, Communication Network, Biological Network, Electrical Network.

1. Introduction

Graph theory began with Euler who was asked to find a nice path across the seven Koningsberg bridges. Graph theory is the study of graphs which are mathematical structures used to model pair wise relation between objects. A graph G is an ordered triple consisting of a nonempty set $V(G)$ of vertices $E(G)$ disjoint from $V(G)$, of edges and an incidence function $\psi(G)$ that associates with each edge of G an unordered pair of vertices of G.

2. Graph Theoretical Terminologies

Definition 2.1. *A simple graph $G = (V, E)$ with vertex partition $V = \{V_1, V_2\}$ is called a bipartite graph if every edge of E joins a vertex in V_1 to a vertex in V_2. In general, a Bipartite graph has two sets of vertices, V_1 and V_2, in which every edge connect any vertex in set V_1 to any vertex in set V_2.*

Definition 2.2. *It is number of edges in a shortest path between Vertex U and Vertex V. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices and it is notated as $d(U, V)$.*

Definition 2.3. *Euler Circuit is a circuit in graph G which traverses every edge of graph exactly once. Euler Circuit is simply a closed path and called as Euler line.*

Definition 2.4. *A connected graph with no cycles is called a tree. The edges of a tree are known as branches.*

Definition 2.5. *Let G be a connected graph, then the sub-graph H of G is called a spanning tree of G if H is a tree and H contains all vertices of G.*

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Definition 2.6. *It is defined as a tree in which there is exactly one vertex of degree two and each of the remaining vertices are of degree one or three.*

Definition 2.7. *Let $G = (V, E)$ be a connected graph. A cut set is a set of edges whose removal from G leaves G disconnected.*

Definition 2.8. *The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph Vertex coloring is one of the most important concepts in graph theory and is used in many real time applications.*

Definition 2.9. *Let $G = (V, E)$ be a graph. A subset K of V is called a vertex covering of G, if every edge of G is incident with a vertex in K. A vertex K of graph G is said to be minimal vertex covering if no vertex can be deleted from K.*

Definition 2.10. *It is the least number of colors needed to color the graph. A graph that can be assigned a proper k-coloring is k-colorable, and it is k-chromatic if its chromatic number is exactly k.*

Definition 2.11. *The chromatic number of a planar graph is no greater than 4.*

Definition 2.12. *The node-incidence matrix (A-matrix) describes the way a circuit is connected. It is very important in computer simulation. The columns in A-matrix correspond to the branches; and the rows correspond to the nodes.*

Definition 2.13. *The basic cutset matrix (Q-matrix) describes the way the basic cutset is chosen. Each column corresponds to a branch and each row corresponds to a basic cutest.*

Definition 2.14. *The basic loop matrix (B-matrix) describes the way the basic loop is chosen. Each column corresponds to a branch and each row corresponds to a basic loop.*

3. Network

A network is simply a collection of connected objects which involves the movement or flow of some commodities such as electrical current, information, products and people. All sorts of systems, in the real world can be represented as a network. Some examples are internet, world wide web, social networks, networks of publications linked by citations, transportation networks, metabolic networks, electrical network, communication networks and so on. When all the elements in a network are replaced by lines with circles or dots at both ends resulting diagram is a graph of that network. Thereby topology of network can be studied by means of its graph. Some examples are electrical network, computer network, biological network, transportation network etc.

Various Network Measures: Degree k_i; which is the number of edges involving node i. Mean path length is the average shortest path between all node pairs. Network Diameter is the longest shortest path.

4. Review of Studies

N. Lakshmi Prasanna [3] in the paper Applications of Graph Labeling in Communication Networks addresses how the concept of graph labeling can be applied to network security, network addressing, channel assignment process, social networks etc. He explains the designing of fault tolerant systems with facility Graphs in which the network is represented in the form of a facility graph. In Automatic channel allocation for small wireless local area networks the correspondence between the channels and the graph is that as the channels listen the messages in regular intervals as the same way the labeling algorithm

should be kept running at regular intervals. He explains how to avoid stealth worms by using vertex covering algorithm.Then an optimal solution is found for worm propagation. Moreover he explains Analysis of Communication Efficiency in sensor networks with Voronoi Graph, Reduction of the Complexity of Algorithms in Compression Networks, Graph Labeling in Communication Relevant to Ad hoc Networks, Effective Communication in Social Networks by Using Graphs etc.

Rishi Pal Singh [5] Application of Graph Theory in Computer Science and Engineering, to analyze the graph theoretic application, two problem areas are considered. Classical problem and Problems from applications, such as data base designing, software engineering, network system, computer hardware, data structure, image processing, data mining, website designing, etc.

B.Sadavare [6] in their review of application of graph theory for network, explores different shortest path algorithms such as Dijkstr a's algorithm, Bellman ford Algorithm and Warshall's algorithm. They reviewed a number of studies related to finding shortest path in different networks like cable network ,water supply system network, transportation network etc.

S.G Shirinivas [9] in their study applications of graph theory in heterogeneous fields mainly focuses on the computer science applications uses graph theoretical concepts.They gave a brief description of algorithms and graph theoretic languages. He says that to identify the chemical components automatically computer language called Dendral has been developed. He explains graph models in Operation Research and chemistry, graph coloring in computer science, job scheduling , aircraft scheduling, bioprocessor tasks and automatic channel allocation issue in small wire less LAN by modelling the network in the form of a graph and solve it using graph coloring methodology. He explained about the clustering of web documents using graph models. Author gives an overview of graph based classification of finger prints.

Berdewad [1] in their study presents a graph model to represent a circuit network. He also presented a graph representation of matrix. In this paper he focuses on application of graph theory to electrical network analysis and matrix approach to electrical network.Through this study he concluded that graph theory has more practical applications in solving electrical network problems.

Samaila Abdhulla [7], circuit models of graphs are represented in logical connection method. Here they formulates matrix method of adjacency and incidence and application of truth table. They conclude that a circuit network can be represented by different methods such as circuit representation, graph model and matrix method by using logical truth table.

Shamim Ahmed [8] explains some applications of graph coloring. By using this coloring technique author explains how an art gallery can be guarded by video cameras. Every part of the gallery must be visible to at least one of them.To place the camera at strategic positions graph coloring techniques are used. In addition to this, they explored the wide application of graph coloring in physical layout segmentation, round robin spot scheduling, air craft scheduling, bi processor tasks, timetable preparation, GSM mobile phone networks, etc.

5. Graph Theory in Computer Network

Graph theoretical concepts has wide scope in computer science areas such as website designing, network security,communication network and so on. A data structure can be designed in the form of a tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using graph concepts. In the same way the most important concept of graph coloring can be utilized in resource allocation, scheduling and so on.

Data Mining: Data mining is process of perceiving required information from huge data with the help of various methods. Mostly the data we deal with in data science can be shaped as graphs. These graphs can be mined utilizing known algorithms and various techniques in graph theory helps to understand them in better way. Graph is captivating model of data backed with a strong theory and a set of quality algorithms to solve related problems.

Website Designing: Graphs can be used to model website designing. Google's successful Web search algorithms are based on the WWW graph, in which web pages are represented by vertices and hyperlinks between them are represented by edges in the graph, thereby we get the web graph. In which a complete bipartite graph plays a vital role that each vertex representing a type of object is connected to every vertex representing other kind of objects.

Figure 1. **Facebook representation as graph theory**

There are many advantages of using graph representation in website development such as, Searching and community discovery, role of directed graph in web site utility evaluation and link structure and finding all connected component and provide easy detection.

Figure 2. **Graphical representation of Google map**

Computer Network Security: To simulate the propagation of stealth worms on large computer networks and to develop strategies to protect the network from virus attacks, scientists Eric Filiol and the French Navy ESCANSIC used the vertex cover algorithm. The idea is to find an optimal solution for worm propagation and an optimal solution for designing the network design strategy .The main aim is to find a minimum vertex cover in the graph whose vertices are the route servers and whose edges are the connection between the routing servers.

Graphical Representation of Communication Network: Graph theory can be used to represent communication networks, which is a collection of terminals, links and nodes which enables communication between users of terminals.

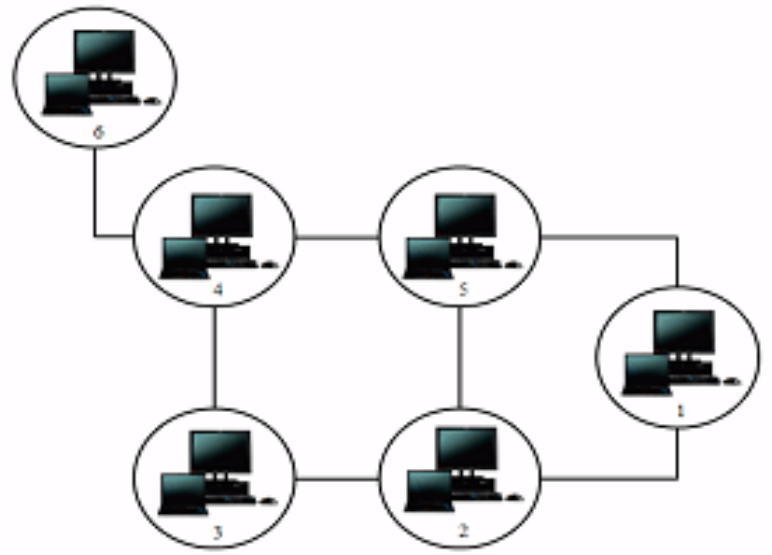

Figure 3. **Graphical representation of Computer network**

Every communication network has three basic components, such as terminals, processors and transmission channels. This network transmits packets of .data between computers telephones or other devices. Graph theory helps to model the communication network by vertices as terminals, processors and edges represent transmission channels through which data flows. Thus the data packets can be transmitted from input to output through a sequence of switches joined by directed edges In communication problem, an edge between two nodes is used to denote the presence of a communication channel between two end points. The Internet is one example of such a network, where the nodes correspond to routers and edges correspond to wires or ?ber between pairs of routers.

To provide fast communication in sensor networks a special kind of graph labelling known as radio labelling can be used which is defined as Let $G = (V(G), E(G))$ be a connected graph. A radio labelling for G is a One one function F from $V(G)$ to N, set of natural numbers union $\{0\}$, in a way that, for any two vertices u and v, $|f(u)f(v)| \geq D(G) - d(u, v) + 1$.

Communication Network as Binary Tree: Communication network can be represented as a complete binary tree in which squares as terminals ,sources or destinations for packets of data, circles as switches that directs the packets through network. There is a unique path between every pair of vertices in an undirected tree.so the switch can receive packets and forward in the complete binary tree in a analogous directed path.

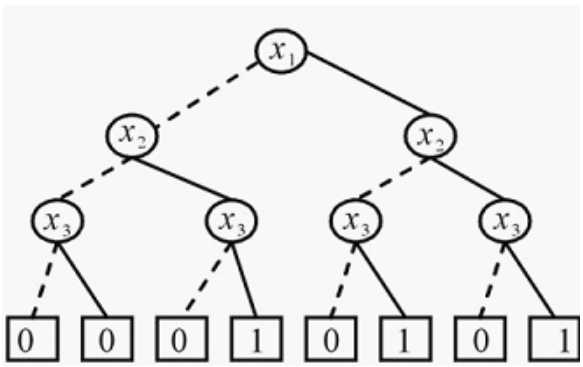

Figure 4. **Communication network as binary tree**

GSM Mobile Phone Network and Four Color Theorem: The first GSM network was launched in 1991 by Radiolinja in Finland with joint technical infrastructure maintenance from Ericsson. GSM is a cellular network and its entire geographical

range is divided into hexagonal cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the immediate vicinity. GSM networks operate in only four different frequency ranges it is because the map of the cellular regions can be properly colored by using only four different colors. So, the vertex coloring algorithm may be used for assigning at most four different frequencies for any GSM mobile phone network.

6. Graph Theory in Biological Network

Graphs are used to represent relationships among species on different physical and micro-biological criteria. For example, the evolutionary relationships among the existing species are expressed in a tree structure called phylogenetic tree. Graphs are also used in problems to analyze biological data.Ecological landscapes can be modeled using graphs. V-represents habitat patches or roosts for birds, E-represent movement between the patches. Graphs help study the structural organization of a landscape, importance of certain nodes, degree of connectivity between them. These properties affect the spread of disease, the vulnerability to disturbance of the landscape, and other issues related to conservation.

The Swiss biochemist Frederich Miescher first observed DNA in the late 1869. DNA is Deoxyribo Nuclic Acid is found in every cell of the organism instructions an organism needs to develop, live and produce.Its sequencing and fragment assembly is the problem of reconstructing full strands of DNA based on the pieces of data recorded. The Eulerian circuit in graph theory has a role to solve the problem of DNA fragment assembly. Graphs are widely used in medical ultra sound projection. It is widely used in medical ultra sound projection and it is non traumatic to human body. It provides real time display and 3d graph based segmentation algorithm can be used to construct a 3D graph .It can generate a set of minimum spanning trees each corresponds to a 3D sub region.Graph theoretical approaches can be used to investigate the organisational principals of brain network with a variety of images including structural MRI, Diffusion MRI and EEG/MEG. Once the brain connectivity information is extracted from neuro imaging data graph, theoretical approaches can be further applied to model brain networks. Brain architecture can be depicted as graphs composed of nodes representing regions and edges representing structural or functional connectivity among the nodes.

7. Graph Theory in Electrical Network

The idea behind the use of graph theory in electrical network is originated with G. Kirchoff in 1847 and is improved by I.C. Monwell in 1892 and a milestone in graph theoretic analysis of electrical network was achieved by W.S. Percieval.

In electrical engineering term branch is used for edge, node for vertex and loop for circuit. Trees with directed edges are of great importance in electrical and computer programming. The basic cutest and loop matrices can be used to formulate the independent Kirchhoff's law equation. This will give much more efficient solution to circuit analysis problems. The importance of basic cutsets is the formulation of independent KCL equations: The importance of basic loops is the formulation of independent KVL equations.

A different choice of tree gives a different set of basic cutsets and basic loops. The set of independent KCL and KVL equations found is not unique. But any set of independent KCL and KVL equations gives essentially the same information about the circuit. So, it doesn't matter which tree is chosen. Once a tree is chosen, a set of independent KCL and KVL equations is found. Any other KCL or KVL equation is derivable from the independent set.Once the tree is chosen from an electrical network, there are two approaches to solve the problem. cutset and loop approach instead of mesh and nodal analysis. Cutset voltage approach aim to find all tree voltages and loop current approach aim to find all cotree currentsSystem of

equations formed by KCL and KVL depends on the structure of graph of the circuit. They depends only on the way the circuit elements are inter connected. Thus several results in electrical network are graph theoretic in nature.

8. Conclusion

The main aim of this paper is to investigate the importance and the relevance of the graph theoretical concepts in different networks. By the reviewed studies author can find out the scope of further research in application of graph theoretic concepts in electrical, computer, biological and transportation networks. As a role of graphs it is showing its easiness to simulate each real life situations by the help of a graph.

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