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Three-Dimensional Couette Flow Through Porous Medium Between Two Infinite Horizontal Parallel Porous Plates

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Abstract: Aim of the paper is to investigate heat transfer through Couette flow of a viscous incrompessible fluid between two infinite horizontal parallel porous flat plates. The lower plate is stationary and the upper plate is moving with uniform velocity U subjected to transverse sinusoidal injection through lower plate and uniform suction through upper plate. Hence, the flow becomes three-dimensional due to fluid injection through lower plate. The governing equations of motion and energy are solved by regular perturbation technique and separation of variables technique. Velocity and temperature distributions are discussed numerically and shown through graphs. Nusselt number and coefficient of skin-friction at the plates are derived, discussed numerically and shown through graphs.

Keywords: Three-dimensionl, Couette flow, porous medium, skin-friction, Nusselt number.

1. Introduction

The flow of fluid through porous medium has many applications in various branches of Science and Technology, that's why this field has attracted the attention of number of scholars. In fact, a porous material containing the fluid is a non-homogeneous medium, but it may be possible to treat it as a homogeneous one. For the sake of analysis, by taking its dynamical properties to be equal to the local averages of the original non-homogeneous continuum. Thus a complicated problem of the flow through a porous medium gets reduced to the flow problem of a homogeneous fluid with some additional resistance. A series of investigations have been made by different scholars where the porous medium is either bounded by horizontal or vertical surfaces. The effect of periodic variation of suction velocity on three-dimensional convective flow and heat transfer through a porous medium was discussed by Gersten and Gross [10]. Ram and Mishra [7] applied the equations of motion to study the unsteady MHD flow of conducting fluid through porous medium. Varshney [6] analysed an oscillatory two-dimensional flow through porous medium bounded by a horizontal porous plate subjected to a variable suction velocity. Raptis [2] investigated the unsteady flow through a porous medium bounded by an infinite porous plate subjected to a constant suction and variable temperature. Raptis and Perdikis [1] studied free convective flow through a porous medium bounded by a vertical porous plate with constant suction when the free stream velocity oscillates in time about a constant mean value.

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On the other hand, flows through porous medium in the channels have numerous Engineering and Geophysical applications, e.g. in the field of chemical engineering for filteration and purification processes; in the field of agriculture engineering to study the underground water resources; in petroleum technology to study the moment of natural gas, oil and water through the oil reservoirs etc. In view of these applications , Singh [9] presented couette flow with transpiration cooling for ordinary medium. Singh and Sharma [8] investigated magnetohydrodynamic three-dimensional couette flow with transpiration cooling. Vidhya and Sundarammal Kesavan [12] presented laminar convection through porous medium between two vertical parallel plates with heat source.The effect of heat source on free convective flow of a incompressible, viscous, electrically conducting fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of a transverse magnetic field was studied by Das, Mishra and Mishra [15]. Khem chand [11] investigated the heat transfer and hydromagnetic boundary layer flow of an electrically conducting viscous, incompressible fluid over a continuous flat surface moving in a parallel free stream. Das, Maity and Das [14] analysed the unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate in a porous medium with constant suction in presence of a uniform transverse magnetic field. Gireesha et.al [4] described three-dimensional Couette flow of a dusty fluid with heat transfer.Jha and Apere [5] explained time-dependent MHD Couette flow of rotating fluid with Hall and ion-slip currents. Das et. al [13] studied the effect of variable suction and radiative heat transfer on MHD couette flow through a porous medium in the slip flow regime. Transient free convection flow past a vertical plate through porous medium with variation in slip flow regime was studied by Mishra [3]. Zhang [16] investigated the effect of wall surface modification in the combined Couette and Poiseuille flows in a nano channel. In the present paper Heat transfer through three-dimensionl couette flow through porous medium bounded between parallel porous plates is investigated.

2. Formulation of the Problem

Consider Couette flow of a viscous incompressible fluid through a porous medium bounded between two infinite parallel flat porous plates. The lower stationary porous plate is lying horizontally on $x^* - z^*$ plane and upper moving porous plate is placed parallel to lower plate at distance 'h'. The y^*-axis is taken perpendicular to the planes of the plates. The lower and the upper plates are maintained at constant temperatures T_0 and T_1, respectively, when $T_1 > T_0$. The upper plate is subjected to a constant suction velocity V, whereas the lower plate is subjected to a transverse sinusoidal injection velocity of the form given by

$$v^*(z^*) = V(1 + \varepsilon \cos \pi z^*/h) \tag{1}$$

where $e(\ll 1)$ is a positive quantity. Without any loss of generality, the distance 'h' between the plates is taken equal to the wave length of the injection velocity. All physical quantities are independent of x^* for fully developed laminar flow and the flow remains three-dimensional due to the periodic injection of fluid through lower plate. The governing equations of continuity, motion and energy in the presence of volumetric rate of heat generation/absorption are

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2}$$

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \upsilon \left[\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right] - \frac{\upsilon u^*}{K^*} \tag{3}$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \upsilon \left[\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right] - \frac{\upsilon v^*}{K^*}, \tag{4}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \upsilon \left[\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right] - \frac{\upsilon w^*}{K^*} \tag{5}$$

$$\rho C_p \left[v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} \right] = \kappa \left[\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right] + Q^*(T^* - T_s) \tag{6}$$

where ρ is the density, p^* the pressure, K^* the permeability of the porous medium, ν the kinematic viscosity, κ the thermal conductivity, C_p the specific heat at constant pressure, Q^* the volumetric rate of heat generation/absorption and T_s the static temperature. The boundary conditions are

$$\begin{aligned} &y^* = 0 : u^* = 0, v^*(z^*) = V(1 + \varepsilon \cos \pi z^*/h), w^* = 0, T^* = T_s + (T_1 - T_s)(1 + \varepsilon \cos \pi z^*/h), \\ &y^* = h : u^* = U, v^* = V, w^* = 0, T^* = T_1 \end{aligned} \tag{7}$$

2.1. Method of Solution

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{h},\ z = \frac{z^*}{h},\ u = \frac{u^*}{U},\ w = \frac{w^*}{V},\ v = \frac{v^*}{V},\ p = \frac{p^*}{\rho V^2},\ \theta = \frac{T^* - Ts}{T_1 - Ts},\ Re = \frac{Vh}{\nu},\ \mathrm{Pr} = \frac{\mu C_p}{\kappa},\ Q = \frac{Q^* h}{\nu} \tag{8}$$

into the equations (2) to (6), we get

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{u}{ReK} \tag{10}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{v}{ReK} \tag{11}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{w}{ReK} \tag{12}$$

$$v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) + Q\theta \tag{13}$$

where Re is the cross-flow Reynolds number, Pr the Prandtl number, K the permeability parameter and Q the volumetric rate of heat generation/absorption. The boundary conditions in non-dimensional form are

$$\begin{aligned} &y = 0 : u = 0, v = 1 + \varepsilon \cos \pi z, w = 0, \theta = 1 + \varepsilon \cos \pi z, \\ &y = 1 : u = 1, v = 1, w = 0, \theta = 1 \end{aligned} \tag{14}$$

Since $e(\ll 1)$ is very small, Therefore assuming

$$f(y, z) = f_0(y) + \varepsilon f_1(y, z) \tag{15}$$

where f stands for u, v, w, p or θ. When $e = 0$, the problem reduces to the two-dimensional Couette flow through porous medium. Using (15) into the equations (9) to (13), and equating the terms of $O(e)$, we get

$$\frac{dv_0}{dy} = 0 \Rightarrow v_0 \quad \text{is independent of y, say } v_0 = 1, \text{ i.e. } v_0 \text{ is constant.} \tag{16}$$

$$\frac{d^2 u_0}{dy^2} - Re\frac{du_0}{dy} - \frac{u_0}{K} = 0 \tag{17}$$

$$\frac{d^2 \theta_0}{dy^2} - RePr\frac{d\theta_0}{dy} + RePrQ\theta_0 = 0 \tag{18}$$

The corresponding boundary conditions are

$$\begin{aligned} &y = 0 : u_0 = 0, v_0 = 1, \theta_0 = 1; \\ &y = 1 : u_0 = 1, v_0 = 1, \theta_0 = 1. \end{aligned} \tag{19}$$

Equations (17) and (18) are ordinary differential equations and solved under the boundary conditions (19). Through straight forward calculations, the solutions of u_0 and θ_0 are known and given by

$$u_0(y) = \frac{\exp(m_1 y) - e(m_2 y)}{\exp(m_1) - \exp(m_2)} \tag{20}$$

$$\theta_0(y) = \frac{\{1 - \exp(\alpha_2)\}\exp(\alpha_1 y) + \{\exp(\alpha_1) - 1\}\exp(\alpha_2 y)}{\exp(\alpha_1) - \exp(\alpha_2)} \tag{21}$$

where $m_1 = \frac{1}{2}\left[Re + \sqrt{Re^2 + \frac{4}{K}}\right]$, $m_2 = \frac{1}{2}\left[Re - \sqrt{Re^2 + \frac{4}{K}}\right]$, $\alpha_1 = \frac{1}{2}\left[RePr + \sqrt{Re^2\,\mathrm{Pr}^2 - 4QRePr}\right]$ and $\alpha_2 = \frac{1}{2}\left[RePr - \sqrt{Re^2\,\mathrm{Pr}^2 - 4QRePr}\right]$. When $e \neq 0$, substituting (15) into the equations (9) to (13) and comparing the coefficients of $O(e)$, we get

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{22}$$

$$v_1\frac{\partial u_0}{\partial y} + v_0\frac{\partial u_1}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{u_1}{ReK} \tag{23}$$

$$\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{v_1}{ReK} \tag{24}$$

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{ReK} \tag{25}$$

$$v_1\frac{\partial \theta_0}{\partial y} + \frac{\partial \theta_1}{\partial y} = \frac{1}{RePr}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) + Q\theta_1 \tag{26}$$

The corresponding boundary conditions become

$$\begin{aligned} &y = 0 : u_1 = 0, v_1 = \cos\pi z, w_1 = 0, \theta_1 = \cos\pi z; \\ &y = 1 : u_1 = 0, v_1 = 0, w_1 = 0, \theta_1 = 0 \end{aligned} \tag{27}$$

The equations (22), (24) and (25) are independent of the mean flow and mean temperature, and equations (23) and (26), respectively. In view of the boundary conditions (27), the following assumptions are made to solve the equations (22), (24) and (25)

$$v_1(y,z) = v_{11}(y)\cos\pi z, \tag{28}$$

$$w_1(y,z) = -\frac{1}{\pi}v_{11}'(y)\sin\pi z \tag{29}$$

$$p_1(y,z) = p_{11}(y)\cos\pi z \tag{30}$$

where the prime denotes differentiation with respect to y. Now it is observed that the continuity equation (22) is satisfied. On substitution of the equations (28) to (30) into the equations (24) and (25), an ordinary differential equation is obtained and solved under the boundary conditions (27). Hence, the expressions of $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ are known and given by

$$v_1(y,z) = \frac{1}{D}\left[D_1\exp(\lambda_1 y) + D_2\exp(\lambda_2 y) + D_3\exp(\pi y) + D_4\exp(-\pi y)\right]\cos\pi z \tag{31}$$

$$w_1(y,z) = \frac{1}{\pi D}\left[-D_1\lambda_1\exp(\lambda_1 y) - D_2\lambda_2\exp(\lambda_2 y) - D_3\pi\exp(\pi y) + D_4\pi\exp(-\pi y)\right]\sin\pi z, \tag{32}$$

$$p_1(y,z) = \frac{1}{Re\pi D}\left[-D_3\left(\pi Re + \frac{1}{K}\right)\exp(\pi y) + D_4\left(-\pi Re + \frac{1}{K}\right)\exp(-\pi y)\right]\cos\pi z. \tag{33}$$

where D, D_1 to D_4, λ_1 and λ_2 are constants and given in Appendix. In order to solve the differential equations (23) and (26) for u_1 and θ_1, respectively, it is assumed that

$$u_1 = u_{11}(y)\cos\pi z \tag{34}$$

$$\theta_1 = \theta_{11}(y)\cos\pi z \tag{35}$$

Using (34) and (35) into the equations (23) and (26), respectively; we get

$$u''_{11} - Reu'_{11} - \left(\pi^2 + \frac{1}{K}\right)u_{11} = Rev_{11}u'_0 \tag{36}$$

$$\theta''_{11} - RePr\theta'_{11} + (RePrQ - \pi^2)\theta_{11} = RePrv_{11}\theta'_0 \tag{37}$$

Here the prime denotes differentiation with respect to y. The corresponding boundary conditions are given by

$$\begin{aligned} &y = 0 : u_{11} = 0, \theta_{11} = 1, \\ &y = 1 : u_{11} = 0, \theta_{11} = 0 \end{aligned} \tag{38}$$

Equations (36) and (37) are ordinary differential equations and solved under the boundary conditions (38). Through straight forward calculations, the solutions of $u_1(y,z)$ and $\theta_1(y,z)$ are known and given by

$$\begin{aligned} u_1(y,z) = &[M_1\exp(\lambda_1 y) + M_2\exp(\lambda_2 y) \\ &+ \frac{\mathrm{Re}m_2}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_1\exp\{(m_2+\lambda_1)y\}}{2m_2\lambda_1+\frac{1}{K}} + \frac{D_2\exp\{(m_2+\lambda_2)y\}}{2m_2\lambda_2+\frac{1}{K}}\right\} \\ &+ \frac{\mathrm{Re}m_2}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_3\exp\{(m_2+\pi)y\} - D_4\exp\{(m_2-\pi)y\}}{\pi(2m_2-\mathrm{Re})}\right\} \\ &- \frac{\mathrm{Re}m_1}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_1\exp\{(m_1+\lambda_1)y\}}{2m_1\lambda_1+\frac{1}{K}} + \frac{D_2\exp\{(m_1+\lambda_2)y\}}{2m_1\lambda_2+\frac{1}{K}}\right\} \\ &- \frac{\mathrm{Re}m_1}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_3\exp\{(m_1+\pi)y\} - D_4\exp\{(m_1-\pi)y\}}{\pi(2m_1-\mathrm{Re})}\right\}]\cos\pi z, \end{aligned} \tag{39}$$

$$\begin{aligned} \theta_1(y,z) = &[N_1\exp(\beta_1 y) + N_2\exp(\beta_2 y) + D_5\left\{\frac{D_1\exp\{(\lambda_1+\alpha_1)y\}}{G_1} + \frac{D_2\exp\{(\lambda_2+\alpha_1)y\}}{G_2}\right\} \\ &+ D_6\left\{\frac{D_1\exp\{(\lambda_1+\alpha_2)y\}}{G_5} + \frac{D_2\exp\{(\lambda_2+\alpha_2)y}{G_6}\right\} + D_5\left\{\frac{D_3\exp\{(\pi+\alpha_1)y\}}{G_3} + \frac{D_4\exp\{(-\pi+\alpha_1)y\}}{G_4}\right\} \\ &+ D_6\left\{\frac{D_3\exp\{(\pi+\alpha_2)y\}}{G_7} + \frac{D_4\exp\{(-\pi+\alpha_2)y\}}{G_8}\right\}]\cos\pi z. \end{aligned} \tag{40}$$

where G_1 to G_8, M_1, M_2, N_1, N_2, α_1, α_2, β_1, β_2 are constants, and their expressions are given in Appendix.

3. Skin-friction Coefficient

Skin-friction coefficient at both the plates is given by

$$C_f = \frac{\tau_\omega h}{\mu U} = \left(\frac{\partial u}{\partial y}\right)_{y=0,1}, \tag{41}$$

where $\tau_\omega = \mu\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)_{y^*=0,h}$. Hence

$$\begin{aligned} (C_f)_0 = &\frac{m_1-m_2}{\exp(m_1)-\exp(m_2)} + \varepsilon[M_1\lambda_1 + M_2\lambda_2 + \frac{\mathrm{Re}m_2}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_1(m_2+\lambda_1)}{2m_2\lambda_1+\frac{1}{K}} + \frac{D_2(m_2+\lambda_2)}{2m_2\lambda_2+\frac{1}{K}}\right\} \\ &+ \frac{\mathrm{Re}m_2}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_3(m_2+\pi) - D_4(m_2-\pi)}{\pi(2m_2-\mathrm{Re})}\right\} - \frac{\mathrm{Re}m_1}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_3(m_1+\pi) - D_4(m_1-\pi)}{\pi(2m_1-\mathrm{Re})}\right\} \\ &- \frac{\mathrm{Re}m_1}{D\{\exp(m_2)-\exp(m_1)\}}\left\{\frac{D_1(m_1+\lambda_1)}{2m_1\lambda_1+\frac{1}{K}} + \frac{D_2(m_1+\lambda_2)}{2m_1\lambda_2+\frac{1}{K}}\right\}]\cos\pi z, \end{aligned} \tag{42}$$

and

$$(C_f)_1 = \frac{m_1\exp(m_1) - m_2\exp(m_2)}{\exp(m_1) - \exp(m_2)} + \varepsilon\cos\pi z[M_1\lambda_1\exp(\lambda_1) + M_2\lambda_2\exp(\lambda_2)$$
$$+ \frac{Rem_2}{D\{\exp(m_2) - \exp(m_1)\}}\left\{\frac{D_1(m_2+\lambda_1)\exp(m_2+\lambda_1)}{2m_2\lambda_1 + \frac{1}{K}} + \frac{D_2(m_2+\lambda_2)\exp(m_2+\lambda_2)}{2m_2\lambda_2 + \frac{1}{K}}\right\}$$
$$- \frac{Rem_1}{D\{\exp(m_2) - \exp(m_1)\}}\left\{\frac{D_1(m_1+\lambda_1)\exp(m_1+\lambda_1)}{2m_1\lambda_1 + \frac{1}{K}} + \frac{D_2(m_1+\lambda_2)\exp(m_1+\lambda_2)}{2m_1\lambda_2 + \frac{1}{K}}\right\}$$
$$+ \frac{Rem_2}{D\{\exp(m_2) - \exp(m_1)\}}\left\{\frac{D_3(m_2+\pi)\exp(m_2+\pi) - D_4(m_2-\pi)\exp(m_2-\pi)}{\pi(2m_2 - Re)}\right\}$$
$$- \frac{Rem_1}{D\{\exp(m_2) - \exp(m_1)\}}\left\{\frac{D_3(m_1+\pi)\exp(m_1+\pi) - D_4(m_1-\pi)\exp(m_1-\pi)}{\pi(2m_1 - Re)}\right\}]. \tag{43}$$

4. Nusselt Number

The rate of heat transfer in terms of Nusselt number at both the plates is given by

$$Nu = \frac{qh}{\kappa(T_1 - Ts)} = -\left(\frac{\partial T}{\partial y}\right)_{y=0,1}, \tag{44}$$

where $q = -\kappa\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0,h}$. Hence

$$Nu_0 = \frac{1}{\exp(\alpha_2) - \exp(\alpha_1)}\{\alpha_1\{1 - \exp(\alpha_2)\} + \alpha_2\{\exp(\alpha_1) - 1\}\} - \varepsilon[\beta_1 N_1 + \beta_2 N_2$$
$$+ D_5\left\{\frac{(\lambda_1+\alpha_1)D_1}{G_1} + \frac{(\lambda_2+\alpha_1)D_2}{G_2} + \frac{(\pi+\alpha_1)D_3}{G_3} + \frac{(-\pi+\alpha_1)D_4}{G_4}\right\}$$
$$+ D_6\left\{\frac{(\lambda_1+\alpha_2)D_1}{G_5} + \frac{(\lambda_2+\alpha_2)D_2}{G_6} + \frac{(\pi+\alpha_2)D_3}{G_7} + \frac{(-\pi+\alpha_2)D_4}{G_8}\right\}]\cos\pi z, \tag{45}$$

and

$$Nu_1 = \frac{1}{\exp(\alpha_2) - \exp(\alpha_1)}\{\alpha_1\{1 - \exp(\alpha_2)\}\exp(\alpha_1) + \alpha_2\{\exp(\alpha_1) - 1\}\exp(\alpha_2)\} - \varepsilon[\beta_1 N_1\exp(\beta_1) + \beta_2 N_2\exp(\beta_2)$$
$$+ D_5\left\{\begin{array}{l}\frac{(\lambda_1+\alpha_1)D_1}{G_1}\exp((\lambda_1+\alpha_1) + \frac{(\lambda_2+\alpha_1)D_2}{G_2}\exp(\lambda_2+\alpha_1)\\ + \frac{(\pi+\alpha_1)D_3}{G_3}\exp(\pi+\alpha_1) + \frac{(-\pi+\alpha_1)D_4}{G_4}\exp(-\pi+\alpha_1)\end{array}\right\}$$
$$+ D_6\left\{\begin{array}{l}\frac{(\lambda_1+\alpha_2)D_1}{G_5}\exp(\lambda_1+\alpha_2) + \frac{(\lambda_2+\alpha_2)D_2}{G_6}\exp(\lambda_2+\alpha_2)\\ + \frac{(\pi+\alpha_2)D_3}{G_7}\exp(\pi+\alpha_2) + \frac{(-\pi+\alpha_2)D_4}{G_8}\exp(-\pi+\alpha_2)\end{array}\right\}]\cos\pi z. \tag{46}$$

5. Results and Discussion

It is observed from figure 1 that fluid velocity decreases due to increase in the cross-flow velocity of fluid and increases due to increase in the peremeability of the medium. It is noted from figure 2 that fluid temperature decreases due to increase in the cross-flow velocity of fluid or the Prandtl number in the presence of heat sink. It is observed from figure 3 that fluid temperature increases due to increase in heat generation parameter, while it decreases due to increase in the Prandtl number or heat sink parameter. It is seen from figure 4 that skin-friction coefficient at the lower plate decreases due to increase in the cross-flow velocity of fluid , while it increases due to increase in the peremeability parameter when other parameters are kept fixed. It is noted from figure 5 that skin-friction coefficient at the upper plate increases due to increase in the cross-flow velocity of fluid , while it decreases due to increase in the peremeability parameter when other parameters are kept fixed. It is observed from figure 6 that the Nusselt number at the lower plate increases due to increase in the Prandtl number, heat

sink parameter or cross-flow velocity of fluid when other parameters are kept fixed. It is seen from figure 7 that the Nusselt number at the upper plate decreases due to increase in the Prandtl number , heat sink parameter or cross-flow velocity of fluid when other parameters are kept fixed.

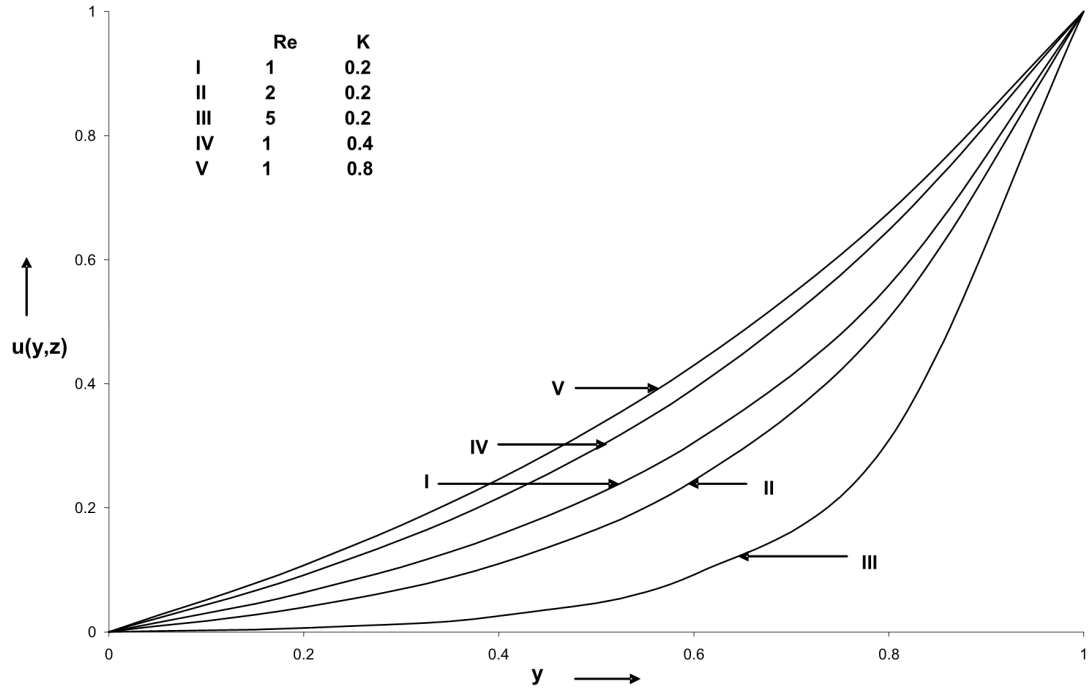

Figure 1. **Velocity distribution versus y when** $\varepsilon = 0.2$, $z = 0.33$.

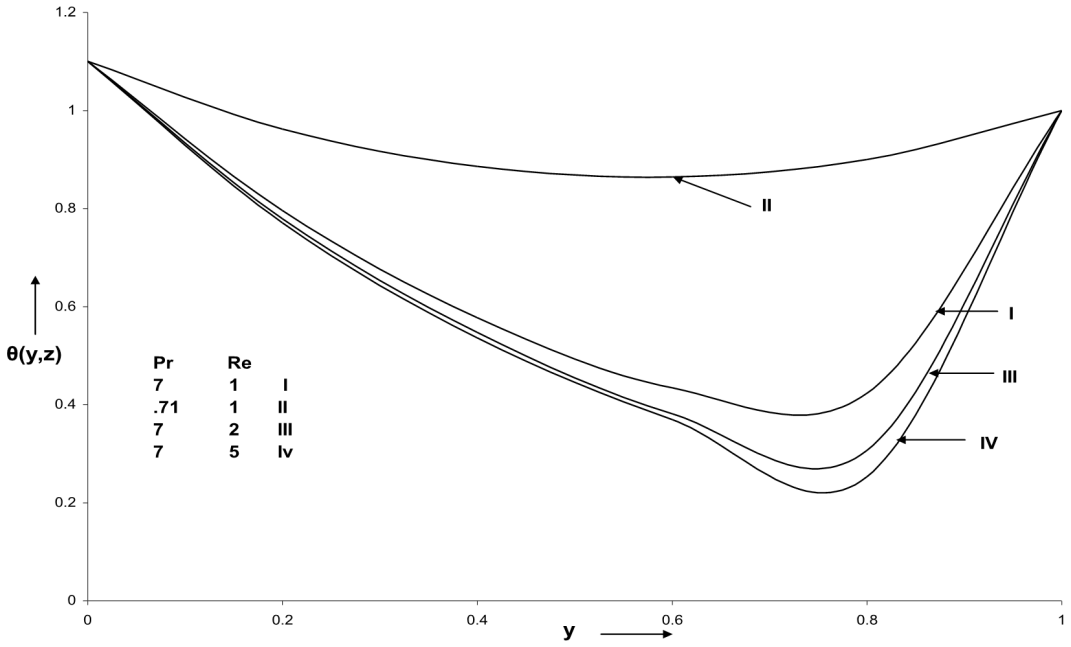

Figure 2. **Temperature distribution versus y when** $\varepsilon = 0.2$, $Q = -2$, $z = 0.33$.

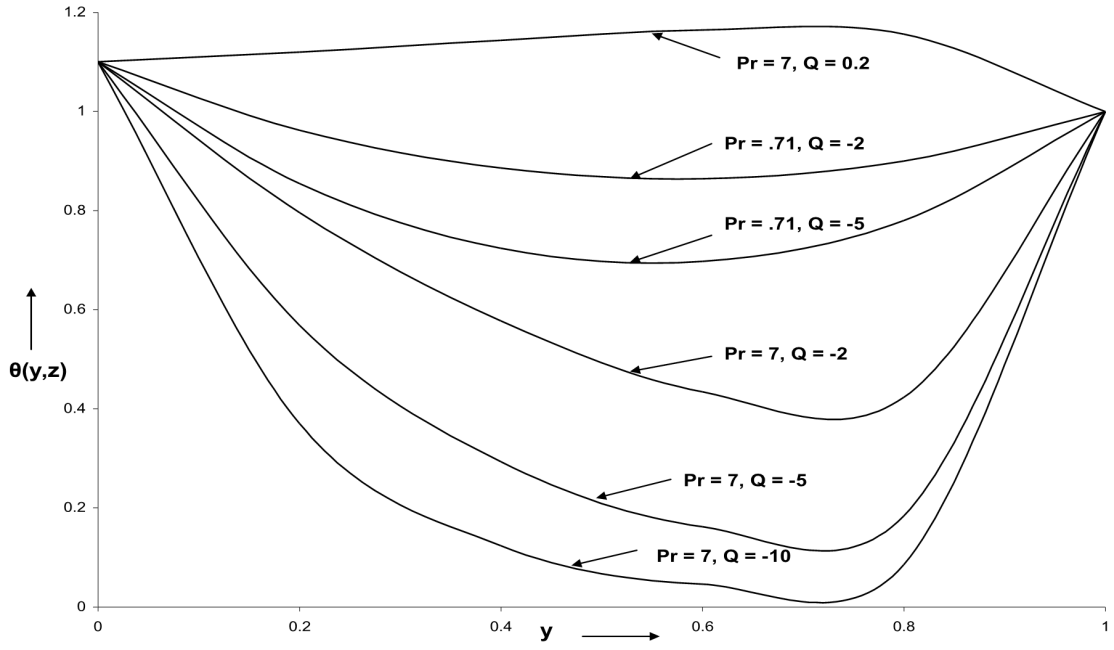

Figure 3. **Temperature distribution versus y when** $\varepsilon = 0.2$, $Re = 1$, $z = 0.33$.

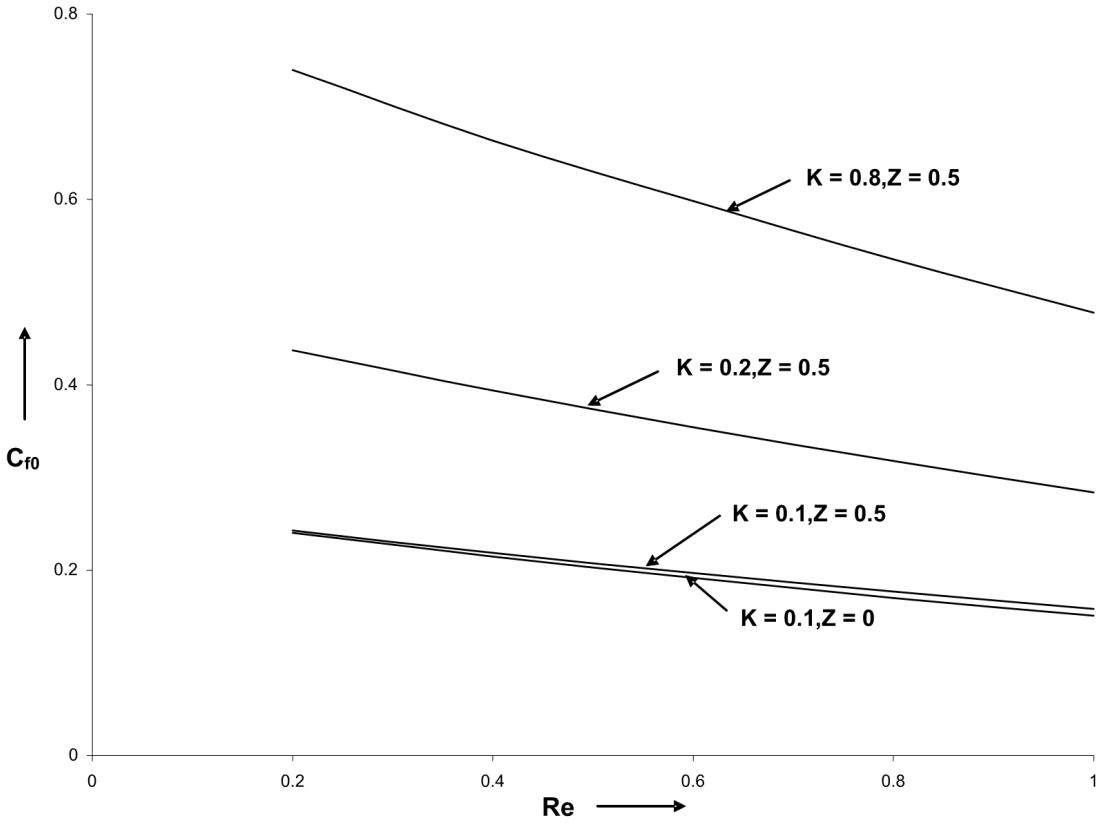

Figure 4. **Skin-friction coefficient** C_{f0} **at lower plate versus Re when** $\varepsilon = 0.2$.

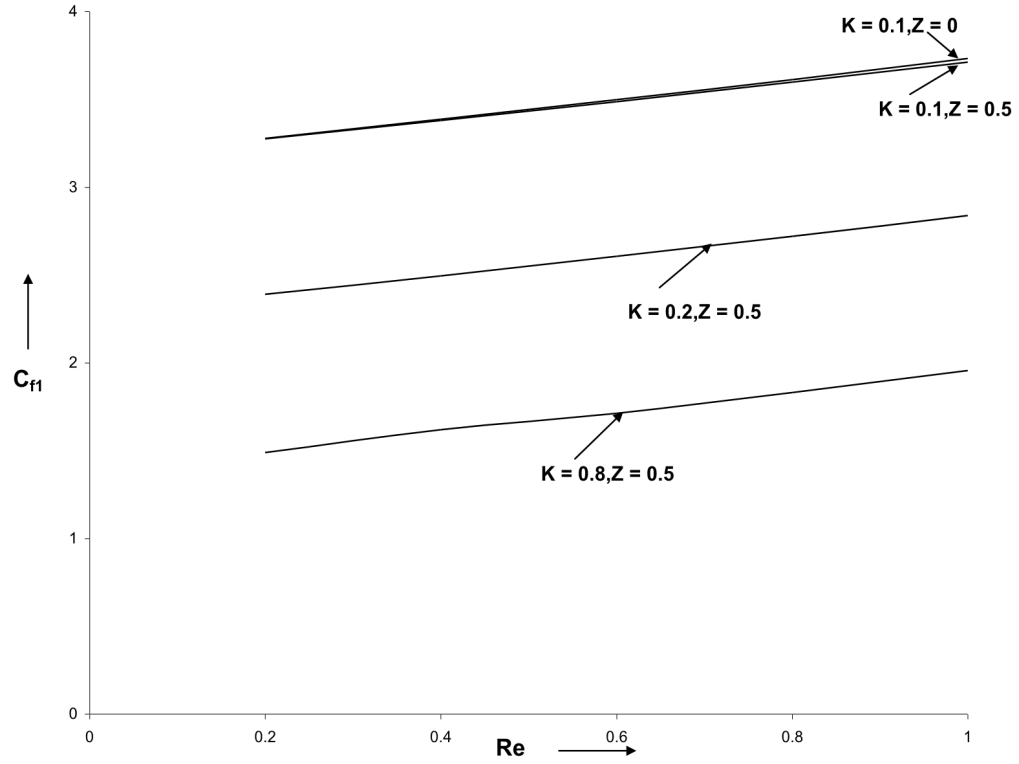

Figure 5. **Skin-friction coefficient C_{f1} at upper plate versus Re when $\varepsilon = 0.2$.**

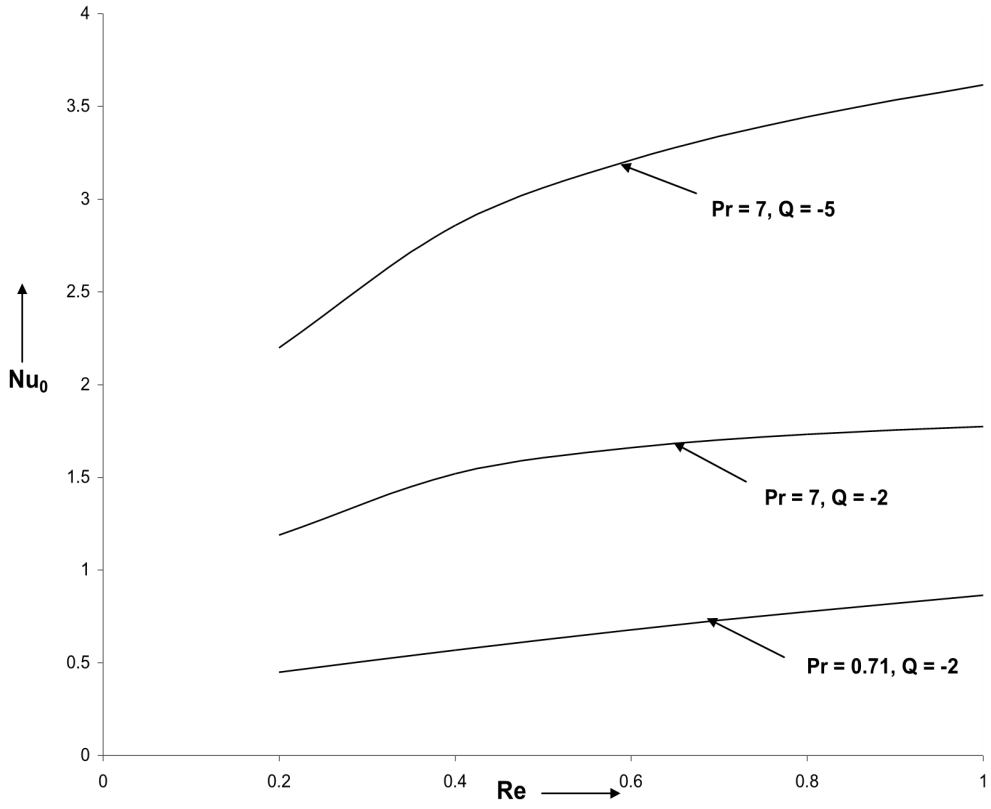

Figure 6. **Nusselt number Nu_0 at lower plate versus Re when $z = 0.5$, $\varepsilon = 0.2$ and $K = 0.2$.**

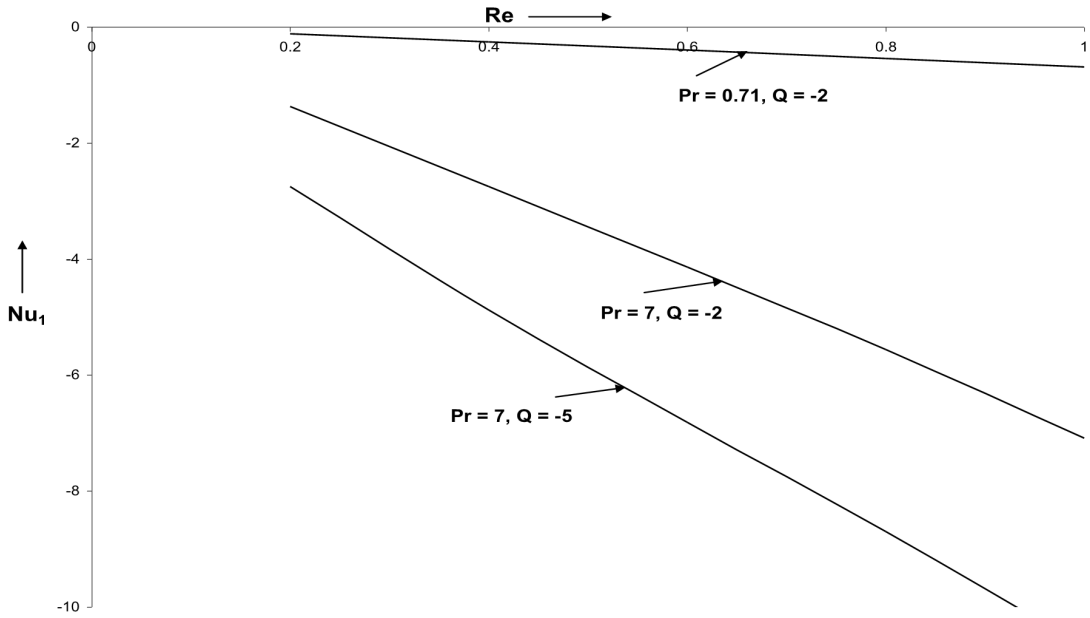

Figure 7. **Nusselt number Nu_1 at upper plate versus Re when $z = 0.5$, $\varepsilon = 0.2$ and $K = 0.2$.**

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Appendix

$$m_1 = \frac{1}{2}\left\{Re + \sqrt{Re^2 + \frac{4}{K}}\right\},$$

$$m_2 = \frac{1}{2}\left\{Re - \sqrt{Re^2 + \frac{4}{K}}\right\},$$

$$\alpha_1 = \frac{1}{2}\left\{RePr + \sqrt{Re^2 \overset{2}{\mathrm{Pr}} - 4QRePr}\right\},$$

$$\alpha_2 = \frac{1}{2}\left\{RePr - \sqrt{Re^2 \overset{2}{\mathrm{Pr}} - 4QRePr}\right\},$$

$$\lambda_1 = \frac{Re + \sqrt{Re^2 + 4\left(\pi^2 + \frac{1}{K}\right)}}{2},$$

$$\lambda_2 = \frac{Re - \sqrt{Re^2 + 4\left(\pi^2 + \frac{1}{K}\right)}}{2},$$

$$\beta_1 = \frac{RePr + \sqrt{Re^2 \mathrm{Pr}^2 - 4(RePrQ - \pi^2)}}{2},$$

$$\beta_2 = \frac{RePr - \sqrt{Re^2 \mathrm{Pr}^2 - 4(RePrQ - \pi^2)}}{2},$$

$$G_1 = (\lambda_1 + \alpha_1)(\lambda_1 + \alpha_1 - RePr) + RePrQ - \pi^2,$$

$$G_2 = (\lambda_2 + \alpha_1)(\lambda_2 + \alpha_1 - RePr) + RePrQ - \pi^2,$$

$$G_3 = (\pi + \alpha_1)(\pi + \alpha_1 - RePr) + RePrQ - \pi^2,$$

$$G_4 = (-\pi + \alpha_1)(-\pi + \alpha_1 - RePr) + RePrQ - \pi^2,$$

$$G_5 = (\lambda_1 + \alpha_2)(\lambda_1 + \alpha_2 - RePr) + RePrQ - \pi^2,$$

$$G_6 = (\lambda_2 + \alpha_2)(\lambda_2 + \alpha_2 - RePr) + RePrQ - \pi^2,$$

$$G_7 = (\pi + \alpha_2)(\pi + \alpha_2 - RePr) + RePrQ - \pi^2,$$

$$G_8 = (-\pi + \alpha_2)(-\pi + \alpha_2 - RePr) + RePrQ - \pi^2,$$

$$\begin{aligned} D = {} & (\pi - \lambda_1)(\pi + \lambda_2)\{\exp(\lambda_2 - \pi) + \exp(\lambda_1 + \pi) - 2\pi(\lambda_2 - \lambda_1)\{\exp(\lambda_1 + \lambda_2) + 1\} \\ & - (\pi + \lambda_1)(\pi - \lambda_2)\{\exp(\lambda_2 + \pi) + \exp(\lambda_1 - \pi)\}, \end{aligned}$$

$$D_1 = -2\pi\lambda_2 + \pi(\lambda_2 + \pi)\exp(\lambda_2 + \pi) - \pi(\pi - \lambda_2)\exp(\lambda_2 - \pi),$$

$$D_2 = 2\pi\lambda_1 - \pi(\pi + \lambda_1)\exp(\lambda_1 - \pi) - \pi(\lambda_1 - \pi)\exp(\lambda_1 + \pi),$$

$$D_3 = -\lambda_1(\lambda_2+\pi)\exp(\lambda_2-\pi) + \lambda_2(\lambda_1+\pi)\exp(\lambda_1-\pi) - \pi(\lambda_2-\lambda_1)\exp(\lambda_1+\lambda_2),$$

$$D_4 = \lambda_1(\lambda_2-\pi)\exp(\lambda_2+\pi) + \lambda_2(\pi-\lambda_1)\exp(\pi+\lambda_1) + \pi(\lambda_1-\lambda_2)\exp(\lambda_1+\lambda_2),$$

$$D_5 = \frac{RePr\alpha_1\{1-\exp(\alpha_2)\}}{D\{\exp(\alpha_1)-\exp(\alpha_2)\}},$$

$$D_6 = \frac{RePr\alpha_2\{\exp(\alpha_1)-1\}}{D\{\exp(\alpha_1)-\exp(\alpha_2)\}},$$

$$D_7 = \frac{Rem_2}{D\{\exp(m_2)-\exp(m_1)\}\{\exp(\lambda_1)-\exp(\lambda_2)\}},$$

$$D_8 = \frac{Rem_1}{D\{\exp(m_2)-\exp(m_1)\}\{\exp(\lambda_1)-\exp(\lambda_2)\}},$$

$$D_9 = \frac{RePr\alpha_1\{1-\exp(\alpha_2)\}}{D\{\exp(\alpha_1)-\exp(\alpha_2)\}\{\exp(\beta_1)-\exp(\beta_2)\}},$$

$$D_{10} = \frac{RePr\alpha_2\{\exp(\alpha_1)-1\}}{D\{\exp(\alpha_1)-\exp(\alpha_2)\}\{\exp(\beta_1)-\exp(\beta_2)\}},$$

$$\begin{aligned}
M_1 = {} & D_7\left\{\frac{D_1\{\exp(\lambda_2)-\exp(m_2+\lambda_1)\}}{2m_2\lambda_1+\frac{1}{K}} + \frac{D_2\{\exp(\lambda_2)-\exp(m_2+\lambda_2)\}}{2m_2\lambda_2+\frac{1}{K}}\right\} \\
& + D_7\left\{\frac{D_3\{\exp(\lambda_2)-\exp(m_2+\pi)\} - D_4\{\exp(\lambda_2)-\exp(m_2-\pi)\}}{\pi(2m_2-Re)}\right\} \\
& - D_8\left\{\frac{D_1\{\exp(\lambda_2)-\exp(m_1+\lambda_1)\}}{2m_1\lambda_1+\frac{1}{K}} + \frac{D_2\{\exp(\lambda_2)-\exp(m_1+\lambda_2)\}}{2m_1\lambda_2+\frac{1}{K}}\right\} \\
& - D_8\left\{\frac{D_3\{\exp(\lambda_2)-\exp(m_1+\pi)\} - D_4\{\exp(\lambda_2)-\exp(m_1-\pi)\}}{\pi(2m_1-Re)}\right\},
\end{aligned}$$

$$\begin{aligned}
M_2 = {} & -D_7\left\{\frac{D_1\{\exp(\lambda_1)-\exp(m_2+\lambda_1)\}}{2m_2\lambda_1+\frac{1}{K}} + \frac{D_2\{\exp(\lambda_1)-\exp(m_2+\lambda_2)\}}{2m_2\lambda_2+\frac{1}{K}}\right\} \\
& - D_7\left\{\frac{D_3\{\exp(\lambda_1)-\exp(m_2+\pi)\} - D_4\{\exp(\lambda_1)-\exp(m_2-\pi)\}}{\pi(2m_2-Re)}\right\} \\
& + D_8\left\{\frac{D_1\{\exp(\lambda_1)-\exp(m_1+\lambda_1)\}}{2m_1\lambda_1+\frac{1}{K}} + \frac{D_2\{\exp(\lambda_1)-\exp(m_1+\lambda_2)\}}{2m_1\lambda_2+\frac{1}{K}}\right\} \\
& + D_8\left\{\frac{D_3\{\exp(\lambda_1)-\exp(m_1+\pi)\} - D_4\{\exp(\lambda_1)-\exp(m_1-\pi)\}}{\pi(2m_1-Re)}\right\},
\end{aligned}$$

$$\begin{aligned}
N_1 = {} & \frac{\exp(\beta_2)}{\exp(\beta_2)-\exp(\beta_1)} \\
& + D_9\left\{\frac{D_1\{\exp(\beta_2)-\exp(\lambda_1+\alpha_1)\}}{G_1} + \frac{D_2\{(\exp(\beta_2)-\exp(\lambda_2+\alpha_1)\}}{G_2}\right\} \\
& + D_9\left\{\frac{D_3\{\exp(\beta_2)-\exp(\pi+\alpha_1)\}}{G_3} + \frac{D_4\{\exp(\beta_2)-\exp(-\pi+\alpha_1)\}}{G_4}\right\} \\
& + D_{10}\left\{\frac{D_1\{\exp(\beta_2)-\exp(\lambda_1+\alpha_2)\}}{G_5} + \frac{D_2\{(\exp(\beta_2)-\exp(\lambda_2+\alpha_2)\}}{G_6}\right\} \\
& + D_{10}\left\{\frac{D_3\{\exp(\beta_2)-\exp(\pi+\alpha_2)\}}{G_7} + \frac{D_4\{\exp(\beta_2)-\exp(-\pi+\alpha_2)\}}{G_8}\right\},
\end{aligned}$$

$$\begin{aligned}
N_2 = {} & \frac{\exp(\beta_1)}{\exp(\beta_1)-\exp(\beta_2)} \\
& - D_9\left\{\frac{D_1\{\exp(\beta_1)-\exp(\lambda_1+\alpha_1)\}}{G_1} + \frac{D_2\{(\exp(\beta_1)-\exp(\lambda_2+\alpha_1)\}}{G_2}\right\} \\
& - D_9\left\{\frac{D_3\{\exp(\beta_1)-\exp(\pi+\alpha_1)\}}{G_3} + \frac{D_4\{\exp(\beta_1)-\exp(-\pi+\alpha_1)\}}{G_4}\right\} \\
& - D_{10}\left\{\frac{D_1\{\exp(\beta_1)-\exp(\lambda_1+\alpha_2)\}}{G_5} + \frac{D_2\{(\exp(\beta_1)-\exp(\lambda_2+\alpha_2)\}}{G_6}\right\} \\
& - D_{10}\left\{\frac{D_3\{\exp(\beta_1)-\exp(\pi+\alpha_2)\}}{G_7} + \frac{D_4\{\exp(\beta_1)-\exp(-\pi+\alpha_2)\}}{G_8}\right\}.
\end{aligned}$$