International Journal of *Mathematics And its Applications*

An Inventory Model for Deteriorating Items with Time Varying Demand and Shortages Using Fuzzy Environment

S. Sasikala[1],*

1 PG and Research Department of mathematics, Adhiyaman Arts & Science College for Women, Krishnagiri, Tamil Nadu, India.

Abstract: Inventory in wider sense is defined as any idle resource of an enterprise. It is physical stock of goods, kept for the purpose of future affairs. Invention scheduling is a difficult task in industry with different operational constraints as well as strategic plan of the business revenue purpose and limit on finishing. To reduce the difficulties we introduce crisp and fuzzy model. Here fuzzy model consist of three methods gradient mean representation method, signed distance method, centroid method to deffuzify the total cost function with the help of triangular fuzzy number. The results obtained by these methods are compared with the help of a numerical example. Sensitivity analysis is also carried out to explore the effect of changes in the values of some of the system parameters. The aim of the study is to minimize the total cost function.

1. Introduction and Preliminaries

Inventory is a stock of items kept on hand used to meet customer demand. A level of inventory is maintained that will meet anticipate demand. If demand is not known with certainty, safety stocks are kept on hand. An inventory model for deteriorating items with shortage is considered where demand, holding cost, unit cost, shortage cost and deteriorating rate are assumed as a triangular fuzzy number [4]. In real life situation inventory system deterioration is critical point. It cannot be ignored Generally deterioration s defined as decay, damage, evaporation, spoilage, loss of utility, loss of marginal values of commodity and item cannot be used for its original purpose. This decreases usefulness of the product. This leads to decreases in usefulness of the product. In our daily life most of the physical goods like liquid, medicine, food grains, alcohols, fresh product, flowers, fruits, vegetables, sea foods under go decay over time. There are various uncertainties mixed up in any inventory system These uncertainties cannot be treated by usual probabilistic model. so here I use fuzzy set theory. The fuzzy set theory which was demonstrated by Zadeh [1]. First they fuzzify the random lead time demand to be fuzzy random variable and then fuzzify the total demand to be triangular fuzzy number and derive the fuzzy total cost. for Defuzzification of total cost function gradient mean representation, signed distance method, centroid method are used. By comparing the results obtained by these method we get better one as an estimate of the total cost in the fuzzy sense in [2].

Definition 1.1. *A Fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(x, \mu_A(x)); x \in X\}$ where $\mu_{\tilde{A}} : X \to [0, 1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called membership value*

---

* *E-mail: sasikala5989@yahoo.com*

or degree of membership of $x \in X$ in the fuzzy set $\tilde{A}$. The larger $\mu_{\tilde{A}}(x)$ indicates the stronger grade of membership form in $\tilde{A}$.

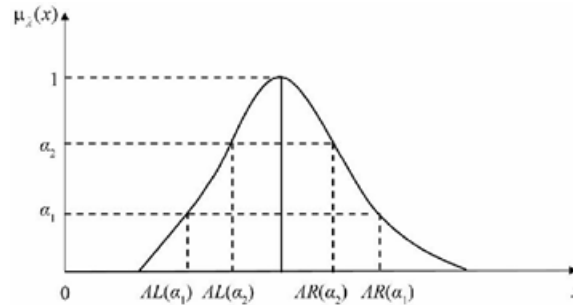**Definition 1.2.** *A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. Fuzzy numbers*

Figure 1. **Fuzzy number $\tilde{A}$ with α-cut**

are represented by two type of membership functions. They are

- *Linear Membership function. For example: Triangular fuzzy number.*

- *Non Linear Membership function. For example: Parabolic fuzzy number.*

Definition 1.3 (Triangular Fuzzy Number). *Triangular fuzzy number $\tilde{A}$, is a fuzzy number with the membership function $\mu_{\tilde{A}}(x)$. Here $\mu_{\tilde{A}} : R \to [0,1]$ is a continuous mapping*

$$\mu_{\tilde{A}} = \begin{cases} 0 & \text{for } -\infty < x \le a \text{ and } c < x < \infty \\ 1 - \frac{b-x}{b-a} & \text{for } a \le x \le b \\ 1 - \frac{x-b}{c-b} & \text{for } b \le x \le c \end{cases}$$
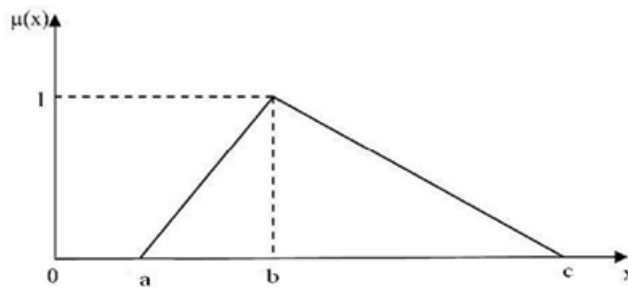

Figure 2. **Triangular Fuzzy Number**

Definition 1.4 (Gradient Mean Integration). *If $\tilde{A} = (a,b,c)$ is a triangular fuzzy number then the gradient mean integration representation of $\tilde{A}$ is defined as*

$$P(\tilde{A}) = \frac{\int_0^{W_A} h\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right) dh}{\int_0^{W_A} h \, dh} \quad \text{with } 0 < h \le W_A \text{ and } 0 < W_A \le 1$$

$$P(\tilde{A}) = \frac{1}{2} \frac{\int_0^1 h[a + h(b-a) + c - h(c-a)] dh}{\int_0^1 h \, dh} = \frac{(a+4b+c)}{6}$$

Definition 1.5 (Signed Distance Method). *If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as*

$$P(\tilde{A}, \tilde{0}) = \int_0^1 d([A_L(\alpha)_\alpha, A_R(\alpha)_\alpha], \tilde{0}) = \frac{1}{4}(a + 2b + c)$$

Definition 1.6 (Centroid Method). *If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number then the centroid method of $\tilde{A}$ is defined as $C\tilde{A} = \frac{a+b+c}{3}$.*

2. Assumption and Notations

2.1. Assumptions

(1). Demand $D(t) = b(1 + at)$ is assumed to be an increasing function of time i.e. where a and b are positive constants and $a > 0$, $0 < b < 1$.

(2). Replenishment is instantaneous and lead -time is zero.

(3). Shortages are allowed and fully backlogged.

2.2. Notations

(1). $D(t)$ is the demand rate at any time t per unit time.

(2). A is the ordering cost per order.

(3). θ is the deterioration rate.

(4). T is the length of the Cycle.

(5). Q is the ordering Quantity per unit.

(6). h is the holding cost per unit per unit time.

(7). S is the shortage Cost per unit time.

(8). C is the unit Cost per unit time.

(9). $K(t_1, T)$ is the total inventory cost per unit time.

(10). $\tilde{D}$ is the fuzzy demand.

(11). θ is the fuzzy deterio ration rate.

(12). h is the fuzzy holding cost per unit per unit time.

(13). $\tilde{S}$ is the fuzzy shortage cost per unit time.

(14). $\tilde{C}$ is the fuzzy unit cost per unit time.

(15). $\tilde{A}(t_1, T)$ is the total fuzzy inventory cost per unit time.

(16). $K_{dG}(t_1, T)$ is the defuzzify value of $\tilde{K}(t_1, T)$ by applying gradient mean integration method.

(17). $K_{dS}(t_1, T)$ is the defuzzify value of $\tilde{K}(t_1, T)$ by applying Signed distance method.

(18). $K_{dC}(t_1, T)$ is the defuzzify value of $\tilde{K}(t_1, T)$ by applying Centroid method.

3. Formulation of Inventory Model

Let $p(t)$ be the on-hand inventory at time t with initial inventory Q. During the period $[0, t_1]$, the on-hand inventory depletes due to demand and deterioration and exhausted at time t_1. The period $[t_1, T]$ is the period of shortages, which are fully backlogged. At any instant of time, the inventory level $p(t)$ is governed by the differential equations.

3.1. Crisp Model

$$\frac{d_p(t)}{dt} + \theta p(t) = -D(t) \ \ 0 \leq t \leq t_1 \ \text{with} \ p(0) = \theta \ \text{and} \ p(t_1) = 0 \tag{1}$$

$$\frac{d_p(t)}{dt} = -D(t) \ \ t_1 \leq t \leq T \ \text{with} \ p(t_1) = 0 \tag{2}$$

$$p(t) = Qe^{-\theta(t)} + \left(\frac{b}{\theta} - \frac{ab}{\theta^2}\right)e^{-\theta t} + \left(\frac{ab}{\theta^2} - \frac{b}{\theta}\right)(1 + at) \tag{3}$$

$$p(t) = b(t_1 - t) + \frac{ab}{2}(t_1^2 - t^2) \tag{4}$$

By using $I(t_1) = 0$, we have

$$Q = \{\frac{b}{\theta}(1 + at_1) - \frac{ab}{\theta^2}\}e^{\theta t_1}).\left(\frac{b}{\theta} - \frac{ab}{\theta^2}\right) \tag{5}$$

Now (5) becomes

$$p(t) = \left[b\left\{(t_1 - t) + \frac{\theta}{2}(t_1 - t)^2\right\} + ab\left\{t_1(t_1 - t) - \frac{(t_1 - t)^2}{2} + \frac{\theta}{2}t_1(t_1 - t)^2 - \frac{\theta}{6}(t_1 - t)^3\right\}\right] \tag{6}$$

Total average number of holding units (I_H) during period $[0, T]$ is given by

$$I_H = \int_0^{t_1} p(t)dt = b\left\{\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right\} + ab\left\{\frac{t_1^3}{3} + \frac{\theta}{8}t_1^4\right\} \tag{7}$$

Total number of deteriorated units (I_S) during period $[0, T]$ is given by $I_D = Q - Total\ Demand$

$$I_D = Q - \int_0^h b(1 + at)dt = \frac{1}{2}b\theta t_1^2 + \frac{1}{3}b\theta t_1^3 \tag{8}$$

Total average number of shortage units (I_S) during period $[0, T]$ is given by

$$I_S = -\int_0^T p(t)dt = \frac{b}{2}(t_1 - T)^2 - \frac{ab}{2}(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3) \tag{9}$$

Total cost of the systems per units is given by $K(t_1, T) = \frac{1}{T}[A + hI_H + CI_D + SI_S]$

$$K(t_1, T) = \frac{1}{T}\left[A + hb\left(\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right) + hab\left(\frac{t_1^3}{3} + \frac{\theta}{8}t_1^4\right) + C\left(\frac{1}{2}b\theta t_1^2 + \frac{1}{3}b\theta t_1^3\right) + S\left\{\frac{b}{2}(t_1 - T)^2 - \frac{ab}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right] \tag{10}$$

3.2. Fuzzy model

Due to uncertainly in the environment it is not easy to define all the parameters precisely, accordingly we assume some of these parameters namely $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}, \tilde{h}$ may change within some limits. Let $\tilde{a} = (a_1, a_2, a_3)$, $\tilde{b} = (b_1, b_2, b_3)$, $\tilde{C} = (C_1, C_2, C_3)$, $\tilde{S} = (S_1, S_2, S_3)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$, $\tilde{h} = (h_1, h_2, h_3)$ are as triangular fuzzy numbers. Total cost of the system per unit time in fuzzy sense is given by

$$\tilde{k}(t_1, T) = \frac{1}{T}\left[A + \tilde{h}\tilde{b}\left(\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right) + \tilde{h}\tilde{a}\tilde{b}\left(\frac{t_1^3}{3} + \frac{\theta}{8}t_1^4\right) + \tilde{C}\left(\frac{1}{2}\tilde{b}\tilde{\theta}t_1^2 + \tilde{a}\tilde{b}\tilde{\theta}t_1^3\right) + \tilde{S}\left\{\frac{\tilde{b}}{2}(t_1 - T)^2\right\} - \frac{\tilde{a}\tilde{b}}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right] \tag{11}$$

We defuzzify the fuzzy total cost $\tilde{k}(t_1, T)$ by Graded Mean Representation, Signed Distance and Centroid Methods.

(i). By Graded Mean Representation Method: Total Cost is given by

$$K_{dG}(t_1, T) = \frac{1}{6}[K_{dG_1}(t_1, T), K_{dG_2}(t_1, T), K_{dG_3}(t_1, T)]$$

Where

$$K_{dG_1}(t_1, T) = \frac{1}{T}\left[A + h_1 b_1\left(\frac{t_1^2}{2} + \frac{\theta_1}{6}t_1^3\right) + h_1 a_1 b_1\left(\frac{t_1^3}{3} + \frac{\theta_1}{8}t_1^4\right) + c_1\left(\frac{1}{2}b_1\theta_1 t_1^2 + \frac{1}{3}b_1\theta_1 t_1^3\right) + S_1\left\{\frac{b_1}{2}(t_1 - T)^2 + \frac{a_1 b_1}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right)\right\} - \frac{S_3 a_3 b_3}{2}(t_1^2 T)\right]$$

$$K_{dG_2}(t_1, T) = \frac{1}{T}\left[A + h_2 b_2\left(\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3\right) + h_2 a_2 b_2\left(\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4\right) + c_2\left(\frac{1}{2}a_2\theta_2 t_1^2 + \frac{1}{3}a_2\theta_2 t_1^3\right) + S_2\left\{\frac{a_1}{2}(t_1 - T)^2 - \frac{a_1 b_1}{2}\left(\frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right]$$

$$K_{dG_3}(t_1, T) = \frac{1}{T}\left[A + h_3 b_3\left(\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3\right) + h_3 a_3 b_3\left(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right) + c_3\left(\frac{1}{2}b_3\theta_3 t_1^2 + \frac{1}{3}b_3\theta_3 t_1^3\right) + S_3\left\{\frac{b_3}{2}(t_1 - T)^2 + \frac{a_3 b_3}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right)\right\} - \frac{S_1 a_1 b_1}{2}(t_1^2 T)\right]$$

$$K_{dG}(t_1, T) = \frac{1}{6}[K_{dG_1}(t_1, T) + 4K_{dG_2}(t_1, T) + K_{dG_3}(t_1, T)] \tag{12}$$

To minimize total cost function per unit time $K_{dG}(t_1, T)$, the optimal value of $t1$ and T can be obtained by solving the following equations:

$$\frac{\partial K_{dG}(t_1, T)}{\partial t_1} = 0 \quad and \quad \frac{\partial K_{dG}(t_1, T)}{\partial T} = 0 \tag{13}$$

Equation (14) is equivalent to $\frac{1}{6T}[h_1 b_1(t_1 + \frac{\theta_1}{2}t_1^2) + h_1 a_1 b_1(t_1^2 + \frac{\theta_1}{2}t_1^3) + C_1(b_1\theta_1 t_1 + a_1 b_1\theta_1 t_1^2) + S_1\{b_1(t_1 - T) + a_1 b_1 t_1^2\} - S_3 a_3 b_3 t_1 T + 4\{h_2 b_2(t_1 + \frac{\theta_2}{2}t_1^2) + h_2 a_2 b_2(t_1^2 + \frac{\theta_2}{2}t_1^3) + C_2(b_2\theta_2 t_1 + a_2 b_2\theta_2 t_1^2) + S_2\{b_2(t_1 - T) - a_2 b_2(t_1 T - t_1^2)\}\} + h_3 b_3(t_1 + \frac{\theta_3}{2}t_1^2) + h_3 a_3 b_3(t_1^2 + \frac{\theta_3}{2}t_1^3) + C_3(b_3\theta_3 t_1 + a_3 b_3\theta_3 t_1^2) + S_3\{b_3(t_1 - T) + a_3 b_3 t_1^2\} - S_1 a_1 b_1 t_1 T] = 0$ and

$$\begin{aligned}
&\left[\frac{1}{6T}\left\{S_1\left\{-b_1(t_1 - T) + \frac{1}{2}a_1 b_1 T^2\right\} - \frac{1}{2}S_3 a_3 b_3 t_1^2 + 4S_2\left\{-b_2(t_1 - T) - \left(\frac{1}{2}a_2 b_2(t_1^2 - T^2)\right)\right\}\right.\right. \\
&+ S_3\left\{-b_3(t_1 - T) + \frac{1}{2}a_3 b_3 T^2\right\} - \frac{1}{2}S_1 a_1 b_1 t_1^2\right\} + \frac{1}{6T^2}\left\{6A + h_1 b_1\left(\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right)\right. \\
&+ h_1 a_1 b_1\left(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right) + C_1\left(\frac{1}{2}b_1\theta_1 t_1^2 + \frac{1}{3}b_1\theta_1 t_1^3\right) + S_1\left\{\frac{b_1}{2}(t_1 - T)^2 + \frac{a_1 b_1}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right)\right\} \\
&- \frac{S_3 a_3 b_3}{2}(t_1^2 T) + 4\left\{h_2 b_2\left(\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3\right) + h_2 a_2 b_2\left(\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4\right) + C_2\left(\frac{1}{2}b_2\theta_2 t_1^2 + \frac{1}{3}b_2\theta_2 t_1^3\right)\right. \\
&+ S_2\left\{\frac{b_2}{2}(t_1 - T)^2 - \frac{a_2 b_2)}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right\} + h_3 b_3\left(\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3\right) \\
&+ h_3 a_3 b_3\left(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right) + C_3\left(\frac{1}{2}b_3\theta_3 t_1^2 + \frac{1}{3}b_3\theta_3 t_1^3\right) + S_3\left\{\frac{b_3}{2}(t_1 - T)^2 + \frac{a_3 b_3}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right) - \frac{S_1 a_1 b_1}{2}(t_1^2 T)\right\}\right] = 0
\end{aligned} \tag{14}$$

further for total cost function $K_{dG}(t_1, T)$ to be convex the following conditions must be as satisfied.

$$\frac{\partial^2 K_{dG}(t_1, T)}{\partial t_1^2} > 0 \quad and \quad \frac{\partial^2 K_{dG}(t_1, T)}{(\partial^2 T)} > 0 \tag{15}$$

$$and \quad \frac{\partial^2 K_{dG}(t_1, T)}{\partial t_1^2}\frac{\partial^2 K_{dG}(t_1, T)}{\partial^2 T} - \frac{\partial^2 K_{dG}(t_1, T)}{\partial t_1 \partial T} > 0 \tag{16}$$

(ii). By Signed Distance Method: Total cost is given by

$$K_{dS}(t_1, T) = \frac{1}{4}[K_{dS_1}(t_1, T), K_{dS_2}(t_1, T), K_{dS_3}(t_1, T)]$$

where

$$K_{dS_1}(t_1, T) = \frac{1}{T}[A + h_1 b_1(\frac{t_1^2}{2} + \frac{\theta_1}{6}t_1^3) + h_1 a_1 b_1(\frac{t_1^3}{3} + \frac{\theta_1}{8}t_1^4) + c_1(\frac{1}{2}b_1\theta_1 t_1^2 + \frac{1}{3}b_1\theta_1 t_1^3) + S_1\{\frac{b_1}{2}(t_1 - T)^2 + \frac{a_1 b_1}{2}(\frac{T^3}{3} + \frac{2}{3}t_1^3)\} - \frac{S_3 a_3 b_3}{2}t_1^2 T)]$$

$$K_{dS_2}(t_1, T) = \frac{1}{T}[A + h_2 b_2(\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3) + h_2 a_2 b_2(\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4) + c_2(\frac{1}{2}b_2\theta_2 t_1^2 + \frac{1}{3}b_2\theta_2 t_1^3) + S_2\{\frac{b_1}{2}(t_1 - T)^2 - \frac{a_1 b_1}{2}(\frac{T^3}{3} - \frac{2}{3}t_1^3)\}]$$

$$K_{dS_3}(t_1,T) = \frac{1}{T}[A + h_3 b_3(\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3) + h_3 a_3 b_3(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4) + c_3(\frac{1}{2}b_3\theta_3 t_1^2 + \frac{1}{3}b_3\theta_3 t_1^3) + S_3\{\frac{b_3}{2}(t_1-T)^2 + \frac{a_3 b_3}{2}(\frac{T^3}{3} + \frac{2}{3}t_1^3)\} - \frac{S_1 a_1 b_1}{2}(t_1^2 T)]$$

$$K_{dS}(t_1,T) = \frac{1}{4}[K_{dS_1}(t_1,T) + 2K_{dS_2}(t_1,T) + K_{dS_3}(t_1,T)] \tag{17}$$

The total cost function $K_{dS}(t_1,T)$ can be minimized. To minimize total cost function per unit time $K_{dG}(t_1,T)$, the optimal value of t_1 and T can be obtained by solving the following equations

$$\frac{\partial K_{dS}(t_1,T)}{\partial t_1} = 0 \ and \ \frac{\partial K_{dS}(t_1,T)}{\partial T} = 0 \tag{18}$$

Equation (19) is equivalent to $\frac{1}{4T}[h_1 b_1(t_1 + \frac{\theta_1}{2}t_1^2) + h_1 a_1 b_1(t_1^2 + \frac{\theta_1}{2}t_1^3) + C_1(b_1\theta_1 t_1 + a_1 b_1\theta_1 t_1^2) + S_1\{b_1(t_1-T) + a_1 b_1 t_1^2\} - S_3 a_3 b_3 t_1 T + 2\{h_2 b_2(t_1 + \frac{\theta_2}{2}t_1^2) + h_2 a_2 b_2(t_1^2 + \frac{\theta_2}{2}t_1^3) + C_2(b_2\theta_2 t_1 + a_2 b_2\theta_2 t_1^2) + S_2\{b_2(t_1-T) - a_2 b_2(t_1 T - t_1^2)\}\} + h_3 b_3(t_1 + \frac{\theta_3}{2}t_1^2) + h_3 a_3 b_3(t_1^2 + \frac{\theta_3}{2}t_1^3) + C_3(b_3\theta_3 t_1 + a_3 b_3\theta_3 t_1^2) + S_3\{b_3(t_1-T) + a_3 b_3 t_1^2\} - S_1 a_1 b_1 t_1 T] = 0$ and

$$
\begin{aligned}
&\left[\frac{1}{4T}\left\{S_1\left\{-b_1(t_1-T) + \frac{1}{2}a_1 b_1 T^2\right\} - \frac{1}{2}S_3 a_3 b_3 t_1^2 + 4S_2\left\{-b_2(t_1-T) - \left(\frac{1}{2}a_2 b_2(t_1^2 - T^2)\right)\right\}\right\}\right.\\
&+ S_3\left\{-b_3(t_1-T) + \frac{1}{2}a_3 b_3 T^2\right\} - \frac{1}{2}S_1 a_1 b_1 t_1^2 + \frac{1}{4T^2}\left\{6A + h_1 b_1\left(\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right)\right.\\
&+ h_1 a_1 b_1\left(\frac{t_1^3}{3} + \frac{\theta_1}{8}t_1^4\right) + C_1\left(\frac{1}{2}b_1\theta_1 t_1^2 + \frac{1}{3}b_1\theta_1 t_1^3\right) + S_1\left\{\frac{b_1}{2}(t_1-T)^2 + \frac{a_1 b_1}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right)\right\}\\
&- \frac{S_3 a_3 b_3}{2}(t_1^2 T) + 4\left\{h_2 b_2\left(\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3\right) + h_2 a_2 b_2\left(\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4\right) + C_2\left(\frac{1}{2}b_2\theta_2 t_1^2 + \frac{1}{3}b_2\theta_2 t_1^3\right)\right.\\
&+ S_2\left\{\frac{a_2}{2}(t_1-T)^2 - \frac{a_2 b_2)}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right\} + h_3 b_3\left(\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3\right)\\
&\left.+ h_3 a_3 b_3\left(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right) + C_3\left(\frac{1}{2}b_3\theta_3 t_1^2 + \frac{1}{3}b_3\theta_3 t_1^3\right) + S_3\left\{\frac{b_3}{2}(t_1-T)^2 + \frac{a_3 b_3}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right) - \frac{S_1 a_1 b_1}{2}(t_1^2 T)\right\}\right] = 0
\end{aligned} \tag{19}
$$

If the total cost function $K_{dS}(t_1,T)$ be convex, the following conditions are satisfied,

$$\frac{\partial^2 K_{dS}(t_1,T)}{\partial t_1^2} > 0 \ \ and \ \ \frac{\partial^2 K_{dS}(t_1,T)}{\partial^2 T} > 0 \tag{20}$$

$$and \ \ \frac{\partial^2 K_{dS}(t_1,T)}{\partial t_1^2}\frac{\partial^2 K_{dS}(t_1,T)}{\partial^2 T} - \frac{\partial^2 K_{dS}(t_1,T)}{\partial t_1 \partial T} > 0 \tag{21}$$

(iii). Centroid Method: Total cost is given by

$$K_{dC}(t_1,T) = \frac{1}{6}[K_{dC_1}(t_1,T), K_{dC_2}(t_1,T), K_{dC_3)}(t_1,T)]$$

Where

$$K_{dC_1}(t_1,T) = \frac{1}{T}[A + h_1 b_1(\frac{t_1^2}{2} + \frac{\theta_1}{6}t_1^3) + h_1 a_1 b_1(\frac{t_1^3}{3} + \frac{\theta_1}{8}t_1^4) + c_1(\frac{1}{2}b_1\theta_1 t_1^2 + \frac{1}{3}b_1\theta_1 t_1^3) + S_1\{\frac{b_1}{2}(t_1-T)^2 + \frac{a_1 b_1}{2}(\frac{T^3}{3} + \frac{2}{3}t_1^3)\} - \frac{S_3 a_3 b_3}{2}(t_1^2 T)]$$

$$K_{dC_2}(t_1,T) = \frac{1}{T}[A + h_2 b_2(\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3) + h_2 a_2 b_2(\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4) + c_2(\frac{1}{2}b_2\theta_2 t_1^2 + \frac{1}{3}b_2\theta_2 t_1^3) + S_2\{\frac{b_1}{2}(t_1-T)^2 - \frac{a_1 b_1}{2}(\frac{T^3}{3} - \frac{2}{3}t_1^3)\}]$$

$$K_{dC_3}(t_1,T) = \frac{1}{T}[A + h_3 b_3(\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3) + h_3 a_3 b_3(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4) + c_3(\frac{1}{2}b_3\theta_3 t_1^2 + \frac{1}{3}b_3\theta_3 t_1^3) + S_3\{\frac{b_3}{2}(t_1-T)^2 + \frac{a_3 b_3}{2}(\frac{T^3}{3} + \frac{2}{3}t_1^3)\} - \frac{S_1 a_1 b_1}{2}(t_1^2 T)]$$

$$K_{dC}(t_1,T) = \frac{1}{3}[K_{dC_1}(t_1,T) + K_{dC_2}(t_1,T) + K_{dC_3}(t_1,T)] \tag{22}$$

Proceeding as before the total cost function $K_{dC}(t_1,T)$ can be minimized. To minimize total cost function per unit time $K_{dC}(t_1,T)$, the optimal value of t_1 and T can be obtained by solving the following equations

$$\frac{\partial K_{dC}(t_1,T)}{\partial t_1} = 0 \ \ and \ \ \frac{\partial K_{dC}(t_1,T)}{\partial T} = 0 \tag{23}$$

Equation (23) is equivalent to $\frac{1}{3T}[h_1b_1(t_1 + \frac{\theta_1}{2}t_1^2) + h_1a_1b_1(t_1^2 + \frac{\theta_1}{2}t_1^3) + C_1(b_1\theta_1t_1 + a_1b_1\theta_1t_1^2) + S_1\{b_1(t_1 - T) + a_1b_1t_1^2\} - S_3a_3b_3t_1T + 2\{h_2b_2(t_1 + \frac{\theta_2}{2}t_1^2) + h_2a_2b_2(t_1^2 + \frac{\theta_2}{2}t_1^3) + C_2(b_2\theta_2t_1 + a_2b_2\theta_2t_1^2) + S_2\{b_2(t_1 - T) - a_2b_2(t_1T - t_1^2)\}\} + h_3b_3(t_1 + \frac{\theta_3}{2}t_1^2) + h_3a_3b_3(t_1^2 + \frac{\theta_3}{2}t_1^3) + C_3(b_3\theta_3t_1 + a_3b_3\theta_3t_1^2) + S_3\{b_3(t_1 - T) + a_3b_3t_1^2\} - S_1a_1b_1t_1T] = 0$ and

$$
\begin{aligned}
&\left[\frac{1}{3T}\left\{S_1\left\{-b_1(t_1 - T) + \frac{1}{2}a_1b_1T^2\right\} - \frac{1}{2}S_3a_3b_3t_1^2 + 4S_2\left\{-b_2(t_1 - T) - \left(\frac{1}{2}a_2b_2(t_1^2 - T^2)\right)\right\}\right\}\right.\\
&+ S_3\left\{-b_3(t_1 - T) + \frac{1}{2}a_3b_3T^2\right\} - \frac{1}{2}S_1a_1b_1t_1^2\right\} + \frac{1}{3T^2}\left\{3A + h_1b_1\left(\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right)\right.\\
&+ h_1a_1b_1\left(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right) + C_1\left(\frac{1}{2}b_1\theta_1t_1^2 + \frac{1}{3}b_1\theta_1t_1^3\right) + S_1\left\{\frac{b_1}{2}(t_1 - T)^2 + \frac{a_1b_1}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right)\right\}\\
&- \frac{S_3a_3b_3}{2}(t_1^2T) + 4\left\{h_2b_2\left(\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3\right) + h_2a_2b_2\left(\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4\right) + C_2\left(\frac{1}{2}b_2\theta_2t_1^2 + \frac{1}{3}b_2\theta_2t_1^3\right)\right.\\
&+ S_2\left\{\frac{b_2}{2}(t_1 - T)^2 - \frac{a_2b_2}{2}\left(t_1^2T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right\} + h_3b_3\left(\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3\right)\\
&\left.\left.+ h_3a_3b_3\left(\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right) + C_3\left(\frac{1}{2}b_3\theta_3t_1^2 + \frac{1}{3}b_3\theta_3t_1^3\right) + S_3\left\{\frac{b_3}{2}(t_1 - T)^2 + \frac{a_3b_3}{2}\left(\frac{T^3}{3} + \frac{2}{3}t_1^3\right) - \frac{S_1a_1b_1}{2}(t_1^2T)\right\}\right\}\right] = 0
\end{aligned}
\tag{24}
$$

If the total cost function $K_{dC}(t_1, T)$ be convex, the following conditions are satisfied,

$$
\frac{\partial^2 K_{dC}(t_1, T)}{\partial t_1^2} > 0 \ \ and \ \ \frac{\partial^2 K_{dC}(t_1, T)}{\partial^2 T)} > 0
\tag{25}
$$

$$
and \ \ \frac{\partial^2 K_{dC}(t_1, T)}{\partial t_1^2}\frac{\partial^2 K_{dC}(t_1, T)}{\partial^2 T} - \frac{\partial^2 K_{dC}(t_1, T)}{\partial t_1 \partial T} > 0
\tag{26}
$$

4. Numerical Examples

4.1. Crisp Model

Consider an inventory system with following parametric values. $A = Rs160/order$, $C = Rs.20/unit$, $h = Rs.10/unit/year$, $T = 0.9636$, $a = 200 units/year$, $b = 0.5 units/year$, $\theta = 0.01/year$, $S = Rs.15/unit/year$, $t_1 = 0.7149$.

$$
\begin{aligned}
K(t_1, T) &= \frac{1}{T}\left[A + hb\left(\frac{t_1^2}{2} + \frac{\theta}{6}t_1^3\right) + hab\left(\frac{t_1^3}{3} + \frac{\theta}{8}t_1^4\right) + C\left(\frac{1}{2}b\theta t_1^2 + \frac{1}{3}b\theta t_1^3\right) + S\left\{\frac{a}{2}(t_1 - T)^2 - \frac{ab}{2}\left(t_1^2T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right]\\
&= Rs.419
\end{aligned}
$$

4.2. Fuzzy Model

$\tilde{a} = (60, 100, 140)$, $\tilde{b} = (.06, .10, .14)$, $\tilde{C} = (16, 20, 24)$, $\tilde{S} = (12, 15, 18)$, $\tilde{\theta} = (.006, .010, .014)$, $\tilde{h} = (3, 5, 7)$. The solution of fuzzy model can be determined by following three methods.

(i). By Graded Mean Representation

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}, \tilde{h}$ are triangular fuzzy numbers $K_{dG}(t_1, T) = Rs.414.5731$, $t_1 = 0.6908$ year, $T = 0.9383$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dG}(t_1, T) = Rs4069852$, $t_1 = 0.7135$ year, $T = 0.9560$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dG}(t_1, T) = Rs.405.5274$, $t_1 = 0.7115$ year, $T = 0.9596$ year.

When $\tilde{a}, \tilde{b}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dG}(t_1, T) = Rs.405.2250$, $t_1 = 0.7120$ year, $T = 0.9603$ year.

When $\tilde{a}, \tilde{b}$ are triangular fuzzy numbers $K_{dG}(t_1, T) = Rs.404.8978$, $t_1 = 0.7131$ year, $T = 0.9611$ year.

(ii). By Signed Distance Method

$\tilde{a} = (60, 100, 140)$, $\tilde{b} = (.06, .10, .14)$, $\tilde{C} = (16, 20, 24)$, $\tilde{S} = (12, 15, 18)$, $\tilde{\theta} = (.006, .010, .014)$, $\tilde{h} = (3, 5, 7)$.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}, \tilde{h}$ all are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.419.6059$, $t_1 = 0.6797$ year, $T = 0.9266$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.408.2810$, $t_1 = 0.7128$ year, $T = 0.9523$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.406.1163$, $t_1 = 0.7093$ year, $T = 0.9576$ year.

When $\tilde{a}, \tilde{b}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.405.6640$, $t_1 = 0.7106$ year, $T = 0.9587$ year.

When $\tilde{a}, \tilde{b}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.405.1742$, $t_1 = 0.7122$ year, $T = 0.9599$ year.

(iii). Centroid Method

$\tilde{a} = (60, 100, 140)$, $\tilde{b} = (.06, .10, .14)$, $\tilde{C} = (16, 20, 24)$, $\tilde{S} = (12, 15, 18)$, $\tilde{\theta} = (.006, .010, .014)$, $\tilde{h} = (3, 5, 7)$.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}, \tilde{h}$ all are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.424.5173$, $t_1 = 0.6691$ year, $T = 0.9153$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dC}(t_1, T) = Rs.409.5606$, $t_1 = 0.7121$ year, $T = 0.9487$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\theta}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.406.7030$, $t_1 = 0.7074$ year, $T = 0.9557$ year.

When $\tilde{a}, \tilde{b}, \tilde{C}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.406.1016$, $t_1 = 0.7092$ year, $T = 0.9571$ year.

When $\tilde{a}, \tilde{b}$ are triangular fuzzy numbers $K_{dS}(t_1, T) = Rs.405.4499$, $t_1 = 0.7113$ year, $T = 0.9587$ year.

4.3. Sensitivity Analysis

A sensitivity analysis is performed to study the effects of changes in fuzzy parameters $\tilde{a}, \tilde{b}$ and $\tilde{\theta}$ on the optimal solution by taking the defuzzify values of these parameters. The results are given below in tabular form:

a (units /year)	t_1(year)	T (year)	$K_{dG}(t_1, T)(Rs)$
60	0.8614	1.1755	328.6584
80	0.7619	1.0367	374.1864
100	0.6908	0.9383	414.6096
120	0.6368	0.8638	451.3349
140	0.5938	0.8049	485.2219

Table 1. Sensitivity Analysis on parameter a

The above table indicates that as the value of a increases, fuzzy cost $K_{dG}(t_1, T)$ increases significantly but t_1 and T decreases drastically.

b (units /year)	t_1(year)	T (year)	$K_{dG}(t_1, T)(Rs)$
0.06	0.7041	0.9564	410.3971
0.08	0.6973	0.9472	412.5235
0.10	0.6908	0.9383	414.6096
0.12	0.6846	0.9299	416.6576
0.14	0.6787	0.9218	418.6696

Table 2. Sensitivity Analysis on parameter b

The above table indicates that as the value of b increases, fuzzy cost $K_{dG}(t_1, T)$ increases significantly but t_1 and T decreases drastically.

θ (units /year)	t_1(year)	T (year)	$K_{dG}(t_1, T)(Rs)$
0.006	0.6978	0.9437	412.2687
0.008	0.69430	0.9410	413.4436
0.010	0.6908	0.9383	414.6096
0.012	0.6874	0.9357	415.7667
0.014	0.6840	0.9331	416.9151

Table 3. Sensitivity Analysis on parameter θ

The above table indicates that as the value of θ increases, fuzzy cost $K_{dG}(t_1, T)$ increases significantly but t_1 and T decreases drastically.

5. Conclusion

The deterioration cost, ordering cost, holding cost, and shortage cost are represented by triangular fuzzy numbers, for de-fuzzification, we used graded mean, signed distance, and centroid method to evaluate the optimal time period of optimistic stock t_1 and total cycle length T which minimizes the total cost. The numerical example shows that graded mean representation method offer minimum cost as compared to signed distance method and centroid method. A sensitivity analysis is also conducted on the parameters a, b, and θ to investigate the effects of fuzziness. Finding suggest that the change in parameters a, b, and θ will result the change in fuzzy cost with some changes in t_1 and T. The increase in values of the parameters will result in increase in fuzzy cost,but decreases t_1 and T. Similarly the decrease in values of these parameters will result in decrease in fuzzy cost, but increases t_1 and T.

References

[1] A.Lotfi Zadeh, *Fuzzy sets*, Information and Control, 8(3)(1965), 338353.

[2] D.Dubois and H.Prade, *Operations on fuzzy numbers*, International Journal of Systems Science, 9(6)(1978), 613-626.

[3] D.Dutta and Pavan Kumar, *Fuzzy inventory model without shortage using triangular fuzzy number with sensitivity analysis*, IOSR-Journal of Mathematics, 4(3)(2012), 32-37.

[4] C.K.Jaggi, S.Pareek and A.Sharma, *Fuzzy inventory model for deteriorating items with time-varying demand and shortages*, American Journal of Operational Research, 2(6)(2012), 81-92.

[5] R.E.Bellman and L.A.Zadeh, *Decision Making in a Fuzzy Environment*, Management Sciences, 17(1970), 141-164.

[6] N.K.Mahapatra and M.Maiti, *Inventory Model for Breakable Item with Uncertain Preparation Time*, Tam-sui Oxford Journal of Management Sciences, 20(2)(2004), 83-102.

[7] N.K.Mahapatra and M.Maiti, *Production-Inventory Model for a Deteriorating Item with Imprecise Preparation Time for Production in finite Time Horizon*, Asia Pacific Journal of Operations Research.

[8] N.K.Mahapatra, K.Das, A.K.Bhunia and M.Maiti, *Multiobjective Inventory Model of Deteriorating Items With Ramp Type Demand Dependent Production, Setup and Unit Costs*, Proceedings of the National Symposium on Recent Advances of Mathematics and Its Applications.

[9] Mitsuo Gen, Yasuhiro Tsujimura and Dazhong Zheng, *An application of fuzzy set theory to inventory control models*, Computers and Industrial Engineering, 33(1997), 553-556.

[10] E.Naddor, *Inventory System*, John Wiley, New York, (1966).

[11] Nagoor A.Gani and S.Maheswari, *Economic order quantity for items with imperfect quality where shortages are backordered in fuzzy environment*, Advances in Fuzzy Mathematics, 5(2)(2010), 91-100.

[12] Pankaj Dutta, Dabjani Chakraborty and Akhil R.Roy, *A single-period inventory model with fuzzy random variable demand*, Mathematical and Computer Modeling, 41(8-9)(2005), 915-922.

[13] K.Roy and M.Maiti, *Multi-Objective Inventory Models of Deteriorating Items with Some Constraints in a Fuzzy Environment*, Computers and Operations Research, 25(12)(1998), 1085-1095.

[14] J.K.Syed and L.A.Aziz, *Fuzzy inventory model without shortages using signed distance method*, Applied Mathematics & Information Sciences, 1(2)(2007), 203-209.