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On Secondary k–Idempotent Bimatrices

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Abstract: The concept of Secondary K–idempotent bimatrices are introduced. Some of characterization on secondary K–idempotent bimatrices are given.

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1. Introduction and Preliminaries

Matrices provide a very powerful tool for dealing with linear models. Bimatrices are an advanced tool which can handle over one linear models at a time. Bimatrices will be useful when time bound comparisons are needed in the analysis of the model [3]. Unlike bimatrices can be of several types. A matrix 'A_B' that satisfies $A_B^2 = A_B$ is called an idempotent bimatrix. A theory for K-real and K-hermitian matrices, as a generalization of secondary real and secondary hermitian matrices, was developed by Hill and Waters [7]. Krishnamoorthy, Rajagopalan and vijayakumar [5] have studied the basic concepts of k-idempotent matrices as generalization of idempotent matrices. The secondary symmetric, skew symmetric and secondary orthogonal matrices have been analyzed by Ann Lee [1] in the year 1976. The secondary k-hermitian matrices are introduced in 2009 by Meenakshi, Krishnamoorthy and Ramesh [6]. In this paper the basic concepts of s-k idempotent matrices are introduced as a generalization of idempotent matrices. Throughout this paper let C^n denotes the unitary space of order n and $C_{n\times n}$ be the space of all complex $n \times n$ bimatrices. For a matrix $A \in C_{n\times n}$, $\bar{A}_B$, A_B^T, A_B^* and $A_{B^{-1}}$ denote conjugate, transpose, conjugate transpose and inverse of the bimatrix A_B respectively. Let 'K_B' be a fixed product of disjoint transpositions in S_n the set of all permutation on $\{1, 2, \ldots, n\}$. Hence it is involuntary(that is k_B^2=identity permutation). Let 'K_B' be the associated permutation bimatrix of K_B and let V_B be the permutation bimatrix with unit in the secondary diagonal. Clearly K_B and V_B satisfies the following properties:

$$K_B = K_B^T = K_B^* = K_B, K_B^2 = I_B$$

Definition 1.1. *A Symmetric matrix A is said to be a idempotent, if $A^2 = A$. For example of 2×2 idempotent matrix is* $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ *and 3×3 idempotent matrix is* $\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$.

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Definition 1.2. *A bimatrix A_B is defined as the union of two rectangular or square array of numbers A_1 and A_2 arranged into rows and columns. It is written as follows $A_B = A_1 \cup A_2$, where $A_1 \neq A_2$ with*

$$A_1 = \begin{bmatrix} a^1_{11} & a^1_{11} & \cdots & a^1_{1n} \\ a^1_{21} & a^1_{22} & \cdots & a^1_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a^1_{m1} & a^1_{m2} & \cdots & a^1_{mn} \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} a^2_{11} & a^2_{11} & \cdots & a^2_{1n} \\ a^2_{21} & a^2_{22} & \cdots & a^2_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a^2_{m1} & a^2_{m2} & \cdots & a^2_{mn} \end{bmatrix}$$

$\cup$ the notational convenience (symbol) only.

Definition 1.3. *A Symmetric bimatrix A_B is said to be a idempotent, if $A_B^2 = A_B$, then $(A_1 \cup A_2)^2 = (A_1 \cup A_2)$.*

Example 1.4. $A_1 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = A_1^2$ *and* $A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = A_2^2 \Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \cup \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \cup \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$

Definition 1.5. *A Matrix $A = a_{ij}$ in $C_{n\times n}$ is said to be s-k idempotent if, $\sum_{t=1}^{n} a_{n-k(j)+1}, t^a t,_{n-k(i)+1} = a_{ij}$, this is equivalent to $KVA^2VK = KVAVK$.*

Definition 1.6. *A Matrix $A_B = a_{ij}$ in $C_{n\times n}$ is said to be s-k idempotent if, $\sum_{t=1}^{n} a_{n-k(j)+1}, t^a t,_{n-k(i)+1} = a_{ij}$ this is equivalent to $K_BV_BA_B^2V_BK_B = K_BV_BA_BV_BK_B$.*

Note 1.7. *It is easy to see that*

(1). $K_BV_BA_B^2V_BK_B = K_BV_BA_BV_BK_B$.

(2). $K_BV_BA_BV_BK_B = K_BV_BA_B^2V_BK_B$.

(3). $V_BK_BA_B^2K_BV_B = V_BK_BA_BK_BV_B$.

(4). $V_BK_BA_BK_BV_B = V_BK_BA_B^2K_BV_B$.

Example 1.8.

(1). $K_BV_BA_B^2V_BK_B = K_BV_BA_BV_BK_B$

$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^2 \cup A_2^2)(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1 \cup A_2)(V_1 \cup V_2)(K_1 \cup K_2)(K_1v_1A_1^2V_1K_1) \cup (K_2V_2A_2^2V_2K_2) = (K_1v_1A_1V_1K_1) \cup (K_2V_2A_2V_2K_2)$

$$K_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = K_2; \quad V_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = V_2 \Rightarrow K_1V_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = K_2V_2; \quad V_1K_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = V_2K_2$$

$$A_1 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = A_1^2; \quad A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = A_2^2$$

(2). $K_BV_BA_BV_BK_B = K_BV_BA_B^2V_BK_B$

$(K_1 \cup K_2)(V_1 \cup V_2)(A_1 \cup A_2)(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^2 \cup A_2^2)(V_1 \cup V_2)(K_1 \cup K_2)$

$(K_1v_1A_1V_1K_1) \cup (K_2V_2A_2V_2K_2) = (K_1v_1A_1^2V_1K_1) \cup (K_2V_2A_2^2V_2K_2)$

$$K_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = K_2; \quad V_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = V_2$$

$$A_1 = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix} = A_1^2; \quad A_2 = \begin{pmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{pmatrix} = A_2^2$$

(3). $V_B K_B A_B^2 K_B V_B = V_B K_B A_B K_B V_B$

$(V_1 \cup V_2)(K_1 \cup K_2)(A_1^2 \cup A_2^2)(K_1 \cup K_2)(V_1 \cup V_2) = (V_1 \cup V_2)(K_1 \cup K_2)(A_1 \cup A_2)(K_1 \cup K_2)(V_1 \cup V_2)$

$(V_1 K_1 A_1^2 K_1 V_1) \cup (V_2 K_2 A_2^2 K_2 V_2) = (V_1 K_1 A_1 K_1 V_1) \cup (V_2 K_2 A_2 K_2 V_2)$

$$A_1 = \begin{pmatrix} 4 & -1 \\ 12 & -3 \end{pmatrix} = A_1^2; \quad A_2 = \begin{pmatrix} 0 & 0 \\ 2 & 1 \end{pmatrix} = A_2^2$$

(4). $V_B K_B A_B K_B V_B = V_B K_B A_B^2 K_B V_B$

$(V_1 \cup V_2)(K_1 \cup K_2)(A_1 \cup A_2)(K_1 \cup K_2)(V_1 \cup V_2) = (V_1 \cup V_2)(K_1 \cup K_2)(A_1^2 \cup A_2^2)(K_1 \cup K_2)(V_1 \cup V_2)$

$(V_1 K_1 A_1 K_1 V_1) \cup (V_2 K_2 A_2 K_2 V_2) = (V_1 K_1 A_1^2 K_1 V_1) \cup (V_2 K_2 A_2^2 K_2 V_2)$

$$A_1 = \begin{pmatrix} 1/4 & 1 \\ 3/16 & 3/4 \end{pmatrix} = A_1^2; \quad A_2 = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} = A_2^2$$

2. Main Results

Theorem 2.1. *If A_B is s-k idempotent then,*

(a). $\bar{A}_B, A_B^T, A_B^*, A_B^S, A_B^{-S}$ and A_B^{-1} (when A_B^{-1}) exists are also s-k idempotent bimatrix.

(b). A_B^n is s-k idempotent for all $n \in N$.

Proof. Now,

(a). Since A_B is s-k idempotent $\Rightarrow K_B V_B A_B^2 V_B K_B = K_B V_B A_B V_B K_B$.

(1).
$$\overline{K_B V_B A_B^2 V_B K_B} = \overline{K_B V_B A_B V_B K_B}$$
$$K_B V_B \bar{A}_B^2 V_B K_B = K_B V_B \bar{A}_B V_B K_B$$
$$K_B V_B (\bar{A}_B)^2 V_B K_B = K_B V_B \bar{A}_B V_B K_B$$

$$(K_1 \cup K_2)(V_1 \cup V_2)(\bar{A}_1 \cup \bar{A}_2)^2 (V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(\bar{A}_1 \cup \bar{A}_2)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 v_1 \bar{A}_1^2 V_1 K_1) \cup (K_2 V_2 \bar{A}_2^2 V_2 K_2) = (K_1 v_1 \bar{A}_1 V_1 K_1) \cup (K_2 V_2 \bar{A}_2 V_2 K_2)$$
$$(K_1 v_1 \bar{A}_1 V_1 K_1) \cup (K_2 V_2 \bar{A}_2 V_2 K_2) = (K_1 v_1 \bar{A}_1 V_1 K_1) \cup (K_2 V_2 \bar{A}_2 V_2 K_2)$$

Hence $\bar{A}_B$ is s-k idempotent.

(2). $(K_B V_B A_B^2 V_B K_B)^T = (K_B V_B A_B V_B K_B)^T$
$$K_B V_B A_B^{2T} V_B K_B = K_B V_B A_B^T V_B K_B$$
$$K_B V_B A_B^{T2} V_B K_B = K_B V_B A_B^T V_B K_B$$

$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^T \cup A_2^T)^2 (V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^T \cup A_2^T)(V_1 \cup V_2)(K_1 \cup K_2)$$

$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^{T2} \cup A_2^{T2})(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^T \cup A_2^T)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 v_1 A_1^{T2} V_1 K_1) \cup (K_2 V_2 A_2^{T2} V_2 K_2) = (K_1 v_1 A_1^T V_1 K_1) \cup (K_2 V_2 A_2^T V_2 K_2)$$
$$(K_1 v_1 A_1^T V_1 K_1) \cup (K_2 V_2 A_2^T V_2 K_2) = (K_1 v_1 A_1^T V_1 K_1) \cup (K_2 V_2 A_2^T V_2 K_2)$$

Hence A_B^T is s-k idempotent.

(3). $(K_B V_B A_B^2 V_B K_B)^* = (K_B V_B A_B V_B K_B)^*$

$K_B V_B A_B^{2*} V_B K_B = K_B V_B A_B^* V_B K_B$

$K_B V_B A_B^{*2} V_B K_B = K_B V_B A_B^* V_B K_B$

$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)^2(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^{*2} \cup A_2^{*2})(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 v_1 A_1^{*2} V_1 K_1) \cup (K_2 V_2 A_2^{*2} V_2 K_2) = (K_1 v_1 A_1^* V_1 K_1) \cup (K_2 V_2 A_2^* V_2 K_2)$$
$$(K_1 v_1 A_1^* V_1 K_1) \cup (K_2 V_2 A_2^* V_2 K_2) = (K_1 v_1 A_1^* V_1 K_1) \cup (K_2 V_2 A_2^* V_2 K_2)$$

Hence A_B^* is s-k idempotent.

(4). $(K_B V_B A_B^2 V_B K_B)^s = (K_B V_B A_B V_B K_B)^s$

$K_B V_B A_B^{2s} V_B K_B = K_B V_B A_B^s V_B K_B$

$K_B V_B A_B^{s2} V_B K_B = K_B V_B A_B^s V_B K_B$

$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^s \cup A_2^s)^2(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^s \cup A_2^s)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^{s2} \cup A_2^{s2})(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^s \cup A_2^s)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 v_1 A_1^s V_1 K_1) \cup (K_2 V_2 A_2^s V_2 K_2) = (K_1 v_1 A_1^s V_1 K_1) \cup (K_2 V_2 A_2^s V_2 K_2)$$

Hence A_B^s is s-k idempotent.

(5). $(\overline{K_B V_B A_B^2 V_B K_B})^S = K_B V_B A_B^{-s} V_B K_B$

$K_B V_B (\bar{A}_B^2)^s V_B K_B = K_B V_B A_B^{-s} V_B K_B$

$K_B V_B (\bar{A}_B^{s2}) V_B K_B = K_B V_B A_B^{-s} V_B K_B$

$$(K_1 \cup K_2)(V_1 \cup V_2)(\bar{A}_1^s \cup \bar{A}_2^s)^2(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^{-s} \cup A_2^{-s})(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1 v_1 \bar{A}_1^{s2} V_1 K_1) \cup (K_2 V_2 \bar{A}^{s2} V_2 K_2) = (K_1 v_1 A_1^{-s} V_1 K_1) \cup (K_2 V_2 A_2^{-s} V_2 K_2)$$
$$(K_1 v_1 \bar{A}_1^s V_1 K_1) \cup (K_2 V_2 \bar{A}_2^s V_2 K_2) = (K_1 v_1 A_1^{-s} V_1 K_1) \cup (K_2 V_2 A_2^{-s} V_2 K_2)$$
$$(K_1 v_1 A_1^{-s} V_1 K_1) \cup (K_2 V_2 A_2^{-s} V_2 K_2) = (K_1 v_1 A_1^{-s} V_1 K_1) \cup (K_2 V_2 A_2^{-s} V_2 K_2)$$

Hence A_B^{-s} is s-k idempotent.

(6). $(K_B V_B A_B^2 V_B K_B)^{-1} = (K_B V_B A_B V_B K_B)^{-1}$

$K_B V_B A_B^{2-1} V_B K_B = K_B V_B A_B^{-1} V_B K_B$

$K_B V_B A_B^{-12} V_B K_B = K_B V_B A_B^{-1} V_B K_B$

$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^{-1} \cup A_2^{-1})^2(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^{-1} \cup A_2^{-1})(V_1 \cup V_2)(K_1 \cup K_2)$$

$$(K_1v_1A_1^{-12}V_1K_1) \cup (K_2V_2A_2^{-12}V_2K_2) = (K_1v_1A_1^{-1}V_1K_1) \cup (K_2V_2A_2^{-1}V_2K_2)$$
$$(K_1v_1A_1^{-1}V_1K_1) \cup (K_2V_2A_2^{-1}V_2K_2) = (K_1v_1A_1^{-1}V_1K_1) \cup (K_2V_2A_2^{-1}V_2K_2)$$

Hence A_B^{-1} is s-k idempotent.

(b). $(K_BV_BA_B^2V_BK_B)^n = (K_BV_BA_BV_BK_B)^n$

$K_BV_BA_B^2V_BK_B.K_BV_BA_B^2V_BK_B \ldots n$ times

$$K_BV_BA_B^{2n}V_BK_B = K_BV_BA_B^nV_BK_B$$
$$K_BV_BA_B^{n2}V_BK_B = K_BV_BA_B^nV_BK_B$$
$$(K_1 \cup K_2)(V_1 \cup V_2)(A_1^n \cup A_2^n)^2(V_1 \cup V_2)(K_1 \cup K_2) = (K_1 \cup K_2)(V_1 \cup V_2)(A_1^n \cup A_2^n)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$(K_1v_1A_1^{n2}V_1K_1) \cup (K_2V_2A_2^{n2}V_2K_2) = (K_1v_1A_1^nV_1K_1) \cup (K_2V_2A_2^nV_2K_2)$$
$$(K_1v_1A_1^nV_1K_1) \cup (K_2V_2A_2^nV_2K_2) = (K_1v_1A_1^nV_1K_1) \cup (K_2V_2A_2^nV_2K_2)$$

Hence A_B^n is s-k idempotent. □

Theorem 2.2. *If A_B is s-k idempotent then*

(a). A_B is periodic with period 4, when AB is an idempotent.

(b). A_B^3 is idempotent Further, if A_B is non-singular, then $A_B^2 = A_B^{-1}.A_B^3 = I_B$.

Proof.

$$\begin{aligned}
K_BV_BA_B^4V_BK_B &= K_BV_BA_B^2.A_B^2V_BK_B \\
&= (K_BV_BA_BV_BK_B).(K_BV_BA_BV_BK_B) \\
K_BV_BA_BV_BK_BK_BV_BA_BV_BK_B &= K_BV_BA_BV_BV_BA_BV_BK_B \\
&= K_BV_BA_B.A_BV_BK_B \\
&= K_BV_BA_B^2V_BK_B \\
&= K_BV_BA_BV_BK_B \\
&= (K_1 \cup K_2)(V_1 \cup V_2)(A_1 \cup A_2)(V_1 \cup V_2)(K_1 \cup K_2) \\
&= (K_1v_1A_1V_1K_1) \cup (K_2V_2A_2V_2K_2) \\
K_BV_BA_B^4V_BK_B &= (K_1v_1A_1V_1K_1) \cup (K_2V_2A_2V_2K_2) \\
(K_BV_BA_B^3V_BK_B) &= (K_BV_BA_B^2.A_BV_BK_B)^2 \\
&= (K_BV_BA_BV_BK_BA_B)^2 \\
&= K_BV_BA_B[V_BK_BA_BK_BV_B]A_BV_BK_BA_B \\
&= K_BV_BA_BV_BK_BA_B^2K_BV_BA_BV_BK_BA_B \\
&= K_BV_BA_BA_B^2V_BK_BK_BV_BA_BV_BK_BA_B \\
&= K_BV_BA_BA_B^2V_BV_BA_BV_BK_BA_B \\
&= K_BV_BA_BA_B^2A_BV_BK_BA_B \\
&= K_BV_BA_B^2A_B^2V_BK_BA_B
\end{aligned}$$

$$= K_BV_B(A_B^2)^2V_BK_BA_B$$
$$= K_BV_BA_B^2V_BK_BA_B$$
$$= K_BV_BA_B^2A_BV_BK_B$$
$$= (K_1 \cup K_2)(V_1 \cup V_2)(A_1^2 \cup A_2^2)(A_1 \cup A_2)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$= (K_1v_1A_1^2A_1V_1K_1) \cup (K_2V_2A_2^2A_2V_2K_2)$$
$$(K_BV_BA_B^3V_BK_B) = (K_1v_1A_1^3V_1K_1) \cup (K_2V_2A_2^3V_2K_2)$$

□

Theorem 2.3. *If A_B and B_B are s-k idempotent bimatrices then*

(a). $A_B + B_B$ is s-k idempotent if and only if $A_BB_B = -B_BA_B$.

(b). A_BB_B is s-k idempotent if and only if $A_BB_B = B_BA_B$.

Proof.

(a). $K_BV_B(A_B + B_B)V_BK_B = K_BV_BA_BV_BK_B + K_BV_BA_BV_BK_B$
$$= K_BV_BA_B^2V_BK_B + K_BV_BA_B^2V_BK_B$$
$$= K_BV_B(A_B^2 + B_B^2)V_BK_B,\ if\ A_BB_B = -B_BA_B$$
$$= (K_1 \cup K_2)(V_1 \cup V_2)(A_1^2 \cup A_2^2 + B_1^2 \cup B_2^2)(V_1 \cup V_2)(K_1 \cup K_2)$$
$$K_BV_B(A_B + B_B)V_BK_B = (K_1v_1(A_1^2 + B_1^2)V_1K_1) \cup (K_2V_2(A_2^2 + B_2^2)V_2K_2)$$

(b). $K_BV_B(A_BB_B)V_BK_B = K_BV_BA_BV_BK_B.K_BV_BA_BV_BK_B$
$$= K_BV_BA_B^2V_BK_B.K_BV_BA_B^2V_BK_B$$
$$= K_BV_B(A_B^2B_B^2)V_BK_B$$
$$= K_BV_B(A_BB_B)^2V_BK_B,\ if\ A_BB_B = -B_BA_B$$
$$= (K_1 \cup K_2)(V_1 \cup V_2)[(A_1 \cup A_2)(B_1 \cup B_2)]^2(V_1 \cup V_2)(K_1 \cup K_2)$$
$$= (K_1 \cup K_2)(V_1 \cup V_2)[A_1^2B_1^2 \cup A_2^2B_2^2](V_1 \cup V_2)(K_1 \cup K_2)$$
$$K_BV_B(A_BB_B)V_BK_B = (K_1v_1(A_1^2B_1^2)V_1K_1) \cup (K_2V_2(A_2^2B_2^2)V_2K_2)$$

□

References

[1] Ann Lee, *Secondary skew symmetric, secondary orthogonal matrices*, Periodica Mathematica Hungarica, 7(1)(1976), 63-76.

[2] N.Elumalai and R.Manikandan, *On S-k normal and con S-k normal bimatrices*, International Journal of Innovative Research and Technology, 3(3)(2016), 102-107.

[3] W.B.Vasanthakandasamy, Florentin Samaranache and K.Ilathendral, *Application of Bimatrices to some fuzzy and Neutrosophic Models*, (2005).

[4] J.H.M.Weddurburn, *Lectures on matrices*, Colloq. Publ. Amer. Math. Soc., 17(1934).

[5] S.Krishnamoorthy, T.Rajagopalan and R.Vijayakumar, *On K-Idempotent Matrices*, J. Anal Comput., 4(2)(2008).

[6] A.R.Meenakshi, S.Krishnamoorthy and G.Ramesh, *On Secondary K-hermitian matrices*, J. Modern Sci., (2009).

[7] R.D.Hill and S.R.Waters, *On k-real and k-hermitian matrices*, Linear Algebra Appl., 169(1992), 17-29.