Int. J. Math. And Appl., 6(1–D)(2018), 825–834
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Intuitionistic Fuzzy 2-(0- or 1-) Prime Ideals in Semirings

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Abstract: The notion of intuitionistic fuzzy set was introduced by Atanassov as a generalization of the notion of fuzzy set. The purpose of this paper is to introduce the concept of three different types of intuitionistic fuzzy prime ideals and it has been shown that A is an intuitionistic fuzzy 2-prime ideal of the semiring S if and only if A^C is an intuitionistic fuzzy m_2-system in S.

MSC: 08A72, 16Y60, 03E72.

Keywords: Semiring, intuitionistic fuzzy ideal (k-ideal), intuitionistic fuzzy 2-(0- or 1-) prime ideal, intuitionistic fuzzy $m_2(m_0 \text{ or } m_1)$-systems.

1. Introduction

The concept of fuzzy set μ of a set X was introduced by Zadeh (see [11]) as a function from X in [0,1]. The concept of fuzzy ideals in a ring was introduced by Liu (see [10]) . Dutta and Biswas (See [3, 4]) studied fuzzy ideals, fuzzy prime ideals of semirings and they defined fuzzy k-ideals and fuzzy prime k-ideals of semirings. Jun et al., (see [7]) extended the concept of an L-fuzzy left (resp. right) ideals of a ring to a semiring. Dheena and Cumaressane (see [5]) introduced the concept of fuzzy 2-(0- or 1-) prime ideals in semiring. One of the most vital generalization of fuzzy set named intuitionistic fuzzy set was introduced by Atanassov (see [1, 2]), as a generalization of the notion of fuzzy set. Kim and Lee (see [8]) studied the intuitionistic fuzzification of the concept of several ideals in a semigroups and investigate some properties of such ideals. Hur et al., (see [6]) investigated an intuitionistic fuzzy k-ideals of a semiring and derived some properties. As a generalization of intuitionistic fuzzy semgroup Kim (see [9]) initiated intuitionistic Q-fuzzy semigroup and applied ideal theory in his concept. This paper contains four sections, the first section is merely introduction. In section 2, notion of intuitionistic prime ideal, intuitionistic fuzzy k-closure and some basic definitions and results which will be used in this article are provided. In section 3, if S is a semiring containing a proper k-ideal $A_{(\alpha,\beta)}$ with $(\alpha,\beta) \neq A(0)$ and if $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy 2-prime ideal of S, then $|Im\mu_A| = 2$ and $|Im\nu_A| = 2$ are shown. The condition that the semiring S contains a proper k-ideal $A_{(\alpha,\beta)}$ with $(\alpha,\beta) \neq A(0)$ is necessary. By an example, it is shown that the result will fail if we drop the condition that the semiring S contains a proper k ideal . In section 4, we have come across that a intuitionistic fuzzy subset A in semiring S holds a property like subgroup, ideal etc., if and only if its level subset $A_{(\alpha,\beta)}$, for all $(\alpha,\beta) \in [0,1]^2$, with $\alpha + \beta \leq 1$ also satisfies the same property in S. However if A is an intuitionistic fuzzy subset of S such that level subset $A_{(\alpha,\beta)}$ is

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an m_2 (m_0 *or* m_1) - system in S, for all $(\alpha, \beta) \in [0,1]^2$, with $\alpha + \beta \leq 1$ then A is not necessarily an intuitionistic fuzzy m_2 (m_0 *or* m_1) - system of S. Nevertheless we have shown that if A is an intuitionistic fuzzy subset in S with $x_1 \in \langle x \rangle_k$ ($x_1 \in \langle x \rangle$) implies $\mu_A(x_1) \geq \mu_A(x)$, and $\nu_A(x_1) \leq \nu_A(x)$ then A is an intuitionistic fuzzy m_2 $(m_0 \; or \; m_1)-$ system in S if and only if $A^{(r,s)} = \langle \mu_A^r, \nu_A^s \rangle = \{x \in S | \mu_A^{(x)} > r, \nu_A^{(x)} < s \; with \; r + s \leq 1\}$ is an m_2 $(m_0 \; or \; m_1)$ system in S.

2. Preliminaries

Definition 2.1. *A non-empty set S together with two binary operation $'+'$ and $'.'$ is said to be a semiring if*

(1). $(S, +)$ is a commutative semigroup,

(2). $(S, .)$ is a semigroup,

(3). $a(b + c) = ab + ac$ and $(a + b)c = ac + bc \; \forall \; a, b, c \in S$.

We say that a semiring S has a zero if there exists an element $0 \in S$ such that $0x = x0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$.

Definition 2.2. *Let S be a semiring. A non-empty subset A of S is said to be a subsemiring of S if A is closed under the operation of addition and multiplication in S.*

Definition 2.3. *A subsemiring of S is called a right $(left)$ ideal of S if for all $r \in S, x \in I, xr \in I (rx \in I)$. A subsemiring I of a semiring S is called an ideal of S if it is both left and right ideal.*

Definition 2.4. *An (A right (left)) ideal I of a semiring S is called a (right (left)) k-ideal of a semiring S if $x + y, y \in I$ implies $x \in I$.*

Definition 2.5. *Let S be a semiring. Then an ideal I of S is said to be a prime if $xy \in I$ implies that $x \in I$ or $y \in I$ for all $x, y \in S$.*

Definition 2.6. *An intuitionistic fuzzy set defined on a non-empty set X is an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$, to the set A, respectively. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy subset $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}$.*
Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in S and let α, $\beta \in [0,1]$ such that $\alpha + \beta \leq 1$. Then the set $A_{(\alpha,\beta)} = \{x \in S | \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$ is called as (α, β)-level subset of A. The set of all $(\alpha, \beta) \in Im(\mu_A) \times Im(\nu_A)$ such that $\alpha + \beta \leq 1$ is called the image of $A = (\mu_A, \nu_A)$ denoted by $Im(A)$. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$, $A^C = \{\langle x, \nu_A(x), \mu_A(x) \rangle : x \in X\}$, $(A \cap B)(x) = (min\{\mu_A(x), \mu_B(x)\}, max\{\lambda_A(x), \lambda_B(x)\})$ and $(A \cup B)(x) = (max\{\mu_A(x), \mu_B(x)\}, min\{\lambda_A(x), \lambda_B(x)\})$.

Definition 2.7. *Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two intuitionistic fuzzy subsets of a semiring S and $x, y, z \in S$. We define sum of A and B as follows:*

$$(A + B)(x) = \begin{cases} \left(\sup\limits_{x=y+z} \{min(\mu_A(y), \mu_B(z))\}, \inf\limits_{x=y+z} \{max(\nu_A(y), \nu_B(z))\} \right) \; if \; x \; is \; expressible \;\; as \; x = y + z, \\ 0, \; otherwise. \end{cases}$$

Definition 2.8. *Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two intuitionistic fuzzy subsets of a semiring S and $x, y, z \in S$. We define composition of A and B as follows:*

$$(A \circ B)(x) = \begin{cases} \left(\sup_{x=yz} \{min(\mu_A(y), \mu_B(z))\}, \inf_{x=yz} \{max(\nu_A(y), \nu_B(z))\} \right) \text{ if } x \text{ is expressible as } x = yz, \\ 0, \text{ otherwise.} \end{cases}$$

Definition 2.9. *A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semiring S is said to be an intuitionistic fuzzy semiring if for all $x, y \in S$:*

(1). $\mu_A(x + y) \geq min\{\mu_A(x), \mu_A(y)\}$

(2). $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

(3). $\nu_A(x + y) \leq max\{\nu_A(x), \nu_A(y)\}$

(4). $\nu_A(xy) \leq max\{\nu_A(x), \nu_A(y)\}$.

Definition 2.10. *A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a semiring S is said to be an intuitionistic fuzzy left ideal(right ideal) of semiring S if for all $x, y \in S$*

(1). $\mu_A(x + y) \geq min\{\mu_A(x), \mu_A(y)\}$

(2). $\mu_A(xy) \geq \mu_A(y)(\mu_A(xy) \geq \mu_A(x))$

(3). $\nu_A(x + y) \leq \max\{\nu_A(x), \nu_A(y)\}$

(4). $\nu_A(xy) \leq \nu_A(y)(\nu_A(xy) \geq \nu_A(x))$.

Lemma 2.11. *Let I be an ideal of a semiring S and $(\alpha, \bar{\alpha}) < (\beta, \bar{\beta}) \neq [0, 1] \in [0, 1]^2$. Then the intuitionistic fuzzy subset defined by*

$$\mu_A(x) = \begin{cases} \beta, & \text{if } x \in I \\ \alpha, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} \bar{\beta}, & \text{if } x \in I \\ \bar{\alpha}, & \text{otherwise.} \end{cases}$$

is an intuitionistic fuzzy ideal of S.

Definition 2.12. *An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ is called an intuitionistic fuzzy k-ideal of a semiring S if for all $x, y, z \in S$, $x + y = z$ implies*

(1). $\mu_A(x) \geq min\{\mu_A(y), \mu_A(z)\}$

(2). $\nu_A(x) \leq max\{\nu_A(y), \nu_A(z)\}$.

Example 2.13.

(1). In a ring, every intuitionistic fuzzy ideal is an intuitionistic fuzzy k-ideal.

(2). Let A be an intuitionistic fuzzy set in the semiring $\mathbb{N}$ defined by: for any $x \in \mathbb{N}$,

$$A(x) = \begin{cases} (0.3, 0.6), & \text{if } x \text{ is odd,} \\ (0.5, 0.4), & \text{if } x \text{ is } non-zero \text{ even,} \\ (1, 0), & \text{if } x = 0. \end{cases}$$

where $\mathbb{N}$ denotes the semiring of non-negative integers under the usual operations. Then A is an intuitionistic fuzzy k-ideal of $\mathbb{N}$.

(3). Let A be an intuitionistic fuzzy set in $\mathbb{N}$ denoted by: for any $x \in \mathbb{N}$,

$$A(x) = \begin{cases} (1,0), & \text{if } x \geq 7, \\ (0.5, 0.4), & \text{if } 5 \leq x < 7, \\ (0,1), & \text{if } 0 \leq x < 5. \end{cases}$$

Then it can bee as easily verified that A is an intuitionistic fuzzy ideal of $\mathbb{N}$ but A is not an intuitionistic fuzzy k-ideal of $\mathbb{N}$.

Definition 2.14. *An intuitionistic fuzzy ideal $A = (\mu_A, \nu_A)$ of a semiring S is called an intuitionistic fuzzy prime ideal of S if (i) $\mu_A(xy) = \max\{\mu_A(x), \nu_A(y)\}$ for all $x, y \in S$, (ii)$\nu_A(xy) = \min\{\mu_A(x), \nu_A(y)\}$ for all $x, y \in S$.*

Theorem 2.15 ([6]). *Let S be a semiring with zero, Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy k-ideal of S and let $S_A = \{x \in S : \mu_A(x) = \mu_A(0),\ \nu_A(x) = \nu_A(0)\}$. Then S_A is a k-ideal of S.*

Definition 2.16 ([5]). *If A is an ideal of S, then $\bar{A} = \{a \in S | a + x \in A \text{ for some } x \in A\}$ is called k-closure of A.*

Lemma 2.17 ([5]). *If A is an ideal of S, then $\bar{A}$ is a k-ideal of S.*

Lemma 2.18 ([5]). *Let A be an ideal of a semiring S. Then A is a k-ideal if and only if $A = \bar{A}$.*

3. Intuitionistic Fuzzy 2-(0- or 1-) Prime Ideal of S

Definition 3.1. *If $A = (\mu_A, \nu_A)$ is an any intuitionistic fuzzy subset of S, then $\overline{A} = (\bar{\mu}_A, \bar{\nu}_A)$ is defined as, for any $a \in S$,*

(1). $\bar{\mu}_A(a) = \sup\limits_{x \in S}\{\min\{\mu_A(a+x), \mu_A(x)\}\}$

(2). $\bar{\nu}_A(a) = \inf\limits_{x \in S}\{\max\{\nu_A(a+x), \nu_A(x)\}\}$.

$\overline{A}$ is called an intuitionistic fuzzy A-closure of A.

Clearly, $\mu_A \leq \bar{\mu}_A$ and $\nu_A \geq \bar{\nu}_A$. When A is an intuitionistic fuzzy ideal of ring S, then $A = \bar{A}$.

Lemma 3.2. *If $A = (\mu_A, \nu_A)$ is any intuitionistic fuzzy ideal of S. Then $\bar{A}$ is an intuitionistic fuzzy k-ideal of S.*

Lemma 3.3. *Suppose A is an intuitionistic fuzzy k-ideal. Then $A = \bar{A}$.*

Lemma 3.4. *Let B and C be an any intuitionistic fuzzy ideals of S and A be an intuitionistic fuzzy k-ideal of S. If $CB \subseteq A$ implies $\bar{C}B \subseteq A$, $C\bar{B} \subseteq A$ and $\bar{C}\bar{B} \subseteq A$.*

Definition 3.5. *An ideal P of a S is called a $0-(2-)$ prime ideal if for any ideals (k-ideals) $A, B \in S$, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$. An ideal P of a S is called a $1-$ prime ideal if for any k-ideals $A \in S$ and for any ideal $B \in R$, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$.*

Definition 3.6. *A subset M of S is called an m_0-system if for every $a, b \in M$, there exists $x \in S$ such that $axb \in M$. A subset M of S is called an m_1-system if for every $a, b \in M$, there exists $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle$ such that $a_1 b_1 \in M$. A subset M of S is called an m_2-system if for every $a, b \in M$, there exists $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $a_1 b_1 \in M$.*

Now we introduce the different types of an intuitionistic fuzzy prime ideals in semiring. These intuitionistic fuzzy prime ideals coincide in rings.

Definition 3.7. *An intuitionistic fuzzy ideal P of S is called an intuitionistic fuzzy 0-(2-) prime ideal if for any intuitionistic fuzzy ideals (k-ideals) $A, B \in S$, $AB \in P$ implies $A \subseteq P$ or $B \subseteq P$. An intuitionistic fuzzy ideal P of S is called a 1-prime ideal if for an any intuitionistic fuzzy k-ideal A, and for an any ideal B of S, $AB \in P$ implies $A \subseteq P$ or $B \subseteq P$.*

Lemma 3.8. *If P is an intuitionistic fuzzy 0-prime ideal of S, then P is an intuitionistic fuzzy 2-prime ideal (intuitionistic fuzzy 1-prime ideal) of S.*

Now we give an example of an intuitionistic fuzzy 2-prime ideal which is not an intuitionistic fuzzy 0-prime ideal.

Example 3.9. *Consider the semiring $S = \{0, 1, 2, 3\}$, where " + " and " $\bullet$ " are defined as follows:*

+	0	1	2	3
0	0	1	2	3
1	1	2	3	3
2	2	3	3	3
3	3	3	3	3

$\bullet$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	3
3	0	3	3	3

Let $P = (\mu_P, \nu_P)$. Define $\mu_P : S \to [0, 1]$ and $\nu_P : S \to [0, 1]$ by

$$\mu_P(x) = \begin{cases} 1, & \text{if } x = 0, 3 \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad \nu_P(x) = \begin{cases} 0, & \text{if } x = 0, 3 \\ 1, & \text{otherwise.} \end{cases}$$

Let $A = (\mu_A, \nu_A)$ and define $\mu_A : S \to [0, 1]$ and $\nu_A : S \to [0, 1]$ by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x = 0, 2, 3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0, & \text{if } x = 0, 2, 3 \\ 1, & \text{otherwise.} \end{cases}$$

Clearly P and A are an intuitionistic fuzzy idealls and $AA \subseteq P$. But $A \nsubseteq P$. Hence P is not an intuitionistic fuzzy 0-prime. However, P is an intuitionistic fuzzy 2-prime ideal of S.

Theorem 3.10. *If P is an intuitionistic fuzzy k-ideal of S, then P is an intuitionistic fuzzy 0-prime ideal if and only if P is an intuitionistic fuzzy 2-prime ideal of S.*

Proof. Assume that P is an intuitionistic fuzzy 2-prime ideal and P is an intuitionistic fuzzy k-ideal of S. Let us assume that A and B are intuitionistic fuzzy ideals of S such that $AB \subseteq P$. By Lemma 3.4, $\bar{A}\bar{B} \subseteq P$. As P is an intuitionistic fuzzy k-ideal of S, and it is 2-prime $\bar{A} \subseteq P$ or $\bar{B} \subseteq P$. But $A \subseteq \bar{A}$ and $B \subseteq \bar{B}$. Thus $A \subseteq P$ or $B \subseteq P$. Hence P is an intuitionistic fuzzy 0-prime ideal of S. Converse part is obvious by Lemma 3.8. □

Definition 3.11. *An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy m_0-system if for any $(t, t'), (s, s') \in [0, 1)^2$ with $t + t' \leq 1$, $s + s' \leq 1$ and $a, b \in S$, if $(\mu_A(a), \nu_A(b)) > (t, t')$, $(\mu_A(a), \nu_A(b)) > (s, s')$, implies that there exists $x \in S$ such that $(\mu_A(axb), \nu_A(axb)) > \max\{(t, t'), (s, s')\}$. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy m_1-system if for any $(t, t'), (s, s') \in [0, 1)^2$ with $t + t' \leq 1$, $s + s' \leq 1$ and $a, b \in S$, if $(\mu_A(a), \nu_A(b)) > (t, t')$, $(\mu_A(a), \nu_A(b)) > (s, s')$, implies that there exists $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle$ such that $(\mu_A(a_1b_1), \nu_A(a_1b_1)) > \max\{(t, t'), (s, s')\}$. An intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of S is said to be an intuitionistic fuzzy m_2-system if for any $(t, t'), (s, s') \in [0, 1)^2$ with $t + t' \leq 1$, $s + s' \leq 1$ and $a, b \in S$, if $(\mu_A(a), \nu_A(b)) > (t, t')$, $(\mu_A(a), \nu_A(b)) > (s, s')$, implies that there exists $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $(\mu_A(a_1b_1), \nu_A(a_1b_1)) > \max\{(t, t'), (s, s')\}$.*

Theorem 3.12. *Every intuitionistic fuzzy m_0-system of S is an m_1-system and m_2-system of S.*

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy m_0-system of S. Let $a, b \in S$ such that $\mu_A(a) > t$, $\mu_A(b) > s$, $\nu_A(a) \leq t$ and $\nu_A(b) \leq s$, such that $(t, t'), (s, s') \in [0, 1)^2$ with $t + t' \leq 1$, $s + s' \leq 1$. As A is an intuitionistic fuzzy m_0-system there exists $x \in S$ such that $(\mu_A(axb), \nu_A(axb)) > \max\{(t, t'), (s, s')\}$. Now $ax = a_1 \in \langle a \rangle_k$, $b_1 = b \in \langle b \rangle_k$. Thus $(\mu_A(a_1b_1), \nu_A(a_1b_1)) > \max\{(t, t'), (s, s')\}$. Hence A is an m_2-system. Similarly, we can prove if A is an m_0-system, then A is an m_1-system. □

The following two Lemmas are easily seen.

Lemma 3.13. *Let A_1 and A_2 be any two intuitionistic fuzzy subsets of S. If $A_1 \leq A$ and $A_2 \leq B$, then $A_1 \circ A_2 \leq A \circ B$ for any intuitionistic fuzzy subsets A and B.*

Lemma 3.14. *Let a_r and b_s be any two intuitionistic fuzzy points of S such that $a_r \in A$ and $b_s \in B$, where A and B are any two intuitionistic fuzzy subset of S. Then $a_r b_s \in AB$.*

Lemma 3.15. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy ideal of S and $a \in S$, then $\mu_A(x) \geq \mu_A(a)$ and $\nu_A(x) \leq \nu_A(a)$ for all $x \in \langle a \rangle$.*

Lemma 3.16. *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy k-ideal of S and $a \in S$ then $\mu_A(x) \geq \mu_A(a)$ and $\nu_A \leq \nu_A(a)$ for all $x \in \langle a \rangle_k$.*

Proof. Suppose $x \in \langle a \rangle_k$. Then by Lemma 2.18, $x+y \in \langle a \rangle$ for some $y \in \langle a \rangle$. By the above Lemma 3.15, $\mu_A(x+y) \geq \mu_A(a)$, $\nu_A(x+y) \leq \nu_A(a)$, $\mu_A(y) \geq \mu_A(a)$ and $\nu_A(y) \leq \nu_A(a)$. Thus $\min\{\mu_A(x+y), \mu_A(y)\} \geq \mu_A(a)$ and $\max\{\nu_A(x+y), \nu_A(y)\} \leq \nu_A(a)$. Since A is an intuitionistic fuzzy k-ideal, we have $\mu_A(x) \geq \mu_A(a)$ and $\nu_A(x) \leq \nu_A(a)$. □

Lemma 3.17. *Let I be a 2-(0- or 1-) prime ideal of S and $(\alpha, \bar{\alpha}) \in [0, 1)^2$ with $\alpha + \bar{\alpha} \leq 1$. If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of S defined by*

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in I \\ \alpha, & \text{otherwise} \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0, & \text{if } x \in I \\ \bar{\alpha}, & \text{otherwise.} \end{cases}$$

Then A is an intuitionistic fuzzy 2-(0- or 1-) prime ideal of S.

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of S. Let I be a 2-prime ideal of S. Clearly A is a non-constant intuitionistic fuzzy ideal of S. Suppose B and C are two intuitionistic fuzzy k-ideals of S such that $BC \subseteq A$, $B \nsubseteq A$ and $C \nsubseteq A$. Then there exist $x, y \in S$ such that $B(x) > A(x)$, and $B(y) \geq A(y)$. This implies that $\mu_A(x) = \mu_A(y) = \alpha$ and $\nu_A(x) = \nu_A(x) = \bar{\alpha}$. Therefore, $x, y \notin I$. Since I is a 2-prime ideal of S, $\langle x \rangle_k \langle y \rangle_k \nsubseteq I$. Hence there exist $c \in \langle x \rangle_k$ and $d \in \langle y \rangle_k$ such that $c.d \notin I$. Let $a = cd$. So $\mu_A(a) = \alpha$ and $\nu_A(a) = \bar{\alpha}$. Hence $(BC)(a) \leq A(a) = (\alpha, \bar{\alpha})$. Now

$$\begin{aligned} (BC)(a) &= (\sup_{a=pq} \{\min\{B(p), C(q)\}\}, \inf_{a=pq} \{\max\{B(p), C(q)\}\}) \\ &\geq (\min\{B(c), C(d)\}, \max\{B(c), C(d)\}) \\ &\geq (\min\{B(x), C(y)\}, \max\{B(x), C(y)\}) \\ &= (\alpha, \bar{\alpha}). \end{aligned}$$

Which contradicts the fact that $BC \subseteq A$. Hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy 2-prime ideal of S. □

Theorem 3.18. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of S and let S contain a proper k-ideal $A_{(\alpha,\beta)}$ with $(\alpha, \beta) \neq A(0)$. If A is an intuitionistic fuzzy 2-prime ideal of S, then $|Im(\mu_A)| = 2$ and $|Im(\nu_A)| = 2$.*

Proof. Since $A = (\mu_A, \nu_A)$ is not constant, $|Im(\mu_A)| \geq 2$ and $|Im(\nu_A)| \geq 2$. Suppose that $|Im(\mu_A)| \geq 3$ and $|Im(\nu_A)| \geq 3$. Let $A(0) = (s_1, s_2)$, $k_1 = g.l.b\{\mu_A(x)|x \in S\}$ and $k_2 = l.u.b\{\nu_A(x)|x \in S\}$. Then there exists $(t_1, t_2) \in [0,1]^2$ such that $(t_1, t_2) < (\alpha, \beta) < (s_1, s_2)$ (Here $(t_1, t_2) < (\alpha, \beta) \Rightarrow t_1 < \alpha \text{ and } t_2 > \beta$) and $(t_1, t_2) \geq (k_1, k_2)$. Let B and C be intuitionistic fuzzy subsets of S such that $B(x) = (\frac{1}{2}(t_1+\alpha), \frac{1}{2}(t_2+\beta))$ for all $x \in S$ and $C(x) = (k_1, k_2)$ if $x \notin A_{(\alpha,\beta)}$; $C(x) = (s_1, s_2)$ if $x \in A_{(\alpha,\beta)}$. Clearly B is an intuitionistic fuzzy k-ideal of S. Now we show that C is an intuitionistic fuzzy k-ideal of S. Clearly C is an intuitionistic fuzzy ideal of S. Let $x, y \in S$. Let us show that for $C(x) = (\mu_C(x), \nu_C(x))$, $\mu_C(x) \geq \min\{\mu_C(x+y), \mu_C(y)\}$ and $\nu_C(x) \leq \max\{\nu_C(x+y), \nu_C(y)\}$. If $C(x) = (s_1, s_2)$, there is nothing to prove. If $C(x) = (k_1, k_2)$, let us show that $\min\{\mu_C(x+y), \mu_C(y)\} = k_1$ and $\max\{\nu_C(x+y), \nu_C(y)\} = k_2$. If not, $\mu_C(x+y) = \mu_C(y) = s_1$ and $\nu_C(x+y) = \nu_C(y) = s_2$, then $y, x+y \in A_{(\alpha,\beta)}$. As $A_{(\alpha,\beta)}$ is a k-ideal of S, $x \in A_{(\alpha,\beta)}$, which is a contradiction. Thus, $\mu_C(x+y) = \mu_C(y) = k_1$ and $\nu_C(x+y) = \nu_C(y) = k_2$. Consequently C is an intuitionistic fuzzy k-ideal of S. We now claim that $BC \subseteq A$. Let $x \in S$. Consider the following cases

(i). $x = 0$. Then $BC(x) = (\sup_{x=uv} \min\{\mu_B(u), \mu_C(v)\}, \inf_{x=uv}\{\max\{\nu_B(u), \nu_C(v)\}\}) \leq (\frac{1}{2}(t_1+\alpha), \frac{1}{2}(t_2+\beta)) < (s_1, s_2) = A(0)$.

(ii). $x \neq 0, x \in A_{(\alpha,\beta)}$. Then $A(x) \geq (\alpha, \beta)$ and $BC(x) = (\sup_{x=uv} \min\{\mu_B(u), \mu_C(v)\}, \inf_{x=uv}\{\max\{\nu_B(u), \nu_C(v)\}\}) \leq (\frac{1}{2}(t_1 + \alpha), \frac{1}{2}(t_2+\beta) < (\alpha, \beta) = A(x)$.

(iii). $x \neq 0, x \notin A_{(\alpha,\beta)}$. Then for any $u, v \in S$ such that $x = uv, u \notin A_{(\alpha,\beta)}$ and $v \notin A_{(\alpha,\beta)}$. Then $C(v) = (\mu_C(v), \nu_C(v)) = (k_1, k_2)$.

Hence $BC(x) = (\sup_{x=uv}\{\min\{\mu_B(u), \mu_C(v)\}\}, \inf_{x=uv}\{\max\{\nu_B(u), \nu_C(v)\}\}) \leq (k_1, k_2) \leq A(x)$. Therefore in all the case, $BC(x) \leq A(x)$. Hence $BC \subseteq A$. Now there exists $u \in S$ such that $A(u) = (t_1, t_2)$. Then $B(u) = (\mu_B(u), \nu_B(v)) = (\frac{1}{2}(t_1+\alpha), \frac{1}{2}(t_2+\beta)) \geq A(u)$. Thus $B \nsubseteq A$. Further there exists $x \in S$ such that $A(x) = (\alpha, \beta)$. Then $x \in A_{(\alpha,\beta)}$ and $C(x) = (s_1, s_2) > (\alpha, \beta) = A(x)$. Hence $C \nsubseteq A$. Thus neither $B \subseteq A$ nor $C \subseteq A$. This implies that A is not an intuitionistic fuzzy 2-prime ideal of S, which contradicts the hypothesis. Hence $|Im\mu_A| = 2$ and $|Im\nu_A| = 2$. □

In Theorem 3.18, the condition that the semiring S contains a proper k-ideal $A_{(\alpha,\beta)}$ with $(\alpha, \beta) \neq A(0)$, is necessary. The following example shows that the theorem will fail if we drop that condition.

Example 3.19. *Consider the semiring $S = \{0, 1, 2, 3\}$, where $+$ and $\bullet$ are defined as in A be an intuitionistic fuzzy subset of S. Let $A = \langle \mu_A, \nu_A \rangle$ Define $\mu_A : S \to [0,1]$ and $\nu_A : S \to [0,1]$ by*

$$\mu_A(x) = \begin{cases} 1, & \text{if } x = 0, \\ 0.33, & \text{if } x = 3, \\ 0.25, & \text{if } x = 2, \\ 0, & \text{if } x = 1 \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0, & \text{if } x = 0, \\ 0.25, & \text{if } x = 3, \\ 0.33, & \text{if } x = 2, \\ 1, & \text{if } x = 1 \end{cases}$$

as $A(0,1)$, $A(0.33, 0.25)$, $A(0.25, 0.33)$ and $A(1,0)$ are ideals and k-ideals are $A(1,0)$ and $A(0,1)$. There is no proper k-ideal $A(\alpha, \beta) \neq A(0,1)$. But A is an intuitionistic fuzzy 2-prime ideal of S such that $|Im(\mu_A)| > 2$ and $|Im(\nu_A)| < 2$.

Theorem 3.20. *Let A be any intuitionistic fuzzy subset of S and let S contain a proper k-ideal $A_{(\alpha,\beta)}$ with $A(\alpha, \beta) \neq A(0,1)$. If A is an intuitionistic fuzzy 2-prime ideal of S, then $A(0,1) = (1,0)$.*

Proof. By Theorem 3.18, $|Im\mu_A| = |Im\nu_A| = 2$ for $A = (\mu_A, \nu_A)$ being intuitionistic fuzzy 2-prime ideal of S. Let $Im\mu_A = \{t_1, s_1\}$ with $t_1 < s_1$ and $Im\nu_A = \{t_2, s_2\}$ with $t_2 < s_2$. Then $A(0,1) = (s_1, s_2)$. Suppose that $(s_1, s_2) \neq (1,0)$. Let $(s_1, s_2) < (n_1, n_2) \leq (1,0)$. Let B and C be intuitionistic fuzzy subsets of S such that $B(x) = (\frac{1}{2}(t_1+n_1), \frac{1}{2}(t_2+n_2))$ for all

$x \in S$ and $C(x) = (t_1, t_2)$ if $x \notin A_{(\alpha,\beta)}$, *and* $C(x) = (n_1, n_2)$ if $x \in A_{(\alpha,\beta)}$. Clearly B is an intuitionistic fuzzy k-ideal of S. Since $A_{(\alpha,\beta)}$ is a k-ideal of S, C is an intuitionistic fuzzy k-ideal of S. It is true that $BC \subseteq A$. As $A(0,1) = (s_1, s_2) < (n_1, n_2) = C(0,1)$. This implies that $C \nsubseteq A$. Also there exists $x \in S$ such that $A(x) = (t_1, t_2) < (\frac{1}{2}(t_1 + n_1), \frac{1}{2}(t_2 + n_2)) = B(x)$. Thus $B \nsubseteq A$. Therefore neither $B \subseteq A$ nor $C \subseteq A$. This is a contradiction to the hypothesis that A is an intuitionistic fuzzy 2-prime ideal of S. Hence $A(0,1) = (1,0)$. □

Theorem 3.21. *Let A be an any intuitionistic fuzzy subset of S. If $|Im\mu(A)| = 2,\ |Im\nu(A)| = 2,\ A(0) = (1,0)$ and the set $S_A = \{x \in S | A(x) = A(0)\}$ is a 2-prime ideal of S, then A is an intuitionistic fuzzy 2-prime ideal of S.*

Proof. Let $|Im(A)| = (\{t_1, 1\}, \{t_2, 0\})$. Clearly $(t_1, t_2) = (1,0)$. Then $A(0) = (1,0)$. Let $x, y \in S$. If $x, y \in S_A$. Then $x + y \in S_A$ and $A(x+y) = (1,0) = \{(\mu_A(x), \nu_A(x)) \cap (\mu_A(y), \nu_A(y)\}$. If $x \in S_A$ and $y \notin S_A$, then we have two cases, viz, $x + y \in S_A$ or $x + y \notin S_A$. In both cases, $A(x+y) \geq \{(\mu_A(x), \nu_A(x)) \cap (\mu_A(y), \nu_A(y)\}$. If $x \notin S_A$ and $y \notin S_A$, then $A(x) = A(y) = (t_1, t_2)$ and thus $A(x+y) \geq \{(\mu_A(x), \nu_A(x)) \cap (\mu_A(y), \nu_A(y)\}$. Hence $A(x+y) \geq \{(\mu_A(x), \nu_A(x)) \cap (\mu_A(y), \nu_A(y))\}$ for all $x, y \in S$. Now if $x \in S_A$, then $xy, yx \in S_A$ and $A(xy) = A(yx) = (1,0)$. If $x \notin S_A$, then $A(xy) \geq A(x) = (t_1, t_2)$ and $A(yx) \geq A(x) = (t_1, t_2)$. Hence A is an intuitionistic fuzzy ideal of S. Let B and C be an intuitionistic fuzzy k-ideals of S such that $BC \subseteq A$. If $B \nsubseteq A$ and $C \nsubseteq A$, then there exist $x, y \in S$ such that $B(x) > A(x)$ and $C(y) > A(y)$. Clearly $A(x) = A(y) = (t_1, t_2)$ implies $x \notin S_A$ and $y \notin S_A$. Now, as S_A is 2-prime ideal of S_A, we have $x_1 \in \langle x \rangle_k$ and $y_1 \in \langle y \rangle_k$ such that $x_1 y_1 \notin S_A$. Hence $A(x) = A(y) = A(x_1 y_1) = (t_1, t_2)$. Now

$$\begin{aligned} BC(x_1y_1) &= (\sup_{x_1y_1=ab} \min\{\mu_B(x_1), \mu_C(y_1)\}, \inf_{x_1y_1=ab} \max\{\nu_B(x_1), \nu_C(y_1)\}) \ \text{(by taking } B = (\mu_B, \nu_B) \text{ and } C = (\mu_C, \nu_C)) \\ &\geq (\min\{\mu_B(x_1), \mu_C(y_1)\}, \max\{\nu_B(x_1), \nu_C(y_1)\}) \\ &> \min\{\mu_B(x), \mu_C(y)\}, \max\{\nu_B(x_1), \nu_C(y_1)\} \ \text{[By Lemma 3.16]} \\ &> (t_1, t_2) = A(x_1y_1). \end{aligned}$$

Hence $BC \nsubseteq A$, which is a contradiction to the fact that $BC \subseteq A$. Thus either $B \subseteq A$ or $C \subseteq A$. This implies that A is a fuzzy 2-prime ideal of S. □

By Theorem 3.20, Theorem 3.21 and Lemma 3.17, the following Theorem is evident.

Theorem 3.22. *Let $A = (\mu_A, \nu_A)$ an intuitionistic fuzzy subset of S and S contain a proper k-ideal $A_{(\alpha,\beta)}$ with $(\alpha, \beta) \neq A(0)$. A is an intuitionistic fuzzy 2-prime ideal of S if and only if $Im\mu_A = \{1, \alpha\}$ where $\alpha \in [0,1)$ and $Im\nu_A = \{0, \bar{\alpha})$ where $\bar{\alpha} \in (0,1]$ and the ideal S_A is a 2-prime ideal of S.*

4. Intuitionistic Fuzzy $m_2(m_0$ *or* $m_1)$-system

In this section intuitionistic fuzzy $m_2(m_0$ *or* $m_1)-$ system is defined and relation between $m_2(m_0$ *or* $m_1)-$ system and 2(0 or 1)-prime ideal is studied.

Example 4.1. *A constant intuitionistic fuzzy subset is an intuitionistic fuzzy $m_2(m_0$ or $m_1)$-system.*

Theorem 4.2. *Let M be a subset of a semiring S. M is an $m_2(m_0$ or $m_1)$-system in S if and only if the characteristic function of M, $\chi_M = (\mu_\chi, \nu_\chi)$ is an intuitionistic fuzzy $m_2(m_0$ or $m_1)$-system in S.*

Proof. Let M be an m_2-system in S. For any $t, t', s, s' \in [0,1)$ with $t + t' \leq 1,\ s + s' \leq 1$, suppose there exist $a, b \in S$ such that $\chi_M(a) > (t, t'),\ \chi_M(b) > (s, s')$. Hence $a, b \in M$. As M is an m_2-system in S, there exist $a_1 \in \langle a \rangle_k, b_1 \in \langle b \rangle_k$ such that

$a_1b_1 \in M$, and hence $\chi_M(a_1b_1) = (1,0)$. Thus $\chi_M(a_1b_1) > \max\{(t,t'),(s,s')\}$.
Conversely, assume that χ_M is an intuitionistic fuzzy m_2-system in S. Let $a, b \in M$. Then $\chi_M(a) = (1,0) = \chi_M(b)$. Thus for any $t, t', s, s' \in [0,1)$ with $t+t' \leq 1$, $s+s' \leq 1$, $\chi_M(a) > (t,t')$, $\chi_M(b) > (s,s')$. Hence there exist $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $\chi_M(a_1b_1) > \max\{(t,t'),(s,s')\}$. Therefore $\chi_M(a_1b_1) = (1,0)$ and hence $a_1b_1 \in M$. Thus M is an $m_2(m_0$ *or* $m_1)$-system in S. □

Remark 4.3. *Let A be an intuitionistic fuzzy subset in S. A holds a property like subgroup, ideal etc., if and only if its level subset $A_{(\alpha,\beta)}$ in S also satisfies the same property in S. However, A is an intuitionistic fuzzy subset in S such that the level subset $A_{(\alpha,\beta)}$ in S is an m_2 (m_0 or m_1)-system in S, for all $(\alpha,\beta) \in [0,1]^2$ with $\alpha+\beta \leq 1$, does not imply A is an intuitionistic fuzzy m_2 (m_0 or m_1)-system of S as the following example shows.*

Example 4.4. *Consider the semiring $S = (\mathbb{Z}_6, \oplus_6, \otimes_6)$. Define $\mu_A : S \to [0,1]$ and $\nu_A : S \to [0,1]$ by*

$$\mu_A(x) = \begin{cases} 1, & \text{if } x = 1 \\ 0.5, & \text{if } x = 3 \\ 0, & \text{if } x = 0,2,4,5 \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0, & \text{if } x = 1 \\ 0.5, & \text{if } x = 3 \\ 1, & \text{if } x = 0,2,4,5. \end{cases}$$

For any $(\alpha,\beta) \in [0,1]^2$ with $\alpha+\beta \leq 1$. Hence $A_{(\alpha,\beta)}$ is an m_2 (m_0 or m_1)-system in S for all $(\alpha,\beta) \in [0,1]^2$ with $\alpha+\beta \leq 1$. But A is not an intuitionistic fuzzy m_2 (m_0 or m_1)-system in S, since $(\mu_A(1),\nu_A(1)) > (0.9,0.1)$ and $(\mu_A(3),\nu_A(3) > (0.4,0.6)$, but there is no $a_1 \in \langle 1 \rangle_k$ and $b_1 \in \langle 3 \rangle_k$ such that $(\mu_A(a_1b_1),\nu_A(a_1b_1)) > (\max\{(0.9,0.1),(0.4,0.6)\}$.

However we have the following Theorem.

Theorem 4.5. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset in S with $x_1 \in \langle x \rangle_k (x_1 \in \langle x \rangle)$ implies $\mu(x_1) \geq \mu(x)$ and $\nu(x_1) \leq \nu(x)$. A is an intuitionistic fuzzy $m_2(m_0$ or $m_1)$-system in S if and only if $A^{(r,s)} = \langle \mu_A^r, \nu_A^s \rangle = \{x \in S | \mu_A^{(x)} > r, \nu_A^{(x)} < s \text{ with } r+s \leq 1\}$ is an $m_2(m_0$ or $m_1)$-system in S for all $r, s \in [0,1)$.*

Proof. Let A be an intuitionistic fuzzy m_2-system in S. Let $x, y \in A^{(r,s)}$ for some $r, s \in [0,1)$. This implies that $\mu_A(x) > r$, $\mu_A(y) > r$, $\nu_A(x) < s$ and $\nu_A(y) < s$. As A is an intuitionistic fuzzy m_2-system in S, there exist $x_1 \in \langle x \rangle_k$ and $y_1 \in \langle y \rangle_k$ such that $\mu_A(x_1y_1) > r$ and $\nu_A(x_1y_1) < s$ implies $x_1y_1 \in \mu_A^{(r,s)}$. Thus $A^{(r,s)}$ is an m_2-system in S.
Conversely, let us assume that $A^{(r,s)}$ is an m_2-system in S for all $r, s \in [0,1)$ with $r+s \leq 1$. If $\mu_A(x) > r$, $\mu_A(y) > r$, $\nu_A(x) < s$ and $\nu_A(y) < s$ for some $r, s \in [0,1)$ and $x, y \in S$. If $(r,s) = (r_1,s_1)$, the result is immediate. So, assume $(r_1,s_1) > (r,s)$. Now $\mu_A(x) > r$, $\mu_A(y) > r_1 > r$, $\nu_A(x) < s$ and $\nu_A(y) < s_1 < s$. Since $A^{(r,s)}$ is an m_2-system in S, then there exist $x_1 \in \langle x \rangle_k$ and $y_1 \in \langle y \rangle_k$ such that $\mu_A(x_1y_1) > r$ and $\nu_A(x_1y_1) < s$. Now $x_1y_1 \in \langle y \rangle_k$ and $\mu_A(x_1y_1) \geq \mu_A(y) > r_1$ and $\nu_A(x_1y_1) \leq \nu_A(y) < s_1$. Thus A is an intuitionistic fuzzy m_2-system of S. □

Theorem 4.6. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy ideal of S and S contain a proper k-ideal $A_{(\alpha,\beta)}$ with $(\alpha,\beta) \neq A(0)$. A is an intuitinistic fuzzy 2-(0- or 1-) prime ideal of S if and only if $A^C = (\mu_A, \nu_A)$ is an intuitionistic fuzzy $m_2(m_0$ or $m_1)$-system of S.*

Proof. Let us assume that A is an intuitionistic fuzzy 2-prime ideal of S. For any $t, t', s, s' \in [0,1)$, with $t+t' \leq 1$, $s+s' \leq 1$, suppose there exist $a, b \in S$ such that $(\nu_A, \mu_A)(a) > (t,t')$ and $(\nu_A, \mu_A)(b) > (s,s') \Rightarrow (\mu_A, \nu_A)(a) < (t',t)$ and $(\mu_A, \nu_A)(b) < (s',s)$. As $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy 2-prime ideal of S, $Im(\mu_A) = \{1, \alpha\}, \alpha \in [0,1)$ and $Im(\nu_A) = \{0, \bar{\alpha}\}, \bar{\alpha} \in (0,1]$. Thus $(\alpha, \bar{\alpha}) < (t',t)$, $(\alpha, \bar{\alpha}) < (s',s)$ and $A(a) = A(b) = (\alpha, \bar{\alpha})$. Let $P = \{x \in S | A(x) = (1,0)\}$. Then by Theorem 3.22, P is a 2-prime ideal in S and $a, b \notin P$. This implies $a, b \in S \setminus P$ which is an m_2-system in S. Thus there exist

$a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $A(a_1b_1) = (\alpha, \bar{\alpha})$. Now $A(a_1b_1) = (\alpha, \bar{\alpha}) < \min\{(t', t), (s', s)\} = (\max\{(t, t'), (s, s')\})^C$. Now $\max\{(t, t'), (s, s')\} = A^C(a_1b_1)$.

Conversely, let us assume that A^C is an intuitionistic fuzzy m_2-system of S. Let A_1, A_2 be two intuitionistic fuzzy k-ideals such that $A_1A_2 \subseteq A$. Suppose that $A_1 \nsubseteq A$ and $A_2 \nsubseteq A$. Now $A_1 = \cup_{a_{(p,p')} \in A_1} a_{(p,p')}$ and $A_2 = \cup_{b_{(q,q')} \in A_2} b_{(q,q')}$. Then there exist $a_{(s,s')} \in A_1$ and $b_{(t,t')} \in A_2$ $s, s', t, t' \in [0, 1)$, such that $A(a) < (s, s')$, $A(b) < (t, t')$. This implies $A^C(a) > (s, s')$ and $A^C(b) > (t, t')$. As A^C is an m_2 system of S, there exist $a_1 \in \langle a \rangle_k$ and $b_1 \in \langle b \rangle_k$ such that $A^C(a_1b_1) > max\{(s', s), (t', t)\} = (min\{(s, s'), (t, t')\})^C$. Thus $A(a_1b_1) < min\{(s, s'), (t, t')\}$ and $(a_1b_1)_{min\{(s,s'),(t,t')\}} \notin A$. Now by Lemma 3.13 and Lemma 3.14, $(a_1b_1)_{min\{(s,s'),(t,t')\}} = (a_1)_{(s,s')}(b_1)_{(t,t')} \in A_1A_2 \subseteq A$, a contradiction. Therefore A is an intuitionistic fuzzy 2-prime ideal of S. □

5. Conclusion

The article presents some study on m-systems and different prime ideals in intuitionistic fuzzy semiring.

Acknowledgement

The first author is thankful to the University Grants Commission, New Delhi-110021, India, for providing OBC National fellowship under grant no: F.4-1/2016-17/NFO-2015-17-OBC-TAM-29759/(SA-III/Website) 01.04.2016.

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