Int. J. Math. And Appl., 6(1–D)(2018), 835–839
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

A Study on Sequencing Problem in Two Different Environments

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Abstract: To represent uncertainty the fuzzy numbers are ideally suited. In the present paper, we have considered the sequence performance measurements and job mean flow time as fuzzy in nature. Using fuzzy technological values, we consider fuzzy sequencing problem in a different method. The concepts of Average High Ranking Method (AHRM) are considered. This idea is used to minimize the total cost in earliness (lateness) at fuzzy environment. It has been shown that fuzzy approaches are used to tackle uncertainty into processing of jobs very effectively. In the procedure to obtain completion time, fuzzy ranking technique and defuzzification methods are involved to compare the fuzzy numbers and to facilitate the effective deterministic algorithm to get the best solution. In the present work, we have compared Crisp sequencing problem and Fuzzy sequencing problem. To support our study, numerical examples are illustrated in this paper.

MSC: 03B52, 90C70

Keywords: Crisp number, fuzzy number, triangular fuzzy number, sequencing problem, fuzzy environment.

1. Introduction

In 1965, Zadeh [20] introduced fuzzy set to deal with uncertainty. In an uncertain environment the fuzzy numbers are used to represent the numerical quantities. The representation of membership function is one of most important criteria that better describes the shape, the nature of the membership function such as triangular fuzzy number, trapezoidal fuzzy number etc. The application of triangular fuzzy number in Matrices is used by Shyamal [18]. Baker [1] describes the sequencing problem with penalties. Cahon [2] has developed fuzzy job sequencing for a job shop. Further the same author in [3] applied fuzzy concepts in production in inventory control. The method to determine the optimal due date on a single machine is given by Chong [4]. The methods to control job lateness were discussed in [5] by Conway. Dueness [6] developed an approach in fuzzy environment to solve multi objective single machine scheduling problem. Single machine problem with fuzzy precedence relation was given by Ishii [9]. Further the same is extended by Xie [19]. Single machine problems under fuzzy processing times are developed in [11, 12]. Ikram [15] developed the concept of minimum lateness in two machine scheduling problem. Fuzzy theory and its operations were discussed in detail by many authors [16, 17, 20]. In this paper, different processing time of each job is considered which helps the contractor to estimate the cost. Also the different due date for each job were considered to satisfy the demand maker to great extend by using the Average High Ranking Method. A new approach to find optimal due date based on processing time of single machine under fuzzy environment is discussed. This article represents a fuzzy mapping on crisp set and fuzzy set to give better solution.

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2. Preliminaries

Definition 2.1. *Let A be a crisp set defined over the universe X. Then for any element x in X, either x is a member of A or not. In a fuzzy set, it is not necessary that x is the full member of the set or not a member. It can be the partial member of the set.*

Definition 2.2. *A fuzzy set* $\widetilde{A}$ *is defined by* $\widetilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. *In the pair* $(x, \mu_A(x))$, *the first element x belong to the classical set A, the second element* $\mu_A(x)$, *belong to the interval [0, 1], called Membership function.*

Definition 2.3. *A fuzzy set* $\widetilde{A}$ *on R must possess at least the following three properties to qualify as a fuzzy number,*

(1). $\widetilde{A}$ *must be a normal fuzzy set;*

(2). $\widetilde{A}$ *must be closed interval for every* $\alpha \in$ *[0,1];*

(3). the support of $\widetilde{A}$, $o\widetilde{A}$, *must be bounded.*

Definition 2.4. *It is a fuzzy number represented with three points as follows:* $\widetilde{A} = (a_1, a_2, a_3)$. *This representation is interpreted as membership functions and holds the following conditions*

(1). a_1 *to* a_2 *is increasing function;*

(2). a_2 *to* a_3 *is decreasing function;*

(3). $a_1 \leq a_2 \leq a_3$.

Definition 2.5. *A sequence is the order in which the jobs are processed. Sequence problems arise when we are concerned with situations where there is a choice in which a number of tasks can be performed. A sequencing problem could involve as Jobs in a manufacturing plant and so on.*

Notation 2.6. *Let*

S_i	*Starting time of job*
C_i	*Completion time of job*
W_i	*Waiting time for job*
R_i	*Relase time of job*
L_i	*The lateness of job*
T_i	$\max(0, c_j - d_j)$
E_i	$\max(0, d_i - c_i)$
A_i	*Average high ranking of the processing time a,b,c*
SI_i	*Slack time of job*
e_i	*Penalty for earliness of job*
l_i	*Penalty for tardiness of job*

Theorem 2.7. *If* $\overline{u} = (a, b, c)$ *is triangular fuzzy number then*

$$V_s f_{(\overline{u})}(\alpha) = s(\alpha)((2b - a - c)\alpha + a + c),$$

$$A_s f_{(\overline{u})}(\alpha) = s(\alpha)(c - a - (c - a)\alpha)$$

In this paper, we found the nearest triangular fuzzy number to $\overline{u}$ with a triangular fuzzy number

$$T(\overline{u}) = (t_1(u), t_2(u), t_3(u)),$$

where $V_s f_{(\overline{u})}(\alpha)$ and $A_s f_{(\overline{u})}(\alpha)$ is nearest to $V_s f_{T(\overline{u})}(\alpha)$ and $A_s f_{T(\overline{u})}(\alpha)$ and because, $Core(\overline{u})$ is a very important for decision maker, hence we want that $Core(\overline{u}) = Core(T(\overline{u}))$, therefore $t_2(u) = \underline{u}(1) = \overline{u}(1)$.

3. Methodology

3.1. Algorithm For Processing Time by Considering All Constraints:

Step 1: Arrange the processing time in increasing order to get optimal sequence.

Step 2: Find the completion time and waiting time $C_i = S_i + P_i$.

Step 3: Find the lateness time and tardiness $T_i = L_i$, if $L_i > 0$ or Otherwise 0.

Step 4: The processing order of each job is given and fixed.

3.2. AHR Algorithm using Triangular Fuzzy Number:

Step 1: Arrange the processing time in triangular fuzzy number.

Step 2: Find the Average High Ranking of the processing time.

Step 3: Find the slack time for all the processing time and due dates $SI_i = |A_i - d_i|$.

Step 4: Then Arrange slack time in increasing order for optimal solution. Suppose the slack time is same then we have to consider them as lowest processing time as before.

Step 5: Find the total penalty of all processing time.

4. Numerical Illustration

Example 4.1. *Consider 6-jobs having processing time and due dates. The following table shows the data.*

Job	1	2	3	4	5	6
P_i	17	10	15	22	26	9
d_i	8	10	9	12	15	11

STP (6-2-3-1-4-5) for each job, the computation for the waiting time using the SPT sequence are below

Job	P_i	S_i	C_i	W_i	d_i	L_i	T_i	Cd_i
6	9	0	9	0	11	-2	0	11
2	10	9	19	9	10	9	9	21
3	15	19	34	19	9	25	25	30
1	17	34	51	34	8	43	43	38
4	22	51	73	51	12	61	61	50
5	26	73	99	73	15	84	84	65

The total penalty cost of job is 165.

Example 4.2. *Consider the 6-job and use the fuzzy triangular number and AHR method to find the penalty coast*

Job	P_i	AHR	d_i	SI_i	e_i	I_i
1	16,17,18	53/3	8	29/3	2	3
2	9,10,11	32/3	10	2/3	2	3
3	14,15,16	47/3	9	20/3	2	3
4	21,22,23	68/3	12	32/3	2	3
5	25,26,27	80/3	15	35/3	2	3
6	8,9,10	29/3	11	4/3	2	3

The 6-jobs having the processing time as fuzzy triangular number are converted into AHR and also used the slack time of all the jobs to find the cost and as per the schedule $S = 2 > 6 > 3 > 1 > 4 > 5$.

Job	Processing Time	SI_i	cost
2	0-32/3	2/3	1.3333
6	32/3-61/3	4/3	4
3	61/3-108/3	20/3	20
1	108/3-161/3	29/3	29
4	161/3-229/3	32/3	32
5	229/3-309/3	35/3	35

The total penalty coast is 121.3333.

5. Conclusion

In this paper, we have compared crisp sequencing problem and fuzzy sequencing problem. Here we have considered triangular fuzzy numbers to represent the fuzzy processing times. The methods involve to compare the fuzzy numbers and to facilitate the effectiveness of the deterministic algorithm to get the best solution. This approach helps the decision maker to identify the process which gives the best alternative.

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