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Stability of Equilibrium Points of the Bigger Primary is a Source of Radiation and the Smaller One is an Oblate Spheroid in Case of Linear

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Abstract: The present paper deals with the equations of motion of photogravitational restricted three body problem in which the bigger primary is the source of radiation and the smaller one is an oblate spheroid.

Keywords: Equilibrium points, triangular, oblatenes, smaller primary.

1. Introduction

The idealized model of restricted three body problem is one of the most celebrated problems of celestial mechanics. The restricted problem specifies the motion of a body of infinitesimal mass under **K** the gravitational attraction of two massive bodies moving about their centre of mass in circular orbit. the developed nations our country has also undertaken space research programme by launching artificial earth satellites for communications and weather forecasting purpose and registered its name as a sixth member nation in the exclusive space club. Indian researchers are contributing their might to make the space research programme successful with the limited resources available at their disposal.

2. Stability of Triangular Equilibrium Point

$$\Omega = \frac{1}{2}\left(x^2+y^2\right) + \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1} + \frac{\mu}{r_2} + \frac{\mu A}{2r_2^3}\right\} \tag{1}$$

$$r_1^2 = (x+\mu)^2 + y^2$$

$$r_2^2 = (x-1+\mu)^2 + y^2 \tag{2}$$

Differentiating (2), we obtain

$$\frac{\partial r_1}{\partial x} = \frac{x+\mu}{r_1};\ \frac{\partial r_1}{\partial y} = \frac{y}{r_1};\ \frac{\partial r_2}{\partial x} = \frac{(x-1+\mu)}{r_2};\ \frac{\partial r_2}{\partial y} = \frac{y}{r_2}$$

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Again differentiating (1) partially with respect to x, we get:

$$\Omega_x = x + \frac{1}{1+\frac{3A}{2}}\left\{-\frac{(1-\mu)(1-q)}{r_1^2}\frac{\partial r_1}{\partial x} - \frac{\mu}{r_2^2}\frac{\partial r_2}{\partial x} - \frac{3\mu A}{2r_2^4}\frac{\partial r_2}{\partial x}\right\}$$

or,

$$\Omega_x = x - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)(x+\mu)}{r_1^3} + \frac{\mu(x-1+\mu)}{r_2^3} + \frac{3\mu A(x-1+\mu)}{2r_2^5}\right\} \tag{3}$$

Differentiating it partially w.r.t. x again, we get

$$\begin{aligned}\Omega_{xx} = 1 - \frac{1}{1+\frac{3A}{2}}&\left\{-\frac{(1-\mu)(1-q)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3\mu A}{2r_2^5} - \frac{3(1-\mu)(1-q)(x+\mu)}{r_1^4}.\frac{\partial r_1}{\partial x}\right.\\ &\left. - \frac{3\mu(x-1+\mu)}{r_2^4}.\frac{\partial r_2}{\partial x} - \frac{15\mu A(x-1+\mu)}{2r_2^6}.\frac{\partial r_2}{\partial x}\right\}\\ = 1 - \frac{1}{1+\frac{3A}{2}}&\left\{\frac{(1-\mu)(1-q)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3\mu A}{2r_2^5} - \frac{3(1-\mu)(1-q)(x+\mu)^2}{r_1^5} - \frac{3\mu(x-1+\mu)^2}{r_2^5} - \frac{15\mu A(x-1+\mu)^2}{2r_2^7}\right\}\end{aligned} \tag{4}$$

Differentiating (3) partially w.r.t. y, we obtain

$$\begin{aligned}\Omega_{xy} &= -\frac{1}{1+\frac{3A}{2}}\left\{-\frac{3(1-\mu)(1-g)(x+\mu)}{r_1^4}.\frac{\partial r_1}{\partial y} - \frac{3\mu(x-1+\mu)}{r_2^4}.\frac{\partial r_2}{\partial y} - \frac{15\mu A(x-1+\mu)}{2r_2^6}.\frac{\partial r_2}{\partial y}\right\}\\ &= \frac{1}{1+\frac{3A}{2}}\left\{\frac{3(1-\mu)(1-q)(x+\mu)}{r_1^5} + \frac{3\mu(x-1+\mu)}{r_2^5} + \frac{15\mu A(x-1+\mu)}{2r_2^7}\right\}y\end{aligned} \tag{5}$$

Differentiating (1) partially w.r.t. y, we obtain

$$\Omega_y = y + \frac{1}{1+\frac{3A}{2}}\left\{-\frac{(1-\mu)(1-q)}{r_1^2}.\frac{\partial r_1}{\partial y} - \frac{\mu}{r_2^2}.\frac{\partial r_2}{\partial y} - \frac{3\mu A}{2r_2^4}.\frac{\partial r_2}{\partial y}\right\}$$

or,

$$\begin{aligned}\Omega_y &= y - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3\mu A}{2r_2^5}\right\}y\\ &= \left\{1 - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3\mu A}{2r_2^5}\right\}\right\}y\end{aligned}$$

Differentiating it partially w.r.t. y

$$\Omega_{yy} = 1 - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3\mu A}{2r_2^5}\right\} + \frac{1}{1+\frac{3A}{2}}\left\{\frac{3(1-\mu)(1-q)}{r_1^5} + \frac{3\mu}{r_2^5} + \frac{15\mu A}{2r_2^7}\right\}y^2 \tag{6}$$

3. Stability of Collinear Equilibrium Points

At the collinear points

$$y = 0$$

$$r_1^2 = (x+\mu)^2$$

$$and \;\; r_2^2 = (x-1+\mu)^2$$

With the help of these relations, it is not difficult to obtain from (4), (5) and (6), the values

$$\Omega_{xx}^{\circ} = 1 + \frac{1}{1+\frac{3A}{2}}\left\{\frac{2(1-\mu)(1-q)}{r_1^3} + \frac{2\mu}{r_2^3} + \frac{6\mu A}{r_2^5}\right\} > 0$$

$$\Omega^{\circ}_{xy} = 0$$

$$and \quad \Omega^{\circ}_{yy} = 1 - \frac{1}{1+\frac{3A}{2}}\left\{\frac{(1-\mu)(1-q)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3\mu A}{r_2^5}\right\} < 0$$

In view of the above equations, we have

$$\lambda^4 - \left[\Omega \overset{\circ}{xx} + \Omega \overset{\circ}{yy} - 4\right]\lambda^2 + \left[\Omega \overset{\circ}{xx}\,\Omega \overset{\circ}{yy} - \Omega \overset{\circ}{xy}\right] = 0 \tag{7}$$

It is easy to note that the roots λ_i $(i = 1, 2, 3, 4)$ of equation (7) are

$$\lambda_{1,2} = \pm\left\{B_1 + \left(B_1^2 + B_2^2\right)^{\frac{1}{2}}\right\}^{\frac{1}{2}} = \pm\lambda$$
$$\lambda_{3,4} = \pm\left\{B_1 - \left(B_1^2 + B_2^2\right)^{\frac{1}{2}}\right\}^{\frac{1}{2}} = \pm \; is$$

where,

$$B_1 = \frac{\Omega^{\circ}_{xx} + \Omega^{\circ}_{yy}}{2} - 2$$
$$B_2^2 = -\Omega^{\circ}_{xx}\Omega^{\circ}_{yy} > 0$$

as $\Omega^{\circ}_{yy} < 0$. It may be noted that $\lambda_{1,2}$ are real while $\lambda_{3,4}$ are pure imaginary. Hence, the collinear points are unstable. Thus we find that the stability of collinear points remain unaffected by the perturbations under consideration.

4. Conclusion

Thus we conclude that the photo gravitational restricted three body problem in which smaller primary is an oblate spheroid and the bigger one is a source of radiation possesses five equilibrium points-two triangular and three collinear. The triangular equilibrium points form nearly equilateral triangles with the primaries and the collinear points lie on the line joining the primaries. The triangular equilibrium points are linearly stable while the collinear points are unstable. The oblateness of the smaller primary as well as the radiation of bigger one affect the location of triangular equilibrium points and introduce a change in their coordinates compared to the classical case. They also reduce the value of critical mass and consequently the range of stability of triangular equilibrium points reduces.

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