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Equilibrium Points and Solutions of the System in Case of Circular Orbit of the Centre of Mass

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Abstract: In this paper, we investigated the effect of the earth's shadow due to solar radiation pressure and the earth's obsoleteness on the motion and stability of a system of two satellites connected by a light flexible and extensible cable in the earth's central gravitational field in the case of a circular orbit of the centre of mass. First of all derived the differential equations of motion of the system for two dimensional case in circular orbit of the centre of mass.

Keywords: Flexible, obsoleteness, Circular orbit, Gravitational field, Jacobian.

1. Introduction

The present study is a direct extension of Sinha and Singh [4] work. studied effect of solar radiation. Pressure on the motion and stability of the system of two interconnected satellites when their centre of mass moves in circular orbit. Again Sinha and Singh [5] could generalize the above problem by considering the centre of mass of the system moving in elliptical orbit. Kumar and Srivastava [7] studied about Evolution and non-evolution motion of a system of two cable-connected artificial satellite under the some perturbative forces. Again Kumar and Prasad [9] studied about non-linear planner oscillation of a cable connected satellites system in non-resonance and many author direct generalization of problem related to measures that our references.

2. Equation of Motion in Case of Circular Orbit

The equation of motion of the system given takes the form of two dimensional cases:

$$\begin{aligned} x'' - 2y' - (3+4B)x + A\psi_1 \cos\epsilon \cos(v-\alpha) + \beta \cos i &= \lambda_\alpha \left[1 - \frac{l_0}{r}\right] x \\ y'' + 2x^1 + By + A\psi_1 \cos\epsilon \sin(v-\alpha) &= \lambda_\alpha \left[1 - \frac{l_0}{r}\right] y \end{aligned} \tag{1}$$

Where

$$A = \frac{\rho^3}{\mu}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)$$

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$$B = \frac{3K_2}{\rho^2}$$
$$\lambda_\alpha = \frac{\rho^3}{\mu} \propto_\propto$$
$$r = \sqrt{x^2 + y^2} \quad (2)$$

in case of circular orbit, the true anomaly V for the elliptic orbit will be replaced by τ whose values is as follows:

$$\tau = wt \quad (3)$$

where w is the angular velocity of the centre of mass of the system in case of circular orbit an 't' is the time of the system respectively the form.

$$x'' - 2y' - (3 + 4B)x + A\psi_1 \cos\epsilon \cos(\tau - \alpha) + \beta \cos i = -\lambda_\alpha \left[1 - \frac{l_0}{r}\right] x$$
$$y'' - 2x' + By - A\psi_1 \cos\epsilon \cos(\tau - \alpha) = -\lambda_\alpha \left[1 - \frac{l_0}{r}\right] y \quad (4)$$

The condition of constraint takes the form

$$x^2 + y^2 \leq l_0^2 \quad (5)$$

Now there will be two types of motion

1. Free motion ($\lambda_\alpha = 0$)
2. Constrained motion ($\lambda_\alpha \neq 0$)

In case of free motion the motion takes with tight string. Thus for free motion we must have from (4)

$$x'' - 2y' - (3 + 4B)x + A\psi_1 \cos\epsilon \cos(\tau - \alpha) + \beta \cos i = 0$$
$$y'' - 2x' + By - A\psi_1 \cos\epsilon \cos(\tau - \alpha) = 0 \quad (6)$$

The system of differential equation (6) can be easily integrated in terms of elementary functions. these integrated clearly indicate that free motion of the satellites of mass m_1 is bound to be converted into constrained motion after some time. Hence here after it is assumed that the system is moving with tight cable like a dumbbell satellites.

The equation of motion can be averaged with respect to t, which varies from 0 to 2π the averaged equations will describe the smaller secular perturbations and long periodic effects due to the periodic forces on the motion of the satellites of mass m_1. And the small secular and long periodic effects of the solar pressure with consideration of the effect of the earth's shadow on the system may be analysed by averaging the periodic term in (4) with respect to τ from θ to $2\pi - \theta$ for the period when the system being under the influence of sun rays directly i.e. for $\psi_1 = 1$ and from $-\theta$ to θ for the period when the system passing through the shadow beam i.e. $\psi_1 = 0$. Thus after averaging the periodic term (4) we have

$$\frac{1}{2\pi}\left[\int_{-\theta}^{\theta} A\cos\epsilon \cos(\tau - \alpha)\, d\tau + \int_{\theta}^{2\pi-\theta} A\cos\epsilon\cos(\tau - \alpha) d\tau\right] = \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi}$$
$$\frac{1}{2\pi}\left[\int_{-\theta}^{\theta} A\cos\epsilon \sin(\tau - \alpha)\, d\tau + \int_{\theta}^{2\pi-\theta} A\cos\epsilon\cos(\tau - \alpha) d\tau\right] = \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} \quad (7)$$

where $\psi_1 = 1$. Thus, the equations of motion (4) of the system are being described by using averaged values (7) in the form.

$$x'' - y' - (3 + 4B)x - \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} = -\lambda_\alpha \left[1 - \frac{l_0}{r}\right] x - \beta\cos i$$

$$y'' + 2x' + By - \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} = -\lambda_\alpha\left[1 - \frac{l_o}{r}\right]y \tag{8}$$

We see that the equation (8) do not contain time explicitly therefore there must exists a Jacobian integral of the problem. Multiply (8) by $2x'$, and $2y'$ and adding both sides. We get after integrating

$$x'^2 + y'^2 - (3+4B)x^2 + By^2 - \frac{-A\cos\epsilon\cos\alpha\sin\theta}{\pi} - \frac{2Ay}{\pi}\cos\epsilon\sin\alpha\sin\theta + \lambda_\alpha\left[\left(x^2+y^2\right) - 2l_0\sqrt{x^2+y^2}\right] = h \tag{9}$$

Where h is the constant of integration equation (9) is below as Jacobian integral of the problem. From equation (9) follows the chance of zero velocity can be obtained in this form

$$(3+4B)x^2 - By^2 + \frac{2Ax\cos\epsilon\cos\alpha\sin\theta}{\pi} + \frac{2Ay\cos\epsilon\sin\alpha\sin\theta}{\pi}\lambda_\alpha\left[\left(x^2+y^2\right) - 2l_0\sqrt{x^2+y^2}\right] = h \tag{10}$$

3. Equilibrium Solution of the Problem

The Equilibrium positions of the system are given by the constant values of the Co-ordinates in the rating from of reference. Now Let $x = x_0$ and $y = y_0$, where x_0 and y_0 are constants give the equilibrium positions then

$$\begin{aligned} x' = x_0' = 0 = x_0'' \\ y' = y_0' = 0 = y_0'' \end{aligned} \tag{11}$$

using (11) in (8) we get

$$\begin{aligned} -(3+4B)x_0 - \frac{A}{\pi}\cos\epsilon\cos\propto\sin\theta &= -\lambda_\alpha\left[1 - \frac{l_0}{r_0}\right]x_0 - \beta\cos i \\ By_0 - \frac{A}{\pi}\cos\epsilon\sin\propto\sin\theta &= -\lambda_\alpha\left[1 - \frac{l_0}{r_0}\right]y_0 \end{aligned} \tag{12}$$

Where, $r_0 = \sqrt{x_0^2 + y_0^2}$. Now we shall discuss the particular solutions of the equation (12) as follows. Thus putting $\alpha = 0$ the equations of motion given (12) takes the form.

$$\begin{aligned} -(3+4B)x_0 - \frac{A}{\pi}\cos\epsilon\sin\theta &= -\lambda_\alpha\left[1 - \frac{l_0}{r_0}\right]x_0 - \beta\cos i \\ By_0 &= -\lambda_\alpha\left[1 - \frac{l_0}{r_0}\right]y_0 \end{aligned} \tag{13}$$

Hence the equilibrium position in this case are given below :

The system may be wholly extended along x - axis and in this case $y = 0$. Thus equilibrium pair is taken as $(a, 0)$. So, putting $x_0 = a$, $y_0 = 0$ in (13), we get.

$$\begin{aligned} -(3+4B)a - \frac{A}{\pi}\cos\epsilon\sin\theta &= -\lambda_\alpha\left[1 - \frac{l_0}{r_0}\right]a - \beta\cos i \\ 0 &= -\lambda_\alpha\left[1 - \frac{l_0}{r_0}\right]0 \end{aligned} \tag{14}$$

Where $r_0 = \sqrt{x_0^2 + 0} = a$. From the first equation of (14), we get

$$\begin{aligned} -(3+4B)a - \frac{A}{\pi}\cos\epsilon\sin\theta &= -\lambda_\alpha a + \lambda_\alpha l_0 - \beta\cos i \\ (\lambda_\alpha - 3 - 4B)a &= \frac{(\lambda_\alpha l_0 - \beta\cos i)\pi + A\cos\epsilon\sin\theta}{\pi} \\ a &= \frac{(\lambda_a l_0 + \lambda_a\beta\cos i)\pi + A\cos\epsilon\sin\theta}{\pi(\lambda_a - 3 - 4B)} \end{aligned}$$

Here the first equilibrium point is given by

$$[a, o] = \left[\frac{(\lambda_\alpha l_0 + \beta\cos i)\pi + \frac{\rho^3}{\mu}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\cos\epsilon\sin\theta}{\pi\left(\lambda_a - 3 - 4\frac{3K_2}{\rho^2}\right)}, 0\right] \tag{15}$$

4. Conclusions

As a result, we conclude that the mass m_1 satellite will move within the boundary of a different zero velocity curves. Represented by (10) for different value of jacobian Constant h. And (15) that the sufficient conditions for the stability of the system at the said equilibrium position $(a, 0)$ in the sense of Liapunov are:-

(i) $-\left(3+4\frac{3K_2}{\rho^2}\right)a-\frac{\frac{\rho^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)}{\pi}\cos\epsilon\sin\theta+\lambda_\alpha a-\lambda_\alpha l_0+\beta\cos i=0$

(ii) $\lambda_\alpha-3-4\frac{3K_2}{\rho^2}-\frac{\lambda_\alpha l_0}{a}>\frac{\beta\cos i}{a}$

(iii) $[a, o]=\left[\frac{(\lambda_\alpha l_0+\beta\cos i)\pi+\frac{\rho^3}{\mu}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\epsilon\sin\theta}{\pi(\lambda_a-3-4\frac{3K_2}{\rho^2})}, 0\right]$

now in order to have the clear the satisfactory of the system at the equilibrium position $(a, 0)$ we have to examine the above condition one by one separately.

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