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Best Proximity for Meir Keeler Contraction in Rectangular Metric Space

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Abstract

In this paper we prove the Best Proximity Point Theorem for a Meir Keeler Contraction in Rectangular M-Metric Space.

Keywords: Best Proximity Point; Meir Keeler Contraction; M-Metric Space; Rectangular M-Metric Space; p-property.

1. Introduction and Preliminaries

One of the topics in Fixed Point Theorem is the singularity of a Fixed Point of Non-Self Map. If a Contraction of Non-Self Mapping $T : A \to B$ does not always result in a Fixed Point $Tx = x$, it makes sense to look into the possibility of x such that d (x, Tx) is least. At this moment, the idea of the ideal closeness emerges. A point x is referred as the Best Proximity Point of

$$T : A \to B \;\; if \;\; d\left(x, Tx\right) = d(A, B) \;\; where \;\; d\left(A, B\right) = \inf\left\{d\left(x, y\right) : x \in A, y \in B\right\}$$

When a Non-Self Mapping T lacks a Fixed Point, a Best Proximity Point denotes the Best Approximation to the exact solution $Tx = x$. If $d(A, B) = 0$. It is evident that a Fixed Point and the Best Proximity Point coincide. The Best Proximity Point Theorem is logical expansions of the Banach Contraction Principle because it shows that a Best Proximity Point reduces to a Fixed Point if the map is believed to be a Self-Mapping [1]. Fan [7] created a Best Approximation Theorem and presented the idea of a Best Proximity in 1969. The well-known Best Contraction Point [1] establishes "the existence and uniqueness of Fixed Point of Contraction Mapping in the context of Metric Space in the field of Metric Fixed Point Theory". The Banach Contraction Principle can be generalized in two different ways [1]. In the first, the active contraction can be changed, and in the second, the Metric Space is changed. Numerous authors, primarily those who studied Partial Metric Space, b-Metric

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Space, Partial b-Metric Space, Branciari Metric Space, Partial Rectangular Metric Space, M-Metric Space, Rectangular M-Metric Space, Rectangular M_b-Metric Space, and others [11-32], generalized the Metric Space. There are numerous generalized contractions found in literature. However, we provide the few names like Meir Keeler Contraction [33], Kannan Contraction [34], Boyd Wong Contraction [35], etc. because of their applicability. Rectangular Metric Space was proposed by Branciari [6] in 2000, and he also came up with a Fixed-Point Theorem. Asadi et al. [8, 36] presented the M-Metric Space in 2014, extending the b-Metric Space, and established numerous Fixed-Point Theorem for the Meir Keeler Contraction and the Banach Contraction Principle. Meir Keeler plays a major role in the Contraction T, and Ozgur [9] extends both Rectangular M-Metric Space and several Fixed-Point Theorem. Here we extend the result for best proximity point theorem. Here We wish to recall some definitions to justify our result.

Definition 1.1 ([2]). *Let C and D be non-empty subsets of metric space (X, d) denoted by C_0 and D_0*

$$C_0 = \{x \in C : d\,(x, y) = d(C, D) \;\; \text{for some} \;\; y \in D\}$$
$$D_0 = \{y \in D : d\,(x, y) = d(C, D) \;\; \text{for some} \;\; x \in C\}$$

Definition 1.2 ([1]). *Let M_s be a non-empty set. A Meir Keeler map $T : M_s \to M_s$ on an RMS (Y, M_s) s.t $\forall\;\; \epsilon > 0, \exists\;\; \delta > 0$ s.t $\exists\;\; c, d\;\; \in Y$ and $\epsilon \le M_s\,(c, d) < \epsilon + \delta \Rightarrow M_s\,(Tc, Td) < \epsilon$.*

Definition 1.3 ([3]). *If Y be a non-empty set. A function $s : Y \times Y \to R^+$ is said to be a Rectangular Metric on Y if it satisfies the following (for all $c, d \in Y$ and for all distinct point $e, f \in Y/\{c, d\}$)*

(i) $s\,(c, d) = 0$ *if and only if* $c = d$

(ii) $s\,(c, d) = s(d, c)$

(iii) $s\,(c, d) \le s\,(c, e) + s\,(e, f) + s(f, d)$

The pair (Y, s) is said to be RMS.

Definition 1.4 ([5]). *If Y be a non-empty set. A function $q : Y \times Y \to R^+$ is said to be a Partial Rectangular Metric on Y, if for $c, d \in Y$ and for all distinct point $e, f \in Y/\{c, d\}$ it satisfies the following*

(i) $c = d$ *if and only if* $q\,(c, c) = q(d, d)$

(ii) $q(c, c) \le q(c, d)$

(iii) $q\,(c, d) = q(d, c)$

(iv) $q\,(c, d) \le q\,(c, e) + q\,(e, f) + q\,(f, d) - q\,(e, e) - q(f, f)$

The pair (Y, q) is said to be Partial Rectangular Metric Space.

Definition 1.5 ([20]). *If Y be a non-empty set. A function $m : Y \times Y \to R^+$ is said to be MM if it satisfies the following*

(i) $m(x,x) = m(y,y) = m(x,y) \Longleftrightarrow x = y$

(ii) $m_{xy} \leq m(x,y)$

(iii) $m(x,y) = m(y,x)$

(iv) $\left(m(x,y) - m_{xy}\right) \leq (m(x,z) - m_{xz}) + (m(z,y) - m_{yz})$

The pair (Y,m) is said to be m-MS.

Definition 1.6 ([11]). *If Y be a non-empty set and $m_s : Y \times Y \to R^+$ a map. If it satisfies the following axioms for all $x, y \in Y$*

(i) $m_s(x,y) = m_{s_{xy}} = M_{s_{xy}} \Longleftrightarrow x = y$

(ii) $m_{s_{xy}} \leq m_s(x,y)$

(iii) $m_s(x,y) = m_s(y,x)$

(iv) $\left(m_s(x,y) - M_{s_{xy}}\right) \leq \left(m_s(x,y) - M_{s_{xy}}\right) + (m_s(u,v) - m_{s_{uv}}) + (m_s(v,y) - m_{s_{vy}})$ *for all* $u, v \in Y/\{x,y\}$

The pair (Y, m_s) is said to be M-Metric Space.

Definition 1.7 ([26]). *Let (Y, m_s) be a Rectangular M-Metric Space. Then*

(i) A sequence $\{x_n\}$ in Y converges to a point x if and only if

$$\lim_{n\to\infty} (m_s(x_n,x) - m_{snx}) = 0 \tag{1}$$

(ii) A sequence $\{x_n\}$ in Y is said to be m_r-CS if and only if $\lim_{n,m\to\infty} (m_s(x_n,x_m) - m_{sx_nx_m}) = 0$ *and*

$$\lim_{n,m\to\infty} (M_s(x_n,x_m) - M_{sx_nx_m}) = 0 \tag{2}$$

(iii) A RMMS is said to be m_s-CS $\{x_n\}$ converges to x s.t $\lim_{n\to\infty} (m_s(x_n,x) - m_{sx_nx}) = 0$ *and*

$$\lim_{n\to\infty} (M_s(x_n,x) - M_{sx_nx}) = 0 \tag{3}$$

Definition 1.8 ([4]). *Let (Y, M_s) be a Rectangular M-Metric Space. A map $T : M_s \to M_s$ is a MKC if* $\forall\ \epsilon > 0, \exists\ \delta > 0$ s.t $\exists\ x, y \in Y$ *and*

$$\epsilon \leq M_s(x,y) < \epsilon + \delta \Rightarrow M_s(Tx,Ty) < \epsilon \tag{4}$$

Definition 1.9 ([37]). *Let (C, D) be a pair of non-empty subsets of Complete Metric Space (X, d) with $C_0 \neq 0$. Then the pair (C, D) is said to be pp if and only if for any $c_1, c_2 \in C_0$ and $d_1, d_2 \in D_0, d(c_1, c_2) = d(C, D) = d(d_1, d_2)$.*

Definition 1.10 ([27]). *Let the function $\phi : [0, \infty) \to [0, \infty)$ which satisfy*

(i) ϕ is continuous and ND

(ii) $\phi(t) = 0$ if and only if $t = 0$.

2. Known Results

We need the following theorems to discuss the main result.

Theorem 2.1 ([13]). *Let (Y, m_s) be a Rectangular M-Metric Space and T be a self-map on Y. If there exists $k \in [0, 1)$ such that*

$$m_s(Tx, Ty) \leq k m_s(x, y) \quad \forall \ x, y \in Y \tag{5}$$

and consider the sequence $\{x_n\}$ defined by $x_{n+1} = Tx_n$. If $x_n \to x$ as $n \to \infty$. Then, $Tx_n \to x_n$ as $n \to \infty$.

Theorem 2.2 ([7]). *Let (Y, m_s) be a Complete Rectangular M-Metric Space and T be a self-map on Y. If there exists $k \in [0, 1)$ such that*

$$m_s(Tx, Ty) \leq k m_s(x, y) \quad \forall \ x, y \in X \tag{6}$$

Then T has a unique fixed point $u \in Y$, where $m_s(u, u) = 0$.

Theorem 2.3 ([14]). *Let (Y, m_s) be a Complete Rectangular M-Metric Space and T be a self-map on Y. If there exists $k \in [0, \frac{1}{2})$ such that*

$$m_s(Tx, Ty) \leq k[m_s(x, Tx) + m_s(y, Ty) \quad \forall \ x, y \in Y \tag{7}$$

Then T has a unique fixed point $u \in Y$, where $m_s(u, u) = 0$.

Theorem 2.4 ([17]). *Let (Y, M_s) be a m_r-Complete Rectangular M-Metric Space and Let T be a Meir Keeler Contraction. Then, T has a unique fixed point $z \in Y$. Moreover, for all $x \in Y$, the sequence $\{T_n(x)\}$ converges to z.*

Theorem 2.5 ([7]). *Let (Y, M_s) be a m_s-Complete Rectangular M-Metric Space and satisfying Meir Keeler Contraction. Let A, B be non-empty continuous set of Y such that $A_0 \neq \phi$. Let $T : A \to B$ be a map satisfying $T(A_0) \subseteq B_0$. Suppose*

$$m_s(Tx, Ty) - d(A, B) \leq m_s(x, y) - d(A, B) \tag{8}$$

for all $x \in A, y \in B$. Then T has a Best Proximity Point.

Theorem 2.6 ([1]). *Let (Y, M_s) be a Complete Rectangular M-Metric Space and let T be a continuous mapping satisfying Meir Keeler Contraction. Let A, B be non-empty closed subsets of Y such that $A_0 \neq \phi$. Let $T : A \to B$ be a map satisfying*

1. *T is continuous and $T(A_0) \subseteq B_0$.*
2. *$\forall\ \epsilon > 0, \exists\ \delta > 0\ \forall\ x \in A, y \in B \in \leq km_s(x, y) - d(x, y,) < \in +\delta \Rightarrow M_s(Tx, Ty) - d(x, y) < \in$ for some $k \in [0, \frac{1}{3})$, then T has a Best Proximity Point.*

Theorem 2.7 ([10]). *Let (Y, M_s) be a Rectangular M-Metric Space and satisfying Meir Keeler Contraction. Let $T : A \to B$ be a map satisfying $T(A_0) \subseteq B_0$. Assume that there exists a function $\phi : [0, \infty) \to [0, \infty)$ satisfying the following:*

1. *$\psi(0) = 0$ and $t > 0 \Rightarrow \phi(t) > 0$*
2. *ϕ is non decreasing and right continuous*
3. *$\forall\ \epsilon > 0, \exists\ \delta > 0$ such that $\in \leq \phi(kc_s(x, y)) - d(x, y,) < \in +\delta \Rightarrow \phi(m_s(Tx, Ty)) - d(x, y) < \in$ for some $k \in [0, \frac{1}{3})$, then T has a Best Proximity Point.*

3. Lemmas

The followings lemmas are in need to prove the main result.

Lemma 3.1 ([9]). *Assume that $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in a Rectangular M-Metric Space (Y, M_s). Then, $\lim_{n\to\infty} \left(m_s(x_n, y_n) - m_{sx_ny_n}\right) = m_s(x, y) - m_{sxy}$.*

Lemma 3.2 ([8]). *Assume $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in a Rectangular M-Metric Space (Y, m_s). Then, $\lim_{n\to\infty} \left(m_s(x_n, y) - m_{sx_ny}\right) = m_s(x, y) - m_{sxy}\quad \forall\quad y \in Y$.*

Lemma 3.3 ([24]). *Assume $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in a Rectangular M-Metric Space (Y, m_s). Then, $\lim_{n\to\infty} \left(m_s(x_n, y_n) - m_{sx_ny_n}\right) = m_s(x, y) - m_{sxy}$.*

Lemma 3.4 ([15]). *Assume $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in a Rectangular M-Metric Space (Y, m_s). Then, $m_s(x, y) = m_{sxy}$. Further if $m_s(y, y)$, then $x = y$.*

Lemma 3.5 ([3]). *Let $\{x_n\}$ be a sequence in a Rectangular M-Metric Space (Y, m_s) such that there exists $s \in [0, \infty)$ s.t*

$$m_s(x_{n+1}, x_n) \leq sm_s(x_n, x_{n-1}) \quad \forall\ \ n \in N \tag{9}$$

Then

(a). $\lim_{n\to\infty} (M_s(x_n, x_{n-1})) = 0$

(b). $\lim_{n\to\infty} (M_s(x_n, x_n)) = 0$

(c). $\lim_{n,m\to\infty} (M_{sx_nx_m}) = 0$

(d). $\{x_n\}$ *is an* m_s*−CS.*

4. Main Results

Theorem 4.1. *Let (Y, M_s) be a m_s-Complete Rectangular M-Metric Space and satisfying Meir Keeler Contraction. Let (C, D) be non-empty closed subsets of Y such that $C_0 \neq \phi$. Let $T : C \to D$ be a map satisfying $T(C_0) \subseteq D_0$. Suppose*

$$M_s\,(Tc, Td) - d\,(C, D) \leq M_s\,(c, d) - d(C, D) \tag{10}$$

for all $c \in C, d \in D$. Then T has a Best Proximity Point.

Proof. Let $c_0 \in C$. Since $Tc_0 \in T(C_0) \subset D_0$, there exists $c_1 \in C_0$ such that $d(c_1, Tc_0) = d(C, D)$. Similarly, $Tc_1 \in T(C_0) \subset D_0$, we choose $c_2 \in C_0$ such that $d(c_2, Tc_1) = d(C, D)$. Again, we get a sequence $\{c_n\}$ in C_0 satisfying

$$d(c_{n+1}, Tc_n) = d(C, D) \quad \forall \;\; n \in N \tag{11}$$

Claim: $d(c_n, c_{n+1}) \to 0$

If $c_n = c_{n+1}$, then c_n is a Best Proximity Point. By pp, we have

$$d(c_{n+1}, c_{n+2}) = d(Tc_n, Tc_{n+1})$$

Suppose $c_n \neq c_{n+1}$ for all $n \in N$. Since $d(c_{n+1}, Tc_n) = d(C, D)$, from (11), we have for all $n \in N$.

$$\begin{aligned} M_s(c_{n+1}, c_{n+2}) &= m_s(Tc_n, Tc_{n+1}) \\ &\leq M_s(c_n, c_{n+1}) - d(C, D) \end{aligned} \tag{12}$$

We get $M_s(c_n, c_{n+1}) = 0$, a contradiction.

We get from (11) that $M_s(c_n, c_{n+1}) = 0$ contradicting our assumption. Therefore $M_s\,(c_{n+1}, c_{n+2}) < M_s(c_n, c_{n+1})$ for any $n \in N$ and $\{M_s(c_n, c_{n+1})\}$ is Non-Decreasing Sequence of Non-Negative Integers, there exists $s \geq 0$ such that $\lim\limits_{n\to\infty} M_s(c_n, c_{n+1}) = s$. We get from (11), for any $n \in N$,

$$M_s(c_{n+1}, c_{n+2}) \leq m_s(c_n, c_{n+1})$$

as $n \to \infty$ in the above equations, and using m_s and M_s, we get $M_s(s) \leq m_s(s)$ this implies $m_s(s) = 0$. Hence

$$\lim_{n\to\infty} M_s(c_n, c_{n+1}) = 0 \tag{13}$$

Next, we show that $\{c_n\}$ is a CS. If otherwise there exists $\varepsilon > 0$, for which we can find two sequences of positive integers (m_k) and (n_k) such that for all positive integers $m_k > n_k > k$, $Ms(c_{m_k}, c_{n_k}) \geq \varepsilon$ and $M_s(c_{m_k}, c_{n_{k-1}}) < \varepsilon$.

Now $\varepsilon \leq M_s(c_{m_k}, c_{n_k}) \leq M_s(c_{m_k}, c_{n_{k-1}}) + Ms(c_{n_{k-1}}, c_{n_k})$, that is $\varepsilon \leq M_s(c_{m_k}, c_{n_k}) < \varepsilon + M_s(c_{n_{k-1}}, c_{n_k})$ as

$k \to \infty$ in the above equation and using (13) we get

$$\lim_{k\to\infty} M_s(c_{m_k}, c_{n_k}) = \varepsilon \tag{14}$$

Again $M_s(c_{m_k}, c_{n_k}) \leq M_s(c_{m_k}, c_{m_{k+1}}) + M_s(c_{m_{k+1}}, c_{n_{k+1}}) + M_s(c_{n_{k+1}}, c_{n_k})$ as $k \to \infty$ in the above equation and using (13) and (14) we get

$$\lim_{k\to\infty} M_s(c_{m_{k+1}}, c_{n_{k+1}}) = \varepsilon \tag{15}$$

Again $M_s(c_{m_k}, c_{n_k}) \leq M_s(c_{m_k}, c_{n_{k+1}}) + M_s(c_{n_{k+1}}, c_{n_k}) \leq M_s(c_{m_k}, c_{n_k}) + M_s(c_{n_k}, c_{n_k})$. Let $k \to \infty$ in the above equation and using (13) and (14) we have

$$\lim_{k\to\infty} M_s(c_{m_k}, c_{n_{k+1}}) = \varepsilon \tag{16}$$

$$\lim_{k\to\infty} M_s(c_{n_k}, c_{m_{k+1}}) = \varepsilon \tag{17}$$

For $c = c_{m_k}$, $d = c_{n_k}$ we have

$$\begin{aligned} M_s(c_{m_k}, Tc_{m_k}) - d(C,D) &\leq M_s(c_{m_k}, c_{m_{k+1}}) + M_s(c_{m_{k+1}}, Tc_{n_k}) - d(C,D) \\ &= M_s(c_{m_k}, c_{m_{k+1}}) \end{aligned}$$

Similarly,

$$M_s(c_{n_k}, Tc_{n_k}) - d(C,D) = M_s(c_{n_k}, c_{n_{k+1}})$$
$$M_s(c_{m_k}, Tc_{n_k}) - d(A,B) = M_s(c_{m_k}, c_{n_{k+1}})$$
$$M_s(c_{n_k}, Tc_{m_k}) - d(C,D) = M_s(c_{n_k}, c_{m_{k+1}})$$

From (12) we have

$$\begin{aligned} M_s(c_{m_{k+1}}, Tc_{n_{k+!}}) &= M_s(c_{m_k}, c_{n_{k+1}}) \\ &\leq M_s\left(c_{m_k}, Tc_{n_k}\right) - d\left(C,D\right) + M_s\left(c_{m_k}, Tc_{m_k}\right) - d\left(C,D\right) + M_s\left(c_{n_k}, Tc_{n_k}\right) - d\left(C,D\right) \\ &+ M_s\left(c_{m_k}, c_{n_k}\right) - d\left(C,D\right) + M_s\left(c_{m_k}, Tc_{m_k}\right) - d\left(C,D\right) + M_s\left(c_{n_k}, Tc_{n_k}\right) - d(C,D) \end{aligned}$$

It follows

$$M_s\left(Tc_{m_k}, Tc_{n_k}\right) \leq M_s\left(c_{m_k}, c_{n_k}\right) + M_s\left(c_{n_k}, Tc_{n_{k+1}}\right) + M_s\left(c_{m_k}, Tc_{m_{k+!}}\right)$$

From (14), (15), (16) and (17) and let $k \to \infty$ in the above equations and using M_s, we get $M_s\left(\varepsilon\right) \leq M_s\left(\varepsilon\right)$, which is a contradiction by M_s. Hence $\{c_n\}$ is a CS.
Since $\{c_n\} \subset C$ and C is a Cauchy Sequence of the Complete Metric Space (X,d), there exists c^* in C such that $c_n \to c^*$. Put c= c_n and d= c^* and since $M_s(c_n, Tc^*) \leq M_s(c_n, c^*) + M_s(c^*, Tc_n)$ and

$M_s(c^*, Tc_n) \leq M_s(c^*, Tc^*) + M_s(Tc^*, Tc_n)$, we have

$$M_s(c_{n+1}, Tc^*) - d(C,D) \leq M_s(Tc_n, Tc^*) \leq m_s\,(c_n, c^*) + M_s(c_n, Tc_n + M_s(c^*, Tc^*) - d(C,D)$$

as $n \to \infty$ in the above equations and using M_s and m_s, we get

$$M_s(c^*, Tc^*) - d(C,D)) \leq m_s(c^*, Tc^*) - d(C,D$$

This implies $M_s(c^*, Tc^*) = d(C,D)$. Hence c^* is a Best Proximity Point of T.
Next to prove uniqueness: Let e and f be two Best Proximity Point and suppose $e \neq f$, then put $c = e$ and $d = f$ in (10), we get

$$M_s\left(T_e, T_f\right) - d\left(C,D\right) \leq m_s\left(e,f\right) - d\left(C,D\right)$$

That is $M_s\left(e,f\right) \leq m_s\left(e,f\right)$ contradiction by Meir Keeler Contraction. Hence $e = f$. □

5. Conclusion

Rectangular M-Metric is a newly developing concept in Fixed Point Theory. Here we have established Meir Keeler Contraction in Rectangular M-Metric Space. We may extend this result to b-metric space in future.

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