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Cordial Labeling of Double Antenna One Point Union Graphs

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Abstract: We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as double tail graph of C_3. i.e. $G = tail(C_3, 2 - P_m)$. In $tail(C_3, 2 - P_m)$ graph double paths are attached at same point of C_3. We restrict our attention to $m = 2, 3, 4$. Further we consider all possible structures of $G^{(k)}$ by changing the common point on G and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures under cordial labeling.

MSC: 05C78.

Keywords: Cordial, labeling, tail graph, invariance, path, one point union.

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1. Introduction and Preliminaries

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West. Cahit [9] introduced the concept of cordial labeling [5]. $f : V(G) \to \{0, 1\}$ be a function. From this label of any edge (uv) is given by $|f(u) - f(v)|$. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e. $v_f(1)$ differ at most by one. Similarly number of edges labeled with 0 i.e. $e_f(0)$ and number of edges labeled with 1 i.e. $e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4). A lot of work has been done in this type of labeling. One may refer dynamic survey by J.Gallian [8].

To obtain a antenna graph or a tail graph we attach one end of P_m to some vertex of G. We denote it by $tail(G, P_m)$ or $ante(G, P_m)$. If there are more paths say $p_{t1}, p_{t2}, \ldots, p_{tk}$ attached to same vertex of G we denote it by $tail(G, p_{t1}, p_{t2}, \ldots, p_{tk})$. If there are m paths of same length say t attached at the same point of G we denote it by $(G, m - p_t)$.

2. Main Results

Theorem 2.1. *All non-isomorphic structures of one point union of $tail(C_3, 2 - P_2)$ graph are cordial.*

Proof. $G = ante(C_3, 2 - P_2)$. Both P_2 are attached at the same point on C_3. G has 5 edges and 5 vertices. Define $f : V(G) \to \{0, 1\}$ as follows. We get different types of labeling units and we use them to obtain a labeled copy of $G^{(k)}$.

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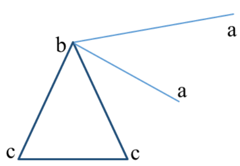

Figure 1: $ante(C_3, 2 - P_2)$

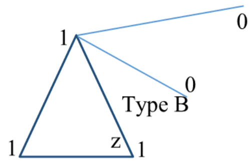

Figure 3: $v_f(0,1) = (2,3)$, $e_f(0,1) = (3,2)$

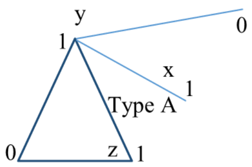

Figure 2: $v_f(0,1) = (2,3)$, $e_f(0,1) = (2,3)$

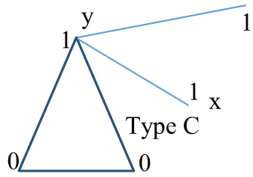

Figure 4: $v_f(0,1) = (2,3)$, $e_f(0,1) = (3,2)$

All above A, B, C type units are cordial. There are three non-isomorphic structures possible on G by taking one point union at 'a', 'b' or 'c'. If we take one point union at vertex 'a' we get **structure 1**.

In that case we fuse Type A label with type C label at point 'x' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'b' we get **structure 2**.

In that case we fuse Type A label with type C label at point 'y' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'c' we get **structure 3**.

In that case we fuse Type A label with type B label at point 'z' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and B type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. In all three structures above the label number distribution is same and in the way given below. On vertices $v_f(0,1) = (2 + 2(k-1), 3 + 3(k-1))$, $k = 1, 2, \ldots$, and on edges if $k = 2x, x = 1, 2, \ldots$, we get $e_f(0,1) = (5x, 5x)$ and if k is of type $k = 2x + 1$, we have $e_f(0,1) = (5x + 2, 5x + 3)$ where $x = 0, 1, 2, \ldots$. Thus the graph is cordial. □

Theorem 2.2. *All non-isomorphic structures of one point union of $ante(C_3, 2 - P_3)$ graph are cordial.*

Proof. $G = ante(C_3, 2 - P_3)$. Both P_3 are attached at the same point on C_3. G has 7 edges and 7 vertices. Define $f : V(G) \to \{0, 1\}$ as follows. We get different types of labeling units with cordial labeling and we use them to obtain a labeled copy of $G^{(k)}$.

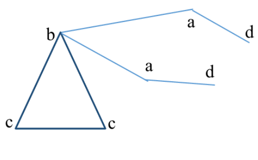

Figure 5: $ante(C_3, 2 - P_2)$

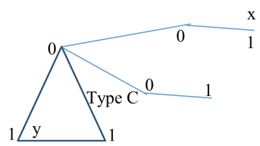

Figure 7: $v_f(0,1) = (3,4), e_f(0,1) = (3,4)$

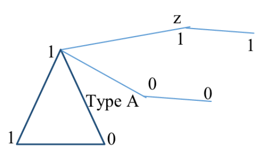

Figure 6: $v_f(0,1) = (3,4), e_f(0,1) = (3,4)$

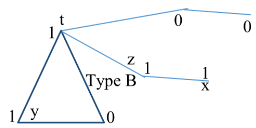

Figure 8: $v_f(0,1) = (3,4), e_f(0,1) = (4,3)$

All above A, B, C type units are cordial. There are four non-isomorphic structures possible on G. If we take one point union at vertex ‘d’ we get **structure 1**.

In that case we fuse Type B label with type C label and fuse it at point ‘x‘ on it. To obtain $G^{(k)}$ we use type B label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex ‘c’ we get **structure 2**.

In that case we fuse Type B label with type C label at point ‘y‘ on it. To obtain $G^{(k)}$ we use type B label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex ‘a’ we get **structure 3**.

In that case we fuse Type B label with type A label at point ‘z‘ on it. To obtain $G^{(k)}$ we use type B label if $i \equiv 1\ (mod\ 2)$ and A type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex ‘b’ we get **structure 4**.

In that case we fuse Type B label with type A label at point ‘t‘ on it. To obtain $G^{(k)}$ we use type B label if $i \equiv 1\ (mod\ 2)$ and A type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. In all four structures the label number distribution is same and in the way given below. On vertices $v_f(0,1) = (3 + 3(k-1), 4 + 3(k-1))$, $k = 1, 2, \ldots,$ and on edges if $k = 2x, x = 1, 2, \ldots,$ we get $e_f(0,1) = (7x, 7x)$ and if k is of type $k = 2x + 1$, we have $e_f(0,1) = (7x + 4, 7x + 3)$ where $x = 0, 1, 2, \ldots$. Thus the graph is cordial. □

Theorem 2.3. *All non-isomorphic structures of one point union of $ante(C_3, 2 - P_4)$ graph are cordial.*

Proof. $G = ante(C_3, 2 - P_3)$. Both P_3 are attached at the same point on C_3. G has 9 edges and 9 vertices. Define $f : V(G) \to \{0, 1\}$ as follows. We get different types of labeling units and we use them to obtain a labeled copy of $G^{(k)}$.

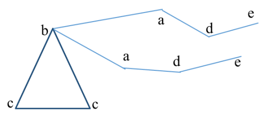

Figure 9: $ante(C_3, 2 - P_2)$

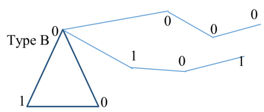

Figure 11: $v_f(0,1) = (6,3), e_f(0,1) = (4,5)$

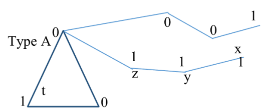

Figure 10: $v_f(0,1) = (4,5), e_f(0,1) = (5,4)$

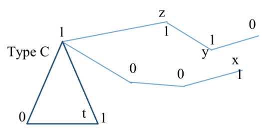

Figure 12: $v_f(0,1) = (4,5), e_f(0,1) = (4,5)$

All above A, B, C type units are cordial. There are five non-isomorphic structures possible on G. If we take one point union at vertex ‘e’ we get **structure 1**.

In that case we fuse Type A label with type C label and fuse it at point ‘x‘ on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex ‘d’ we get **structure 2**.

In that case we fuse Type A label with type C label at point ‘y‘ on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex ‘a’ we get **structure 3**.

In that case we fuse Type A label with type C label at point ‘z‘ on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex ‘c’ we get **structure 4**.

In that case we fuse Type A label with type C label at point ‘t‘ on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. In all four structures the label number distribution is same and in the way given below. On vertices $v_f(0,1) = (4 + 4(k-1), 5 + 4(k-1))$, $k = 1, 2, \ldots,$ and on edges if $k = 2x, x = 1, 2, \ldots,$ we get $e_f(0,1) = (9x, 9x)$ and if k is of type $k = 2x + 1$, we have $e_f(0,1) = (5 + 9x, 4 + 9x)$ where $x = 0, 1, 2, \ldots$. If we take one point union at vertex ‘b’ we get **structure 5**.

In that case we fuse Type A label with type B label at point ‘s‘ on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$

and B type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. The label numbers observed are as follows: On vertices $v_f(0,1) = (4+8x, 5+8x)$ for $k = 2x+1, x = 0, 1, 2, \ldots,$ and if k is of type $2x$ we have $v_f(0,1) = (8x+1, 8x)$. On edges if $k = 2x+1$, $x = 0, 1, 2, \ldots,$ we get $e_f(0,1) = (5+9x, 4+9x)$ and if k is of type $k = 2x$ we have $e_f(0,1) = (9x, 9x)$ where $x = 1, 2, \ldots$. Thus the graph is cordial. □

Theorem 2.4. *All non-isomorphic structures of one point union of $tail(C_3, 2-P_5)$ graph are cordial.*

Proof. Let $G = ante(C_3, 2-P_5)$. Both P_5 are attached at the same point on C_3. G has 11 edges and 11 vertices. Define $f : V(G) \to \{0,1\}$ as follows. We get different three types of labeling units, all cordial and we use them to obtain a labeled copy of G.

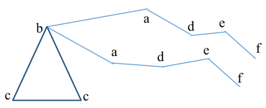

Figure 13: $ante(C_3, 2-P_2)$

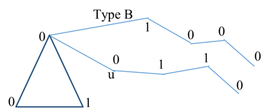

Figure 15: $v_f(0,1) = (6,5), e_f(0,1) = (5,6)$

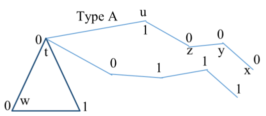

Figure 14: $v_f(0,1) = (6,5), e_f(0,1) = (6,5)$

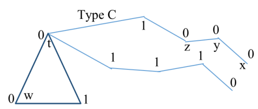

Figure 16: $v_f(0,1) = (6,5), e_f(0,1) = (5,6)$

All above A, B, C type units are cordial. There are six non-isomorphic structures possible on G. If we take one point union at vertex 'f' we get **structure 1**.

In that case we fuse Type A label with type C label at point 'x' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'e' we get **structure 2**.

In that case we fuse Type A label with type C label at point 'y' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'd' we get **structure 3**.

In that case we fuse Type A label with type C label at point 'z' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'b' we get **structure 4**.

In that case we fuse Type A label with type C label at point 't' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'c' we get **structure 5**.

In that case we fuse Type A label with type C label at point 'w' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'a' we get **structure 6**.

In that case we fuse Type A label with type B label at point 'u' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and B type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. In all six structures above the label number distribution is same and in the way given below. On vertices $v_f(0,1) = (1+5k, 5k)$ for all k and for $k = 2x+1$ on edges we have $e_f(0,1) = (6+11x, 5+11x)$, $x = 0, 1, 2, \ldots$ and $e_f(0,1) = (11x, 11x)$ for k is of type $k = 2x$ where $x = 1, 2, \ldots$. Thus the graph is cordial. □

Theorem 2.5. *All non-isomorphic structures of one point union of $ante(C_3, 2-P_6)$ graph are cordial.*

Proof. Let $G = ante(C_3, 2-P_6)$. Both P_6 are attached at the same point on C_3. G has 13 edges and 13 vertices. Define $f : V(G) \to \{0,1\}$ as follows. We get different types of labeling units and we use them to obtain a labeled copy of G.

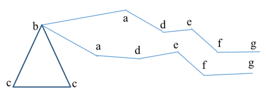

Figure 17: $ante(C_3, 2-P_2)$

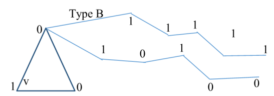

Figure 19: $v_f(0,1) = (5,8), e_f(0,1) = (6,7)$

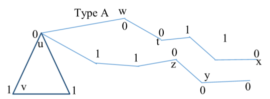

Figure 18: $v_f(0,1) = (7,6), e_f(0,1) = (7,6)$

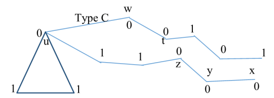

Figure 20: $v_f(0,1) = (7,6), e_f(0,1) = (6,7)$

All above A, B, C type units are cordial. There are 7 non-isomorphic structures possible on $G^{(k)}$. If we take one point union at vertex 'g' we get **structure 1**.

In that case we fuse Type A label with type C label at point 'x' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'f' we get **structure 2**.

In that case we fuse Type A label with type C label at point 'y' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'e' we get **structure 3**.

In that case we fuse Type A label with type C label at point 'z' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'd' we get **structure 4**.

In that case we fuse Type A label with type C label at point 't' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'a' we get **structure 5**.

In that case we fuse Type A label with type C label at point 'w' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and C type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. If we take one point union at vertex 'b' we get **structure 6**.

In that case we fuse Type A label with type B label at point 'u' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and B type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. In all six structures above the label number distribution is same and in the way given below. The label number distribution is on vertices $v_f(0,1) = (7+12x, 6+12x)$ on edges we have $e_f(0,1) = (7+13x, 6+13x)$, when k is of type $k = 2x+1, x = 0, 1, 2, \ldots$ and on vertices $v_f(0,1) = (12x+1, 12x)$ on edges we have $e_f(0,1) = (13x, 13x)$ for k is of type $k = 2x$ where $x = 1, 2, \ldots$. If we take one point union at vertex 'c' we get **structure 7**.

In that case we fuse Type A label with type B label at point 'v' on it. To obtain $G^{(k)}$ we use type A label if $i \equiv 1\ (mod\ 2)$ and B type if $i \equiv 0\ (mod\ 2)$, $i = 1, 2, \ldots, k$. The label number distribution is on vertices $v_f(0,1) = (7+12x, 6+12x)$ on edges we have $e_f(0,1) = (7+13x, 6+13x)$, when m is of type $k = 2x+1$, $x = 0, 1, 2, \ldots$ and on vertices $v_f(0,1) = (12x, 1+12x)$ on edges we have $e_f(0,1) = (13x, 13x)$ for k is of type $k = 2x$ where $x = 1, 2, \ldots$. Thus given graph is cordial. □

3. Conclusion

We have shown that different structures on one point union of $tail(C_3, 2P_m)$ (or $ante(G, 2p_m)$ graph are cordial. We have taken $m = 2, 3, 4, 5$. It is necessary to investigate the cordiality for all m and k. We expect that all non-isomorphic structures of one point union on k-copies of $tail(C_3, P_m)$ are cordial.

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